Edexcel Maths FP3

Past Paper Pack

2009-2013

Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	9	/	0	1	Signature	

Paper Reference(s)

6669/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced/Advanced Subsidiary

Tuesday 23 June 2009 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
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Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Orange)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

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Check that you have the correct question paper.
Answer ALL the questions. You must write your answer to each question in the space following the question.
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Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

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1. Solve the equation

$$7\text{sech}\, x - \tanh x = 5$$

Give your answers in the form $\ln a$ where a is a rational number.

(5)

Leave blank

2.

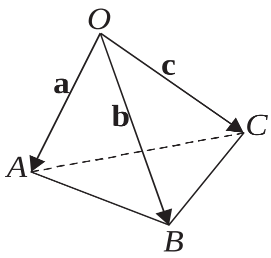

Figure 1

The points A, B and C have position vectors **a**, **b** and **c** respectively, relative to a fixed origin O, as shown in Figure 1.

It is given that

$$\mathbf{a} = \mathbf{i} + \mathbf{j}, \quad \mathbf{b} = 3\mathbf{i} - \mathbf{j} + \mathbf{k} \quad \text{and} \quad \mathbf{c} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}.$$

Calculate

(a) $\mathbf{b} \times \mathbf{c}$, **(3)**

(b) $\mathbf{a}.(\mathbf{b} \times \mathbf{c})$, **(2)**

(c) the area of triangle OBC, **(2)**

(d) the volume of the tetrahedron $OABC$. **(1)**

Leave blank

Question 2 continued

(Total 8 marks)

Q2

Leave blank

3.

$$\mathbf{M} = \begin{pmatrix} 6 & 1 & -1 \\ 0 & 7 & 0 \\ 3 & -1 & 2 \end{pmatrix}$$

(a) Show that 7 is an eigenvalue of the matrix **M** and find the other two eigenvalues of **M**.

(5)

(b) Find an eigenvector corresponding to the eigenvalue 7.

(4)

Leave blank

Question 3 continued

Leave blank

4. Given that $y = \text{arsinh}(\sqrt{x}), \quad x > 0,$

(a) find $\dfrac{dy}{dx}$, giving your answer as a simplified fraction.

(3)

(b) Hence, or otherwise, find

$$\int_{\frac{1}{4}}^{4} \frac{1}{\sqrt{[x(x+1)]}}\, dx,$$

giving your answer in the form $\ln\left(\dfrac{a+b\sqrt{5}}{2}\right)$, where a and b are integers.

(6)

Leave blank

5.

$$I_n = \int_0^5 \frac{x^n}{\sqrt{(25 - x^2)}}\,dx, \qquad n \geqslant 0$$

(a) Find an expression for $\int \frac{x}{\sqrt{(25 - x^2)}}\,dx, \quad 0 \leqslant x \leqslant 5.$

(2)

(b) Using your answer to part (a), or otherwise, show that

$$I_n = \frac{25(n-1)}{n} I_{n-2} \qquad n \geqslant 2$$

(5)

(c) Find I_4 in the form $k\pi$, where k is a fraction.

(4)

Leave blank

Question 5 continued

Leave blank

6. The hyperbola H has equation $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, where a and b are constants.

The line L has equation $y = mx + c$, where m and c are constants.

(a) Given that L and H meet, show that the x-coordinates of the points of intersection are the roots of the equation

$$(a^2m^2 - b^2)x^2 + 2a^2mcx + a^2(c^2 + b^2) = 0$$

(2)

Hence, given that L is a tangent to H,

(b) show that $a^2m^2 = b^2 + c^2$.

(2)

The hyperbola H' has equation $\dfrac{x^2}{25} - \dfrac{y^2}{16} = 1$.

(c) Find the equations of the tangents to H' which pass through the point (1, 4).

(7)

Leave blank

Question 6 continued

Leave blank

7. The lines l_1 and l_2 have equations

$$\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix} \alpha \\ -4 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}.$$

If the lines l_1 and l_2 intersect, find

(a) the value of α,

(4)

(b) an equation for the plane containing the lines l_1 and l_2, giving your answer in the form $ax + by + cz + d = 0$, where a, b, c and d are constants.

(4)

For other values of α, the lines l_1 and l_2 do not intersect and are skew lines.

Given that $\alpha = 2$,

(c) find the shortest distance between the lines l_1 and l_2.

(3)

Leave blank

Question 7 continued

Leave blank

8. A curve, which is part of an ellipse, has parametric equations

$$x = 3\cos\theta, \quad y = 5\sin\theta, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}.$$

The curve is rotated through 2π radians about the x-axis.

(a) Show that the area of the surface generated is given by the integral

$$k\pi\int_0^{\alpha}\sqrt{(16c^2+9)}\,\mathrm{d}c, \quad \text{where } c = \cos\theta,$$

and where k and α are constants to be found.

(6)

(b) Using the substitution $c = \frac{3}{4}\sinh u$, or otherwise, evaluate the integral, showing all of your working and giving the final answer to 3 significant figures.

(5)

Leave blank

Question 8 continued

Q8

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

END

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Paper Reference(s)

6669/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced/Advanced Subsidiary

Monday 28 June 2010 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

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1. The line $x = 8$ is a directrix of the ellipse with equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad a > 0,\ b > 0,$$

and the point (2, 0) is the corresponding focus.

Find the value of a and the value of b.

(5)

Leave blank

2. Use calculus to find the exact value of $\int_{-2}^{1} \frac{1}{x^2+4x+13}\,dx$.

(5)

Leave blank

3. (a) Starting from the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, prove that

$$\cosh 2x = 1 + 2\sinh^2 x$$

(3)

(b) Solve the equation

$$\cosh 2x - 3\sinh x = 15,$$

giving your answers as exact logarithms.

(5)

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4. $I_n = \int_0^a (a - x)^n \cos x \, dx, \quad a > 0, \quad n \geqslant 0$

(a) Show that, for $n \geqslant 2$,

$$I_n = na^{n-1} - n(n-1)I_{n-2}$$

(5)

(b) Hence evaluate $\int_0^{\frac{\pi}{2}} \left(\frac{\pi}{2} - x\right)^2 \cos x \, dx$.

(3)

Leave blank

Question 4 continued

Leave blank

5. Given that $y = (\text{arcosh } 3x)^2$, where $3x > 1$, show that

(a) $(9x^2 - 1)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = 36y$,

(5)

(b) $(9x^2 - 1)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 9x\frac{\mathrm{d}y}{\mathrm{d}x} = 18$.

(4)

Leave blank

Question 5 continued

Leave blank

6. $$\mathbf{M} = \begin{pmatrix} 1 & 0 & 3 \\ 0 & -2 & 1 \\ k & 0 & 1 \end{pmatrix}, \text{ where } k \text{ is a constant.}$$

Given that $\begin{pmatrix} 6 \\ 1 \\ 6 \end{pmatrix}$ is an eigenvector of **M**,

(a) find the eigenvalue of **M** corresponding to $\begin{pmatrix} 6 \\ 1 \\ 6 \end{pmatrix}$, **(2)**

(b) show that $k = 3$, **(2)**

(c) show that **M** has exactly two eigenvalues. **(4)**

A transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is represented by **M**.

The transformation T maps the line l_1, with cartesian equations $\dfrac{x-2}{1} = \dfrac{y}{-3} = \dfrac{z+1}{4}$, onto the line l_2.

(d) Taking $k = 3$, find cartesian equations of l_2. **(5)**

Leave blank

Question 6 continued

Leave blank

7. The plane Π has vector equation

$$\mathbf{r} = 3\mathbf{i} + \mathbf{k} + \lambda(-4\mathbf{i} + \mathbf{j}) + \mu(6\mathbf{i} - 2\mathbf{j} + \mathbf{k})$$

(a) Find an equation of Π in the form $\mathbf{r.n} = p$, where $\mathbf{n}$ is a vector perpendicular to Π and p is a constant.

(5)

The point P has coordinates $(6, 13, 5)$. The line l passes through P and is perpendicular to Π. The line l intersects Π at the point N.

(b) Show that the coordinates of N are $(3, 1, -1)$.

(4)

The point R lies on Π and has coordinates $(1, 0, 2)$.

(c) Find the perpendicular distance from N to the line PR. Give your answer to 3 significant figures.

(5)

Leave blank

Question 7 continued

Leave blank

8. The hyperbola H has equation $\frac{x^2}{16} - \frac{y^2}{4} = 1$.

The line l_1 is the tangent to H at the point $P(4\sec t, 2\tan t)$.

(a) Use calculus to show that an equation of l_1 is

$$2y\sin t = x - 4\cos t$$

(5)

The line l_2 passes through the origin and is perpendicular to l_1.

The lines l_1 and l_2 intersect at the point Q.

(b) Show that, as t varies, an equation of the locus of Q is

$$\left(x^2 + y^2\right)^2 = 16x^2 - 4y^2$$

(8)

Leave blank

Question 8 continued

Q8

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

END

Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 9 / 0 1	Signature	

Paper Reference(s)

6669/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced/Advanced Subsidiary

Friday 24 June 2011 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only		

Team Leader's use only		

Question Number	Leave Blank
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Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

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1. The curve C has equation $y = 2x^3$, $0 \leqslant x \leqslant 2$.

The curve C is rotated through 2π radians about the x-axis.

Using calculus, find the area of the surface generated, giving your answer to 3 significant figures.

(5)

Leave blank

2. (a) Given that $y = x \arcsin x$, $0 \leqslant x \leqslant 1$, find

(i) an expression for $\dfrac{dy}{dx}$,

(ii) the exact value of $\dfrac{dy}{dx}$ when $x = \dfrac{1}{2}$.

(3)

(b) Given that $y = \arctan(3e^{2x})$, show that

$$\frac{dy}{dx} = \frac{3}{5\cosh 2x + 4\sinh 2x}$$

(5)

Leave blank

3. Show that

(a) $\displaystyle\int_5^8 \frac{1}{x^2 - 10x + 34}\,\mathrm{d}x = k\pi$, giving the value of the fraction k,

(5)

(b) $\displaystyle\int_5^8 \frac{1}{\sqrt{(x^2 - 10x + 34)}}\,\mathrm{d}x = \ln(A + \sqrt{n})$, giving the values of the integers A and n.

(4)

Leave blank

4.

$$I_n = \int_1^{e} x^2 (\ln x)^n \, dx, \qquad n \geqslant 0$$

(a) Prove that, for $n \geqslant 1$,

$$I_n = \frac{e^3}{3} - \frac{n}{3} I_{n-1}$$

(4)

(b) Find the exact value of I_3.

(4)

Leave blank

Question 4 continued

Leave blank

5. The curve C_1 has equation $y = 3\sinh 2x$, and the curve C_2 has equation $y = 13 - 3e^{2x}$.

(a) Sketch the graph of the curves C_1 and C_2 on one set of axes, giving the equation of any asymptote and the coordinates of points where the curves cross the axes.

(4)

(b) Solve the equation $3\sinh 2x = 13 - 3e^{2x}$, giving your answer in the form $\frac{1}{2}\ln k$, where k is an integer.

(5)

Leave blank

Question 5 continued

Leave blank

6. The plane P has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix}$$

(a) Find a vector perpendicular to the plane P.

(2)

The line l passes through the point A (1, 3, 3) and meets P at (3, 1, 2).

The acute angle between the plane P and the line l is α.

(b) Find α to the nearest degree.

(4)

(c) Find the perpendicular distance from A to the plane P.

(4)

Leave blank

Question 6 continued

Leave blank

7. The matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} k & -1 & 1 \\ 1 & 0 & -1 \\ 3 & -2 & 1 \end{pmatrix}, \quad k \neq 1$$

(a) Show that $\det \mathbf{M} = 2 - 2k$.

(2)

(b) Find $\mathbf{M}^{-1}$, in terms of k.

(5)

The straight line l_1 is mapped onto the straight line l_2 by the transformation represented by the matrix $\begin{pmatrix} 2 & -1 & 1 \\ 1 & 0 & -1 \\ 3 & -2 & 1 \end{pmatrix}$.

The equation of l_2 is $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = 0$, where $\mathbf{a} = 4\mathbf{i} + \mathbf{j} + 7\mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} + \mathbf{j} + 3\mathbf{k}$.

(c) Find a vector equation for the line l_1.

(5)

Leave blank

Question 7 continued

Leave blank

8. The hyperbola H has equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

(a) Use calculus to show that the equation of the tangent to H at the point $(a\cosh\theta,\ b\sinh\theta)$ may be written in the form

$$xb\cosh\theta - ya\sinh\theta = ab$$

(4)

The line l_1 is the tangent to H at the point $(a\cosh\theta,\ b\sinh\theta)$, $\theta \neq 0$.
Given that l_1 meets the x-axis at the point P,

(b) find, in terms of a and θ, the coordinates of P.

(2)

The line l_2 is the tangent to H at the point $(a\ ,\ 0)$.
Given that l_1 and l_2 meet at the point Q,

(c) find, in terms of a, b and θ, the coordinates of Q.

(2)

(d) Show that, as θ varies, the locus of the mid-point of PQ has equation

$$x(4y^2 + b^2) = ab^2$$

(6)

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Question 8 continued

Q8

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END

Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 9 / 0 1	Signature	

Paper Reference(s)

6669/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced/Advanced Subsidiary

Monday 25 June 2012 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
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1. The hyperbola H has equation

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

Find

(a) the coordinates of the foci of H,

(3)

(b) the equations of the directrices of H.

(2)

Leave blank

2.

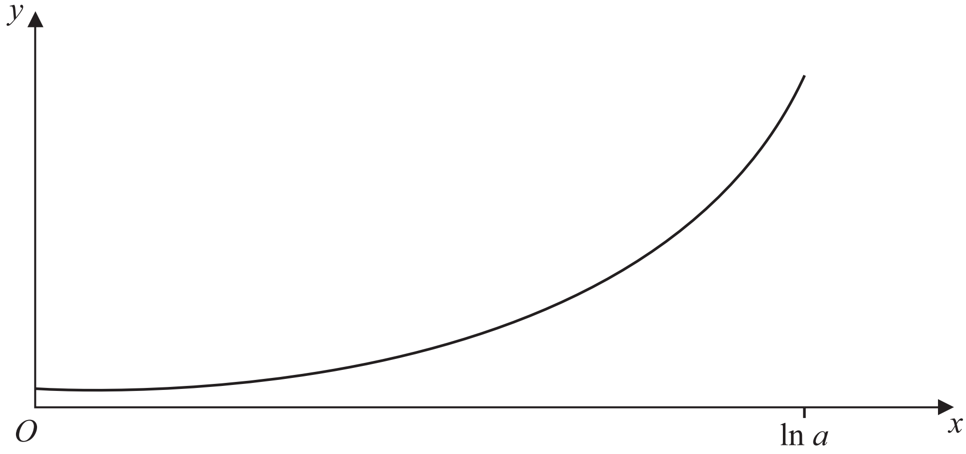

Figure 1

The curve C, shown in Figure 1, has equation

$$y = \frac{1}{3}\cosh 3x, \qquad 0 \leqslant x \leqslant \ln a$$

where a is a constant and $a > 1$

Using calculus, show that the length of curve C is

$$k\left(a^3 - \frac{1}{a^3}\right)$$

and state the value of the constant k.

(6)

Leave blank

3. The position vectors of the points A, B and C relative to an origin O are $\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$, $7\mathbf{i} - 3\mathbf{k}$ and $4\mathbf{i} + 4\mathbf{j}$ respectively.

Find

(a) $\overrightarrow{AC} \times \overrightarrow{BC}$,

(4)

(b) the area of triangle ABC,

(2)

(c) an equation of the plane ABC in the form $\mathbf{r.n} = p$

(2)

Leave blank

Question 3 continued

Leave blank

4. $$I_n = \int_0^{\frac{\pi}{4}} x^n \sin 2x \, dx, \qquad n \geqslant 0$$

(a) Prove that, for $n \geqslant 2$,

$$I_n = \frac{1}{4}n\left(\frac{\pi}{4}\right)^{n-1} - \frac{1}{4}n(n-1)I_{n-2}$$

(5)

(b) Find the exact value of I_2

(4)

(c) Show that $I_4 = \frac{1}{64}(\pi^3 - 24\pi + 48)$

(2)

Leave blank

Question 4 continued

Leave blank

5. (a) Differentiate $x \operatorname{arsinh} 2x$ with respect to x.

(3)

(b) Hence, or otherwise, find the exact value of

$$\int_0^{\sqrt{2}} \operatorname{arsinh} 2x \, \mathrm{d}x$$

giving your answer in the form $A \ln B + C$, where A, B and C are real.

(7)

Leave blank

Question 5 continued

Leave blank

6. The ellipse E has equation

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

The line l_1 is a tangent to E at the point $P\ (a\cos\theta,\ b\sin\theta)$.

(a) Using calculus, show that an equation for l_1 is

$$\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$$

(4)

The circle C has equation

$$x^2+y^2=a^2$$

The line l_2 is a tangent to C at the point $Q\ (a\cos\theta,\ a\sin\theta)$.

(b) Find an equation for the line l_2.

(2)

Given that l_1 and l_2 meet at the point R,

(c) find, in terms of a, b and θ, the coordinates of R.

(3)

(d) Find the locus of R, as θ varies.

(2)

Leave blank

Question 6 continued

Leave blank

7. $$\mathrm{f}(x) = 5\cosh x - 4\sinh x, \qquad x \in \mathbb{R}$$

(a) Show that $\mathrm{f}(x) = \frac{1}{2}(\mathrm{e}^x + 9\mathrm{e}^{-x})$

(2)

Hence

(b) solve $\mathrm{f}(x) = 5$

(4)

(c) show that $\int_{\frac{1}{2}\ln 3}^{\ln 3} \frac{1}{5\cosh x - 4\sinh x}\,\mathrm{d}x = \frac{\pi}{18}$

(5)

Leave blank

Question 7 continued

Leave blank

8. The matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ -1 & 0 & 4 \end{pmatrix}$$

(a) Show that 4 is an eigenvalue of **M**, and find the other two eigenvalues.

(5)

(b) For the eigenvalue 4, find a corresponding eigenvector.

(3)

The straight line l_1 is mapped onto the straight line l_2 by the transformation represented by the matrix **M**.

The equation of l_1 is $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = 0$, where $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$.

(c) Find a vector equation for the line l_2.

(5)

Leave blank

Question 8 continued

Q8

(Total 13 marks)

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Monday 24 June 2013 – Afternoon

Time: 1 hour 30 minutes

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You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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PEARSON

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1. The hyperbola H has foci at (5, 0) and (–5, 0) and directrices with equations

$x = \frac{9}{5}$ and $x = -\frac{9}{5}$.

Find a cartesian equation for H.

(7)

Leave blank

2. Two skew lines l_1 and l_2 have equations

$$l_1: \mathbf{r} = (\mathbf{i} - \mathbf{j} + \mathbf{k}) + \lambda(4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$$

$$l_2: \mathbf{r} = (3\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}) + \mu(-4\mathbf{i} + 6\mathbf{j} + \mathbf{k})$$

respectively, where λ and μ are real parameters.

(a) Find a vector in the direction of the common perpendicular to l_1 and l_2

(2)

(b) Find the shortest distance between these two lines.

(5)

Leave blank

Question 2 continued

Leave blank

3. The point P lies on the ellipse E with equation

$$\frac{x^2}{36} + \frac{y^2}{9} = 1$$

N is the foot of the perpendicular from point P to the line $x = 8$

M is the midpoint of PN.

(a) Sketch the graph of the ellipse E, showing also the line $x = 8$ and a possible position for the line PN.

(1)

(b) Find an equation of the locus of M as P moves around the ellipse.

(4)

(c) Show that this locus is a circle and state its centre and radius.

(3)

Leave blank

Question 3 continued

Leave blank

4. The plane Π_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + s\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix},$$

where s and t are real parameters.

The plane Π_1 is transformed to the plane Π_2 by the transformation represented by the matrix $\mathbf{T}$, where

$$\mathbf{T} = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 2 & -1 \\ 0 & 1 & 2 \end{pmatrix}$$

Find an equation of the plane Π_2 in the form $\mathbf{r.n} = p$

(9)

Leave blank

Question 4 continued

P 4 2 9 5 6 A 0 1 3 3 2

Turn over

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5.

$$I_n = \int_1^5 x^n (2x - 1)^{-\frac{1}{2}}\,dx, \qquad n \geqslant 0$$

(a) Prove that, for $n \geqslant 1$,

$$(2n + 1)I_n = nI_{n-1} + 3 \times 5^n - 1$$

(5)

(b) Using the reduction formula given in part (a), find the exact value of I_2

(5)

Leave blank

Question 5 continued

Leave blank

6. It is given that $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ is an eigenvector of the matrix **A**, where

$$\mathbf{A} = \begin{pmatrix} 4 & 2 & 3 \\ 2 & b & 0 \\ a & 1 & 8 \end{pmatrix}$$

and a and b are constants.

(a) Find the eigenvalue of **A** corresponding to the eigenvector $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$. **(3)**

(b) Find the values of a and b. **(3)**

(c) Find the other eigenvalues of **A**. **(5)**

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Question 6 continued

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7.

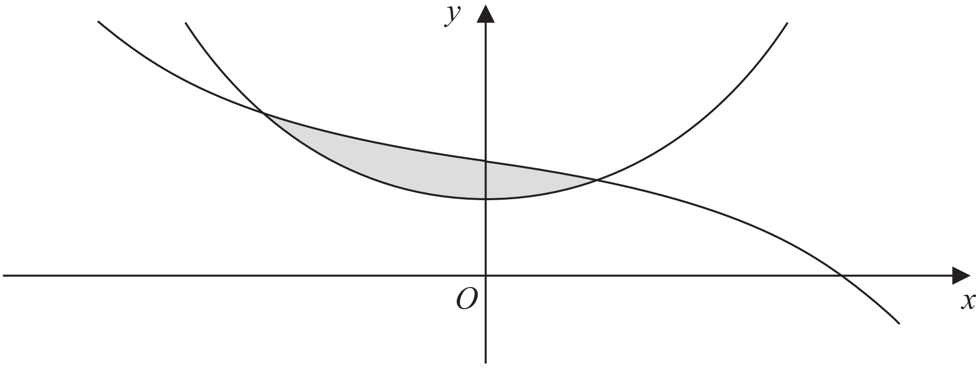

Figure 1

The curves shown in Figure 1 have equations

$$y = 6\cosh x \quad \text{and} \quad y = 9 - 2\sinh x$$

(a) Using the definitions of $\sinh x$ and $\cosh x$ in terms of e^x, find exact values for the x-coordinates of the two points where the curves intersect.

(6)

The finite region between the two curves is shown shaded in Figure 1.

(b) Using calculus, find the area of the shaded region, giving your answer in the form $a \ln b + c$, where a, b and c are integers.

(6)

Leave blank

Question 7 continued

Leave blank

8.

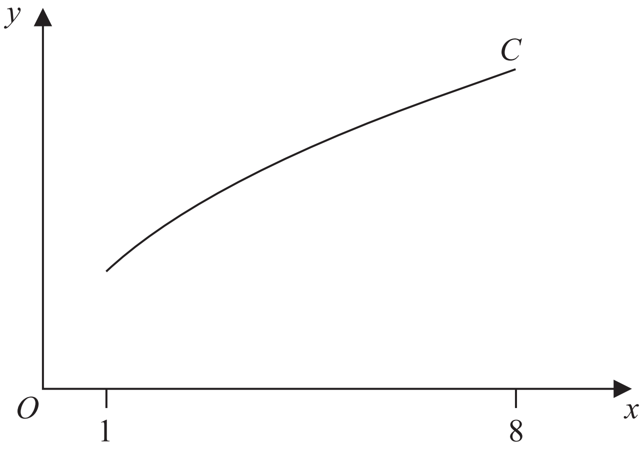

Figure 2

The curve C, shown in Figure 2, has equation

$$y = 2x^{\frac{1}{2}}, \qquad 1 \leqslant x \leqslant 8$$

(a) Show that the length s of curve C is given by the equation

$$s = \int_1^8 \sqrt{\left(1 + \frac{1}{x}\right)}\,dx$$

(2)

(b) Using the substitution $x = \sinh^2 u$, or otherwise, find an exact value for s.

Give your answer in the form $a\sqrt{2} + \ln(b + c\sqrt{2})$ where a, b and c are integers.

(9)

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Question 8 continued

Q8

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

END

Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 9 / 0 1	Signature	

Paper Reference(s)

6669/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced/Advanced Subsidiary

Monday 24 June 2013 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
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Total	

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

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P 4 3 1 4 3 A 0 1 2 8

Turn over

PEARSON

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1. A hyperbola H has equation

$$\frac{x^2}{a^2} - \frac{y^2}{25} = 1, \qquad \text{where } a \text{ is a positive constant.}$$

The foci of H are at the points with coordinates (13, 0) and (–13, 0).

Find

(a) the value of the constant a,

(3)

(b) the equations of the directrices of H.

(3)

2. (a) Find

$$\int \frac{1}{\sqrt{(4x^2 + 9)}}\,\mathrm{d}x$$

(2)

(b) Use your answer to part (a) to find the exact value of

$$\int_{-3}^{3} \frac{1}{\sqrt{(4x^2 + 9)}}\,\mathrm{d}x$$

giving your answer in the form $k\ln(a + b\sqrt{5})$, where a and b are integers and k is a constant.

(3)

Leave blank

3. The curve with parametric equations

$$x = \cosh 2\theta, \quad y = 4\sinh\theta, \quad 0 \leqslant \theta \leqslant 1$$

is rotated through 2π radians about the x-axis.

Show that the area of the surface generated is $\lambda(\cosh^3 \alpha - 1)$, where $\alpha = 1$ and λ is a constant to be found.

(7)

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4.

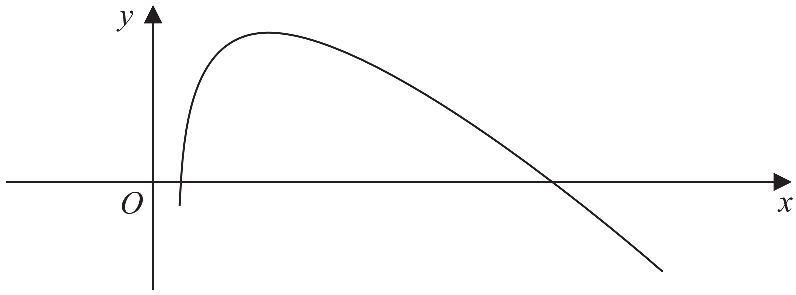

Figure 1

Figure 1 shows part of the curve with equation

$$y = 40 \text{ arcosh } x - 9x, \qquad x \geqslant 1$$

Use calculus to find the exact coordinates of the turning point of the curve, giving your answer in the form $\left(\dfrac{p}{q}, r\ln 3 + s\right)$, where p, q, r and s are integers.

(7)

Leave blank

Question 4 continued

Q4

(Total 7 marks)

Leave blank

5. The matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & a \\ 2 & b & c \\ -1 & 0 & 1 \end{pmatrix}, \text{ where } a, b \text{ and } c \text{ are constants.}$$

(a) Given that **j** + **k** and **i** – **k** are two of the eigenvectors of **M**,

find

(i) the values of a, b and c,

(ii) the eigenvalues which correspond to the two given eigenvectors.

(8)

(b) The matrix **P** is given by

$$\mathbf{P} = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & d \\ -1 & 0 & 1 \end{pmatrix}, \text{ where } d \text{ is constant, } d \neq -1$$

Find

(i) the determinant of **P** in terms of d,

(ii) the matrix $\mathbf{P}^{-1}$ in terms of d.

(5)

Leave blank

Question 5 continued

Leave blank

6. Given that

$$I_n = \int_0^4 x^n \sqrt{(16 - x^2)}\,dx, \quad n \geqslant 0,$$

(a) prove that, for $n \geqslant 2$,

$$(n+2)I_n = 16(n-1)I_{n-2}$$

(6)

(b) Hence, showing each step of your working, find the exact value of I_5

(5)

Leave blank

Question 6 continued

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7. The ellipse E has equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \qquad a > b > 0$$

The line l is a normal to E at a point $P(a\cos\theta, b\sin\theta)$, $\quad 0 < \theta < \frac{\pi}{2}$

(a) Using calculus, show that an equation for l is

$$ax\sin\theta - by\cos\theta = (a^2 - b^2)\sin\theta\cos\theta$$

(5)

The line l meets the x-axis at A and the y-axis at B.

(b) Show that the area of the triangle OAB, where O is the origin, may be written as $k\sin 2\theta$, giving the value of the constant k in terms of a and b.

(4)

(c) Find, in terms of a and b, the exact coordinates of the point P, for which the area of the triangle OAB is a maximum.

(3)

Leave blank

Question 7 continued

Leave blank

8. The plane Π_1 has vector equation

$$\mathbf{r}.(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) = 5$$

(a) Find the perpendicular distance from the point (6, 2, 12) to the plane Π_1

(3)

The plane Π_2 has vector equation

$$\mathbf{r} = \lambda(2\mathbf{i} + \mathbf{j} + 5\mathbf{k}) + \mu(\mathbf{i} - \mathbf{j} - 2\mathbf{k}), \text{ where } \lambda \text{ and } \mu \text{ are scalar parameters.}$$

(b) Find the acute angle between Π_1 and Π_2 giving your answer to the nearest degree.

(5)

(c) Find an equation of the line of intersection of the two planes in the form $\mathbf{r} \times \mathbf{a} = \mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors.

(6)

Leave blank

Question 8 continued

Q8

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END

Further Pure Mathematics FP3

Candidates sitting FP3 may also require those formulae listed under Further Pure Mathematics FP1, and Core Mathematics C1–C4.

Vectors

The resolved part of **a** in the direction of **b** is $\dfrac{\mathbf{a.b}}{|\mathbf{b}|}$

The point dividing *AB* in the ratio $\lambda : \mu$ is $\dfrac{\mu\mathbf{a}+\lambda\mathbf{b}}{\lambda+\mu}$

Vector product: $\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$

$$\mathbf{a.}(\mathbf{b}\times\mathbf{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \mathbf{b.}(\mathbf{c}\times\mathbf{a}) = \mathbf{c.}(\mathbf{a}\times\mathbf{b})$$

If *A* is the point with position vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and the direction vector **b** is given by $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, then the straight line through *A* with direction vector **b** has cartesian equation

$$\frac{x-a_1}{b_1} = \frac{y-a_2}{b_2} = \frac{z-a_3}{b_3}\ (=\lambda)$$

The plane through *A* with normal vector $\mathbf{n} = n_1\mathbf{i} + n_2\mathbf{j} + n_3\mathbf{k}$ has cartesian equation

$$n_1x + n_2y + n_3z + d = 0 \text{ where } d = -\mathbf{a.n}$$

The plane through non-collinear points *A*, *B* and *C* has vector equation

$$\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b}-\mathbf{a}) + \mu(\mathbf{c}-\mathbf{a}) = (1-\lambda-\mu)\mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$$

The plane through the point with position vector **a** and parallel to **b** and **c** has equation

$$\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$$

The perpendicular distance of (α, β, γ) from $n_1x + n_2y + n_3z + d = 0$ is $\dfrac{|n_1\alpha + n_2\beta + n_3\gamma + d|}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$.

Hyperbolic functions

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh 2x = 2\sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\text{arcosh } x = \ln\left\{x + \sqrt{x^2 - 1}\right\} \qquad (x \geq 1)$$

$$\text{arsinh } x = \ln\left\{x + \sqrt{x^2 + 1}\right\}$$

$$\text{artanh } x = \tfrac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \qquad (|x| < 1)$$

Conics

	Ellipse	**Parabola**	**Hyperbola**	**Rectangular Hyperbola**
Standard Form	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$y^2 = 4ax$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$xy = c^2$
Parametric Form	$(a\cos\theta, b\sin\theta)$	$(at^2, 2at)$	$(a\sec\theta, b\tan\theta)$ $(\pm a\cosh\theta, b\sinh\theta)$	$\left(ct, \frac{c}{t}\right)$
Eccentricity	$e < 1$ $b^2 = a^2(1 - e^2)$	$e = 1$	$e > 1$ $b^2 = a^2(e^2 - 1)$	$e = \sqrt{2}$
Foci	$(\pm ae, 0)$	$(a, 0)$	$(\pm ae, 0)$	$(\pm\sqrt{2}c, \pm\sqrt{2}c)$
Directrices	$x = \pm\frac{a}{e}$	$x = -a$	$x = \pm\frac{a}{e}$	$x + y = \pm\sqrt{2}c$
Asymptotes	none	none	$\frac{x}{a} = \pm\frac{y}{b}$	$x = 0, y = 0$

Differentiation

$\mathbf{f}(x)$	$\mathbf{f}'(x)$
$\arcsin x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\dfrac{1}{1+x^2}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\operatorname{arsinh} x$	$\dfrac{1}{\sqrt{1+x^2}}$
$\operatorname{arcosh} x$	$\dfrac{1}{\sqrt{x^2-1}}$
$\operatorname{artanh} x$	$\dfrac{1}{1-x^2}$

***Integration* (+ *constant*; $a > 0$ *where relevant*)**

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\ \mathbf{d}x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln \cosh x$
$\dfrac{1}{\sqrt{a^2-x^2}}$	$\arcsin\left(\dfrac{x}{a}\right) \quad (\lvert x\rvert < a)$
$\dfrac{1}{a^2+x^2}$	$\dfrac{1}{a}\arctan\left(\dfrac{x}{a}\right)$
$\dfrac{1}{\sqrt{x^2-a^2}}$	$\operatorname{arcosh}\left(\dfrac{x}{a}\right), \quad \ln\left\{x+\sqrt{x^2-a^2}\right\} \quad (x > a)$
$\dfrac{1}{\sqrt{a^2+x^2}}$	$\operatorname{arsinh}\left(\dfrac{x}{a}\right), \quad \ln\left\{x+\sqrt{x^2+a^2}\right\}$
$\dfrac{1}{a^2-x^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{a+x}{a-x}\right\rvert = \dfrac{1}{a}\operatorname{artanh}\left(\dfrac{x}{a}\right) \quad (\lvert x\rvert < a)$
$\dfrac{1}{x^2-a^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{x-a}{x+a}\right\rvert$

Arc length

$$s = \int \sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x \qquad \text{(cartesian coordinates)}$$

$$s = \int \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t \qquad \text{(parametric form)}$$

Surface area of revolution

$$S_x = 2\pi\int y \ \ \mathrm{d}s \ = 2\pi\int y\sqrt{\left(1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)}\mathrm{d}x$$

$$= 2\pi\int y\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t$$

Further Pure Mathematics FP1

Candidates sitting FP1 may also require those formulae listed under Core Mathematics C1 and C2.

Summations

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

Numerical solution of equations

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Conics

	Parabola	**Rectangular Hyperbola**
Standard Form	$y^2 = 4ax$	$xy = c^2$
Parametric Form	$(at^2, 2at)$	$\left(ct, \dfrac{c}{t}\right)$
Foci	$(a, 0)$	Not required
Directrices	$x = -a$	Not required

Matrix transformations

Anticlockwise rotation through θ about O: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Reflection in the line $y = (\tan\theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

In FP1, θ will be a multiple of 45°.

Core Mathematics C4

Candidates sitting C4 may also require those formulae listed under Core Mathematics C1, C2 and C3.

Integration (+ *constant*)

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\,\mathrm{d}x$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\tan x$	$\ln\lvert\sec x\rvert$
$\cot x$	$\ln\lvert\sin x\rvert$
$\operatorname{cosec} x$	$-\ln\lvert\operatorname{cosec} x+\cot x\rvert, \quad \ln\lvert\tan(\tfrac{1}{2}x)\rvert$
$\sec x$	$\ln\lvert\sec x+\tan x\rvert, \quad \ln\lvert\tan(\tfrac{1}{2}x+\tfrac{1}{4}\pi)\rvert$

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

Core Mathematics C3

Candidates sitting C3 may also require those formulae listed under Core Mathematics C1 and C2.

Logarithms and exponentials

$$e^{x\ln a} = a^x$$

Trigonometric identities

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

Differentiation

f(x)	**f′(x)**
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x\tan x$
$\cot x$	$-\text{cosec}^2\, x$
$\text{cosec}\, x$	$-\text{cosec}\, x\cot x$
$\dfrac{f(x)}{g(x)}$	$\dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$a^2 = b^2 + c^2 - 2bc \cos A$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^n\mathrm{C}_r = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1 \times 2 \times \ldots \times r}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$u_n = ar^{n-1}$

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Numerical integration

The trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + ... + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

Core Mathematics C1

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic series

$u_n = a + (n - 1)d$

$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$