Paper Reference(s)

6669/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced

Mock Paper

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae	Answer Booklet

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 4 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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Turn over

1. Find the exact values of x for which

$$4\tanh^2 x - 2\operatorname{sech}^2 x = 3,$$

giving your answers in the form $\pm \ln a$, where a is real.

(Total 6 marks)

2.

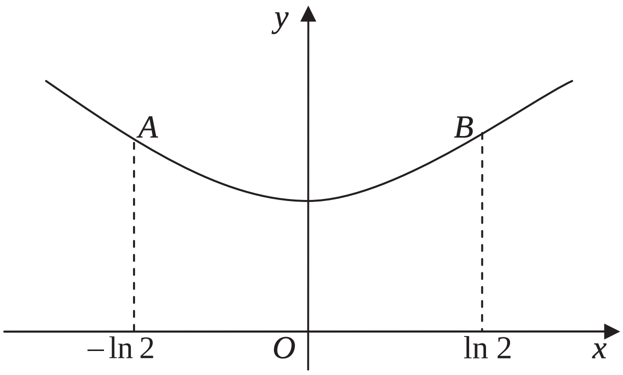

Figure 1

Figure 1 shows part of the curve with equation $y = 2\cosh\left(\frac{1}{2}x\right)$. The points A and B lie on the curve and have x-coordinates $-\ln 2$ and $\ln 2$ respectively. The arc of the curve joining A and B is rotated through 2π radians about the x-axis.

Find the exact area of the curved surface area formed.

(Total 7 marks)

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3. Using the substitution $x = \dfrac{3}{\sinh\theta}$, or otherwise, find the exact value of

$$\int_4^{3\sqrt{3}} \frac{1}{x\sqrt{(x^2+9)}}\,dx,$$

giving your answer in the form $a \ln b$, where a and b are rational numbers.

(Total 8 marks)

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4. $$y = \arctan\left(\sqrt{x}\right), \quad x > 0, \; 0 < y < \frac{\pi}{2}.$$

(a) Find the value of $\frac{dy}{dx}$ at $x = \frac{1}{4}$.

(3)

(b) Show that $2x(1+x)\frac{d^2y}{dx^2} + (1+3x)\frac{dy}{dx} = 0$.

(6)

(Total 9 marks)

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5. $$I_n = \int_0^{\frac{\pi}{2}} \sin^n x \, dx, \; n \geqslant 0$$

(a) Show that $I_n = \frac{n-1}{n} I_{n-2}$, for $n \geqslant 2$

(5)

(b) Using the result in part (a), find the exact value of

$$\int_0^{\frac{\pi}{2}} x \sin^5 x \cos x \, dx.$$

(5)

(Total 10 marks)

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6. Referred to a fixed origin O, the points P, Q and R have coordinates $(\mathbf{i} - 3\mathbf{j} + \mathbf{k})$,$(-2\mathbf{i} + \mathbf{j} - 3\mathbf{k})$ and $(3\mathbf{j} - 5\mathbf{k})$ respectively. The plane Π_1 passes through P, Q and R. Find

(a) $\overrightarrow{PQ} \times \overrightarrow{QR}$,

(4)

(b) a cartesian equation of Π_1.

(2)

The plane Π_2 has equation $\mathbf{r}.(\mathbf{i} + \mathbf{j} - \mathbf{k}) = 6$. The planes Π_1 and Π_2 intersect in the line l.

(c) Find a vector equation of l, giving your answer in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$.

(5)

(Total 11 marks)

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7. $$\mathbf{A} = \begin{pmatrix} 2 & k & 0 \\ 1 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}, \text{ where } k \text{ is a constant.}$$

Given that $\begin{pmatrix} 9 \\ 3 \\ -2 \end{pmatrix}$ is an eigenvector of **A**,

(a) show that $k = 6$,

(3)

(b) find the eigenvalues of **A**.

(4)

A transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is represented by the matrix **A**.

The point P has coordinates $(t - 2, t, 2t)$ where t is a parameter.

(c) Show that, for any value of t, the transformation T maps P onto a point on the line with equation $x - 4y - 4 = 0$

(5)

(Total 12 marks)

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8. The point P $(5\sec u, 3\tan u)$ lies on the hyperbola H with equation $\frac{x^2}{25} - \frac{y^2}{9} = 1$.

The tangent to H at P intersects the asymptote of H with equation $y = \frac{3}{5}x$ at the point R and the asymptote with equation $y = -\frac{3}{5}x$ at the point S.

(a) Use differentiation to show that an equation of the tangent to H at P is

$$3x = 5y\sin u + 15\cos u.$$

(4)

(b) Prove that P is the mid-point of RS.

(8)

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END