PMT

Paper Reference(s)
6669/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced

Tuesday 23 June 2009 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP3), the paper reference (6669), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

M35145A

1. Solve the equation

$$7 \operatorname{sech} x - \tanh x = 5$$

Give your answers in the form ln *a*, where *a* is a rational number.

(5)

2.

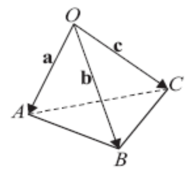

Figure 1

The points *A*, *B* and *C* have position vectors **a**, **b** and **c** respectively, relative to a fixed origin *O*, as shown in Figure 1.

It is given that

$$\mathbf{a} = \mathbf{i} + \mathbf{j}, \quad \mathbf{b} = 3\mathbf{i} - \mathbf{j} + \mathbf{k} \quad \text{and} \quad \mathbf{c} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}.$$

Calculate

(*a*) $\mathbf{b} \times \mathbf{c}$,

(3)

(*b*) $\mathbf{a} . (\mathbf{b} \times \mathbf{c})$,

(2)

(*c*) the area of triangle *OBC*,

(2)

(*d*) the volume of the tetrahedron *OABC*.

(1)

PMT

3. $$\mathbf{M} = \begin{pmatrix} 6 & 1 & -1 \\ 0 & 7 & 0 \\ 3 & -1 & 2 \end{pmatrix}$$

(*a*) Show that 7 is an eigenvalue of the matrix **M** and find the other two eigenvalues of **M**. **(5)**

(*b*) Find an eigenvector corresponding to the eigenvalue 7. **(4)**

---

4. Given that $y = \text{arsinh}(\sqrt{x})$, $x > 0$,

(*a*) find $\frac{\mathrm{d}y}{\mathrm{d}x}$, giving your answer as a simplified fraction. **(3)**

(*b*) Hence, or otherwise, find

$$\int_{\frac{1}{4}}^{4} \frac{1}{\sqrt{[x(x+1)]}}\,\mathrm{d}x,$$

giving your answer in the form $\ln\left(\frac{a+b\sqrt{5}}{2}\right)$, where a and b are integers. **(6)**

---

5. $$I_n = \int_0^5 \frac{x^n}{\sqrt{(25-x^2)}}\,\mathrm{d}x, \quad n \geq 0.$$

(*a*) Find an expression for $\int \frac{x}{\sqrt{(25-x^2)}}\,\mathrm{d}x, \quad 0 \leq x \leq 5.$ **(2)**

(*b*) Using your answer to part (*a*), or otherwise, show that

$$I_n = \frac{25(n-1)}{n} I_{n-2}, \quad n \geq 2.$$

(5)

(*c*) Find I_4 in the form $k\pi$, where k is a fraction. **(4)**

---

6. The hyperbola H has equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where a and b are constants.

The line L has equation $y = mx + c$, where m and c are constants.

(*a*) Given that L and H meet, show that the x-coordinates of the points of intersection are the roots of the equation

$$(a^2m^2 - b^2)x^2 + 2a^2mcx + a^2(c^2 + b^2) = 0.$$

(2)

Hence, given that L is a tangent to H,

(*b*) show that $a^2m^2 = b^2 + c^2$. **(2)**

The hyperbola H' has equation $\frac{x^2}{25} - \frac{y^2}{16} = 1$.

(*c*) Find the equations of the tangents to H' which pass through the point (1, 4). **(7)**

---

7. The lines l_1 and l_2 have equations

$$\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} \alpha \\ -4 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}.$$

If the lines l_1 and l_2 intersect, find

(*a*) the value of α, **(4)**

(*b*) an equation for the plane containing the lines l_1 and l_2, giving your answer in the form $ax + by + cz + d = 0$, where a, b, c and d are constants. **(4)**

For other values of α, the lines l_1 and l_2 do not intersect and are skew lines.

Given that $\alpha = 2$,

(*c*) find the shortest distance between the lines l_1 and l_2. **(3)**

PMT

8. A curve, which is part of an ellipse, has parametric equations

$$x = 3\cos\theta, \quad y = 5\sin\theta, \quad 0 \leq \theta \leq \frac{\pi}{2}.$$

The curve is rotated through 2π radians about the x-axis.

(a) Show that the area of the surface generated is given by the integral

$$k\pi \int_0^{\alpha} \sqrt{(16c^2 + 9)} \ \mathrm{d}c, \text{ where } c = \cos\theta,$$

and where k and α are constants to be found.

(6)

(b) Using the substitution $c = \frac{3}{4}\sinh u$, or otherwise, evaluate the integral, showing all of your working and giving the final answer to 3 significant figures.

(5)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6669/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced

Thursday 24 June 2010 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

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N35389RA

PMT

1. The line $x = 8$ is a directrix of the ellipse with equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad a > 0, \quad b > 0,$$

and the point (2, 0) is the corresponding focus.

Find the value of a and the value of b.

(5)

---

2. Use calculus to find the exact value of $\int_{-2}^{1} \frac{1}{x^2 + 4x + 13}\, dx$.

(5)

---

3. (*a*) Starting from the definitions of sinh x and cosh x in terms of exponentials, prove that

$$\cosh 2x = 1 + 2\sinh^2 x$$

(3)

(*b*) Solve the equation

$$\cosh 2x - 3\sinh x = 15,$$

giving your answers as exact logarithms.

(1)

4.

$$I_n = \int_0^a (a - x)^n \cos x \; dx, \quad a \geq 0, \quad n \geq 0.$$

(*a*) Show that, for $n \geq 2$,

$$I_n = na^{n-1} - n(n-1)I_{n-2}$$

(5)

(*b*) Hence evaluate $\int_0^{\frac{\pi}{2}} \left(\frac{\pi}{2} - x\right)^2 \cos x \, dx$.

(3)

---

5. Given that $y = (\text{arcosh } 3x)^2$, where $3x > 1$, show that

(*a*) $(9x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 36y$,

(5)

(*b*) $(9x^2 - 1)\frac{d^2y}{dx^2} + 9x\frac{dy}{dx} = 18$.

(4)

6.

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 3 \\ 0 & -2 & 1 \\ k & 0 & 1 \end{pmatrix}, \text{ where } k \text{ is a constant.}$$

Given that $\begin{pmatrix} 6 \\ 1 \\ 6 \end{pmatrix}$ is an eigenvector of **M**,

(*a*) find the eigenvalue of **M** corresponding to $\begin{pmatrix} 6 \\ 1 \\ 6 \end{pmatrix}$,

(2)

(*b*) show that $k = 3$,

(2)

(*c*) show that **M** has exactly two eigenvalues.

(4)

A transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is represented by **M**.

The transformation T maps the line l_1, with cartesian equations $\frac{x-2}{1} = \frac{y}{-3} = \frac{z+1}{4}$, onto the line l_2.

(*d*) Taking $k = 3$, find cartesian equations of l_2.

(5)

PMT

7. The plane Π has vector equation

$$\mathbf{r} = 3\mathbf{i} + \mathbf{k} + \lambda(-4\mathbf{i} + \mathbf{j}) + \mu(6\mathbf{i} - 2\mathbf{j} + \mathbf{k})$$

(*a*) Find an equation of Π in the form $\mathbf{r.n} = p$, where $\mathbf{n}$ is a vector perpendicular to Π and p is a constant.
(5)

The point P has coordinates (6, 13, 5). The line l passes through P and is perpendicular to Π. The line l intersects Π at the point N.

(*b*) Show that the coordinates of N are (3, 1, –1).
(4)

The point R lies on Π and has coordinates (1, 0, 2).

(*c*) Find the perpendicular distance from N to the line PR. Give your answer to 3 significant figures.
(5)

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8. The hyperbola H has equation $\frac{x^2}{16} - \frac{y^2}{4} = 1$.

The line l_1 is the tangent to H at the point P ($4 \sec t$, $2 \tan t$).

(*a*) Use calculus to show that an equation of l_1 is

$$2y \sin t = x - 4 \cos t$$

(5)

The line l_2 passes through the origin and is perpendicular to l_1.

The lines l_1 and l_2 intersect at the point Q.

(*b*) Show that, as t varies, an equation of the locus of Q is

$$(x^2 + y^2)^2 = 16x^2 - 4y^2$$

(2)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6669/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced Level

Friday 24 June 2011 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

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Instructions to Candidates

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Advice to Candidates

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P35414A

PMT

1. The curve C has equation $y = 2x^3, \quad 0 \leq x \leq 2$.

The curve C is rotated through 2π radians about the x-axis.

Using calculus, find the area of the surface generated, giving your answer to 3 significant figures.
(5)

---

2. (*a*) Given that $y = x \arcsin x, \; 0 \leq x \leq 1$, find

(i) an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$,

(ii) the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = \frac{1}{2}$.
(3)

(*b*) Given that $y = \arctan (3\mathrm{e}^{2x})$, show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{5\cosh 2x + 4\sinh 2x}.$$
(5)

---

3. Show that

(*a*) $\int_5^8 \frac{1}{x^2 - 10x + 34} \, \mathrm{d}x = k\pi$, giving the value of the fraction k,
(5)

(*b*) $\int_5^8 \frac{1}{\sqrt{(x^2 - 10x + 34)}} \, \mathrm{d}x = \ln (A + \sqrt{n})$, giving the values of the integers A and n.
(4)

4.

$$I_n = \int_1^{\mathrm{e}} x^2 (\ln x)^n \, \mathrm{d}x, \quad n \geq 0.$$

(*a*) Prove that, for $n \geq 1$,

$$I_n = \frac{\mathrm{e}^3}{3} - \frac{n}{3} I_{n-1}.$$
(4)

(*b*) Find the exact value of I_3.
(4)

---

5. The curve C_1 has equation $y = 3 \sinh 2x$, and the curve C_2 has equation $y = 13 - 3\mathrm{e}^{2x}$.

(*a*) Sketch the graph of the curves C_1 and C_2 on one set of axes, giving the equation of any asymptote and the coordinates of points where the curves cross the axes.
(4)

(*b*) Solve the equation $3 \sinh 2x = 13 - 3\mathrm{e}^{2x}$, giving your answer in the form $\frac{1}{2} \ln k$, where k is an integer.
(5)

---

6. The plane P has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix}$$

(*a*) Find a vector perpendicular to the plane P.
(2)

The line l passes through the point A (1, 3, 3) and meets P at (3, 1, 2).

The acute angle between the plane P and the line l is α.

(*b*) Find α to the nearest degree.
(4)

(*c*) Find the perpendicular distance from A to the plane P.
(4)

PMT

7. The matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{pmatrix} k & -1 & 1 \\ 1 & 0 & -1 \\ 3 & -2 & 1 \end{pmatrix}, \quad k \neq 1.$$

(*a*) Show that det $\mathbf{M} = 2 - 2k$.

(2)

(*b*) Find $\mathbf{M}^{-1}$, in terms of k.

(5)

The straight line l_1 is mapped onto the straight line l_2 by the transformation represented by the matrix

$$\begin{pmatrix} 2 & -1 & 1 \\ 1 & 0 & -1 \\ 3 & -2 & 1 \end{pmatrix}$$

The equation of l_2 is $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = 0$, where $\mathbf{a} = 4\mathbf{i} + \mathbf{j} + 7\mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} + \mathbf{j} + 3\mathbf{k}$.

(*c*) Find a vector equation for the line l_1.

(5)

8. The hyperbola H has equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

(*a*) Use calculus to show that the equation of the tangent to H at the point $(a \cosh \theta, b \sinh \theta)$ may be written in the form

$$xb \cosh \theta - ya \sinh \theta = ab.$$

(4)

The line l_1 is the tangent to H at the point $(a \cosh \theta, b \sinh \theta)$, $\theta \neq 0$.

Given that l_1 meets the x-axis at the point P,

(*b*) find, in terms of a and θ, the coordinates of P.

(2)

The line l_2 is the tangent to H at the point $(a, 0)$.

Given that l_1 and l_2 meet at the point Q,

(*c*) find, in terms of a, b and θ, the coordinates of Q.

(2)

(*d*) Show that, as θ varies, the locus of the mid-point of PQ has equation

$$x(4y^2 + b^2) = ab^2.$$

(6)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6669/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced Level

Monday 25 June 2012 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics FP3), the paper reference (6669), your surname, initials and signature.

Information for Candidates

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Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

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PMT

1. The hyperbola H has equation

$$\frac{x^2}{16} - \frac{y^2}{9} = 1.$$

Find

(*a*) the coordinates of the foci of H,
(3)

(*b*) the equations of the directrices of H.
(2)

2.

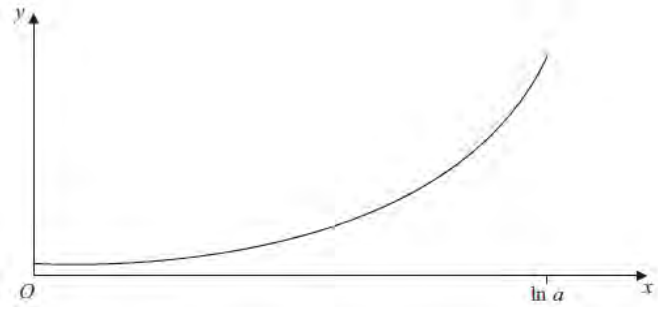

Figure 1

The curve C, shown in Figure 1, has equation

$$y = \frac{1}{3}\cosh 3x, \qquad 0 \le x \le \ln a,$$

where a is a constant and $a > 1$.

Using calculus, show that the length of curve C is

$$k\left(a^3 - \frac{1}{a^3}\right)$$

and state the value of the constant k.
(6)

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3. The position vectors of the points A, B and C relative to an origin O are $\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$, $7\mathbf{i} - 3\mathbf{k}$ and $4\mathbf{i} + 4\mathbf{j}$ respectively.

Find

(*a*) $\overrightarrow{AC} \times \overrightarrow{BC}$,

(4)

(*b*) the area of triangle ABC,

(2)

(*c*) an equation of the plane ABC in the form $\mathbf{r} . \mathbf{n} = p$.

(2)

4.

$$I_n = \int_0^{\frac{\pi}{4}} x^n \sin 2x \, \mathrm{d}x, \quad n \geq 0.$$

(*a*) Prove that, for $n \geq 2$,

$$I_n = \frac{1}{4} n \left(\frac{\pi}{4}\right)^{n-1} - \frac{1}{4} n(n-1) I_{n-2}.$$

(5)

(*b*) Find the exact value of I_2.

(4)

(*c*) Show that $I_4 = \frac{1}{64}(\pi^3 - 24\pi + 48)$.

(2)

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5. (*a*) Differentiate $x \operatorname{arsinh} 2x$ with respect to x.

(3)

(*b*) Hence, or otherwise, find the exact value of

$$\int_0^{\sqrt{2}} \operatorname{arsinh} 2x \, \mathrm{d}x.$$

giving your answer in the form $A \ln B + C$, where A, B and C are real.

(7)

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6. The ellipse E has equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

The line l_1 is a tangent to E at the point $P(a \cos \theta, b \sin \theta)$.

(*a*) Using calculus, show that an equation for l_1 is

$$\frac{x \cos \theta}{a} + \frac{y \sin \theta}{b} = 1.$$

(4)

The circle C has equation

$$x^2 + y^2 = a^2.$$

The line l_2 is a tangent to C at the point $Q(a \cos \theta, a \sin \theta)$.

(*b*) Find an equation for the line l_2.

(2)

Given that l_1 and l_2 meet at the point R,

(*c*) find, in terms of a, b and θ, the coordinates of R.

(3)

(*d*) Find the locus of R, as θ varies.

(2)

7.

$$\mathrm{f}(x) = 5 \cosh x - 4 \sinh x, \quad x \in \mathbb{R}.$$

(*a*) Show that $\mathrm{f}(x) = \frac{1}{2}(\mathrm{e}^x + 9\mathrm{e}^{-x})$.

(2)

Hence

(*b*) solve $\mathrm{f}(x) = 5$,

(4)

(*c*) show that $\int_{\frac{1}{2}\ln 3}^{\ln 3} \frac{1}{5\cosh x - 4\sinh x} \mathrm{d}x = \frac{\pi}{18}$.

(5)

PMT

8. The matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ -1 & 0 & 4 \end{pmatrix}.$$

(*a*) Show that 4 is an eigenvalue of $\mathbf{M}$, and find the other two eigenvalues.
(5)

(*b*) For the eigenvalue 4, find a corresponding eigenvector.
(3)

The straight line l_1 is mapped onto the straight line l_2 by the transformation represented by the matrix $\mathbf{M}$.

The equation of l_1 is $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = 0$, where $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$.

(*c*) Find a vector equation for the line l_2.
(5)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6669/01R

Edexcel GCE

Further Pure Mathematics FP3 (R)

Advanced/Advanced Subsidiary

Monday 24 June 2013 – Afternoon

Time: 1 hour 30 minutes

<u>Materials required for examination</u>	<u>Items included with question papers</u>
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

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P42956A

1. The hyperbola H has foci at (5, 0) and (–5, 0) and directrices with equations

$x = \frac{9}{5}$ and $x = -\frac{9}{5}$.

Find a cartesian equation for H.

(7)

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2. Two skew lines l_1 and l_2 have equations

$$l_1\text{: } \mathbf{r} = (\mathbf{i} - \mathbf{j} + \mathbf{k}) + \lambda(4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$$

$$l_2\text{: } \mathbf{r} = (3\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}) + \mu(-4\mathbf{i} + 6\mathbf{j} + \mathbf{k})$$

respectively, where λ and μ are real parameters.

(*a*) Find a vector in the direction of the common perpendicular to l_1 and l_2.

(2)

(*b*) Find the shortest distance between these two lines.

(5)

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3. The point P lies on the ellipse E with equation

$$\frac{x^2}{36} + \frac{y^2}{9} = 1$$

N is the foot of the perpendicular from point P to the line $x = 8$.

M is the midpoint of PN.

(*a*) Sketch the graph of the ellipse E, showing also the line $x = 8$ and a possible position for the line PN.

(1)

(*b*) Find an equation of the locus of M as P moves around the ellipse.

(4)

(*c*) Show that this locus is a circle and state its centre and radius.

(3)

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4. The plane Π_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + s\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix},$$

where s and t are real parameters.

The plane Π_1 is transformed to the plane Π_2 by the transformation represented by the matrix $\mathbf{T}$, where

$$\mathbf{T} = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 2 & -1 \\ 0 & 1 & 2 \end{pmatrix}$$

Find an equation of the plane in the form $\mathbf{r}.\ \mathbf{n} = p$.

(9)

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5. $$I_n = \int_1^5 x^n (2x-1)^{-\frac{1}{2}}\, dx, \qquad n \geq 0$$

(*a*) Prove that, for $n \geq 1$,

$$(2n+1)I_n = nI_{n-1} + 3 \times 5^n - 1$$

(5)

(*b*) Using the reduction formula given in part (*a*), find the exact value of I_2.

(5)

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6. It is given that $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ is an eigenvector of the matrix **A**, where

$$\mathbf{A} = \begin{pmatrix} 4 & 2 & 3 \\ 2 & b & 0 \\ a & 1 & 8 \end{pmatrix}$$

and a and b are constants.

(*a*) Find the eigenvalue of **A** corresponding to the eigenvector $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$.

(3)

(*b*) Find the values of a and b.

(3)

(*c*) Find the other eigenvalues of **A**.

(5)

7.

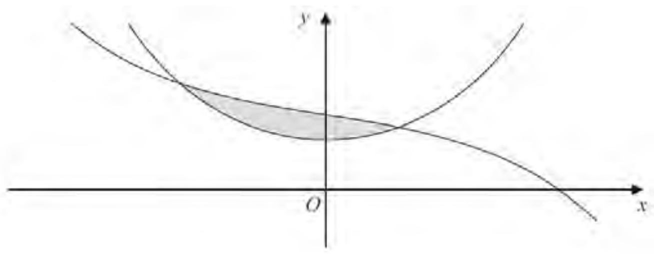

Figure 1

The curves shown in Figure 1 have equations

$$y = 6\cosh x \qquad \text{and} \qquad y = 9 - 2\sinh x$$

(*a*) Using the definitions of sinh x and cosh x in terms of e^x, find exact values for the x-coordinates of the two points where the curves intersect.

(6)

The finite region between the two curves is shown shaded in Figure 1.

(*b*) Using calculus, find the area of the shaded region, giving your answer in the form $a \ln b + c$, where a, b and c are integers.

(6)

PMT

8.

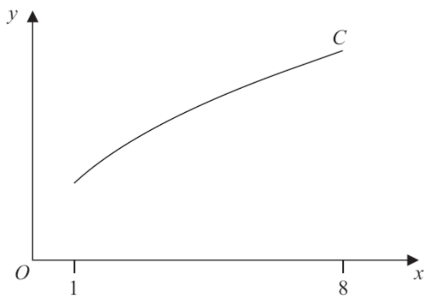

Figure 2

The curve C, shown in Figure 2, has equation

$$y = 2x^{\frac{1}{2}}, \qquad 1 \leq x \leq 8$$

(a) Show that the length s of curve C is given by the equation

$$s = \int_1^8 \sqrt{\left(1 + \frac{1}{x}\right)}\, \mathrm{d}x$$

(2)

(b) Using the substitution $x = \sinh^2 u$, or otherwise, find an exact value for s.

Give your answer in the form $a\sqrt{2} + \ln(b + c\sqrt{2})$ where a, b and c are integers.

(9)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6669/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced/Advanced Subsidiary

Monday 24 June 2013 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P43143A

1. A hyperbola H has equation

$$\frac{x^2}{a^2} - \frac{y^2}{25} = 1, \qquad \text{where } a \text{ is a positive constant.}$$

The foci of H are at the points with coordinates (13, 0) and (–13, 0).

Find

(*a*) the value of the constant a,

(3)

(*b*) the equations of the directrices of H.

(3)

---

2. (*a*) Find

$$\int \frac{1}{\sqrt{(4x^2+9)}}\,dx$$

(2)

(*b*) Use your answer to part (*a*) to find the exact value of

$$\int_{-3}^{3} \frac{1}{\sqrt{(4x^2+9)}}\,dx$$

giving your answer in the form $k \ln(a + b\sqrt{5})$, where a and b are integers and k is a constant.

(3)

---

3. The curve with parametric equations

$$x = \cosh 2\theta, \quad y = 4\sinh\theta, \quad 0 \le \theta \le 1$$

is rotated through 2π radians about the x-axis.

Show that the area of the surface generated is $\lambda(\cosh^3 \alpha - 1)$, where $\alpha = 1$ and λ is a constant to be found.

(7)

4.

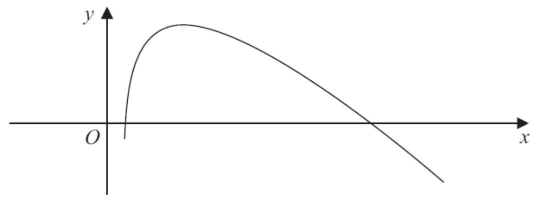

Figure 1

Figure 1 shows part of the curve with equation

$$y = 40 \operatorname{arcosh} x - 9x, \qquad x \ge 1$$

Use calculus to find the exact coordinates of the turning point of the curve, giving your answer in the form $\left(\frac{p}{q}, r\ln 3 + s\right)$ where p, q, r and s are integers.

(7)

---

5. The matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & a \\ 2 & b & c \\ -1 & 0 & 1 \end{pmatrix}, \text{ where } a, b \text{ and } c \text{ are constants.}$$

(*a*) Given that **j** + **k** and **i** – **k** are two of the eigenvectors of **M**,

find

(i) the values of a, b and c,

(ii) the eigenvalues which correspond to the two given eigenvectors.

(8)

(*b*) The matrix **P** is given by

$$\mathbf{P} = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & d \\ -1 & 0 & 1 \end{pmatrix}, \text{ where } d \text{ is constant, } d \neq -1$$

Find

(i) the determinant of **P** in terms of d,

(ii) the matrix $\mathbf{P}^{-1}$ in terms of d.

(5)

---

6. Given that

$$I_n = \int_0^4 x^n \sqrt{(16 - x^2)}\,dx, \qquad n \geq 0,$$

(*a*) prove that, for $n \geq 2$,

$$(n+2)I_n = 16(n-1)I_{n-2}$$

(6)

(*b*) Hence, showing each step of your working, find the exact value of I_5.

(5)

---

7. The ellipse E has equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \qquad a > b > 0$$

The line l is a normal to E at a point $P(a\cos\theta, b\sin\theta)$, $\quad 0 < \theta < \frac{\pi}{2}$

(*a*) Using calculus, show that an equation for l is

$$ax\sin\theta - by\cos\theta = (a^2 - b^2)\sin\theta\cos\theta$$

(5)

The line l meets the x-axis at A and the y-axis at B.

(*b*) Show that the area of the triangle OAB, where O is the origin, may be written as $k\sin 2\theta$, giving the value of the constant k in terms of a and b.

(4)

(*c*) Find, in terms of a and b, the exact coordinates of the point P, for which the area of the triangle OAB is a maximum.

(3)

---

8. The plane Π_1 has vector equation

$$\mathbf{r}.(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) = 5$$

(*a*) Find the perpendicular distance from the point (6, 2, 12) to the plane Π_1.

(3)

The plane Π_2 has vector equation

$$\mathbf{r} = \lambda(2\mathbf{i} + \mathbf{j} + 5\mathbf{k}) + \mu(\mathbf{i} - \mathbf{j} - 2\mathbf{k}), \text{ where } \lambda \text{ and } \mu \text{ are scalar parameters.}$$

(*b*) Find the acute angle between Π_1 and Π_2 giving your answer to the nearest degree.

(5)

(*c*) Find an equation of the line of intersection of the two planes in the form $\mathbf{r} \times \mathbf{a} = \mathbf{b}$, where **a** and **b** are constant vectors.

(6)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

WFM03/01

Pearson Edexcel
International Advanced Level

Further Pure Mathematics F3

Advanced/Advanced Subsidiary

Tuesday 10 June 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Blue)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

P44518A

1. Given that $y = \arctan\left(\frac{2x}{3}\right)$,

(*a*) find $\frac{dy}{dx}$, giving your answer in its simplest form.

(2)

(*b*) Use integration by parts to find

$$\int \arctan\left(\frac{2x}{3}\right) dx$$

(4)

---

2. The line with equation $x = 9$ is a directrix of an ellipse with equation

$$\frac{x^2}{a^2} + \frac{y^2}{8} = 1$$

where a is a positive constant.

Find the two possible exact values of the constant a.

(6)

---

3. Using the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials,

(*a*) prove that

$$\cosh^2 x - \sinh^2 x \equiv 1$$

(2)

(*b*) find algebraically the exact solutions of the equation

$$2 \sinh x + 7 \cosh x = 9$$

giving your answers as natural logarithms.

(5)

---

4. A non-singular matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 3 & k & 0 \\ k & 2 & 0 \\ k & 0 & 1 \end{pmatrix}, \text{ where } k \text{ is a constant.}$$

(*a*) Find, in terms of *k*, the inverse of the matrix **M**.

(5)

The point *A* is mapped onto the point (–5, 10, 7) by the transformation represented by the matrix

$$\begin{pmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

(*b*) Find the coordinates of the point *A*.

(3)

---

5. Given that

$$I_n = \int_0^{\frac{\pi}{4}} \cos^n \theta \, \mathrm{d}\theta, \qquad n \geq 0$$

(*a*) prove that, for $n \geq 2$,

$$nI_n = \left(\frac{1}{\sqrt{2}}\right)^n + (n-1)I_{n-2}$$

(6)

(*b*) Hence find the exact value of I_5, showing each step of your working.

(5)

---

6. The hyperbola *H* has equation

$$\frac{x^2}{16} - \frac{y^2}{4} = 1$$

The line *l* is a tangent to *H* at the point *P* ($4 \cosh \alpha$, $2 \sinh \alpha$), where α is a constant, $\alpha \neq 0$.

(*a*) Using calculus, show that an equation for *l* is

$$2y \sinh \alpha - x \cosh \alpha + 4 = 0$$

(4)

The line *l* cuts the *y*-axis at the point *A*.

(*b*) Find the coordinates of *A* in terms of α.

(2)

The point *B* has coordinates (0, $10 \sinh \alpha$) and the point *S* is the focus of *H* for which $x > 0$.

(*c*) Show that the line segment *AS* is perpendicular to the line segment *BS*.

(5)

---

7. The curve *C* has parametric equations

$$x = 3t^2, \quad y = 12t, \quad 0 \leq t \leq 4$$

The curve *C* is rotated through 2π radians about the *x*-axis.

(*a*) Show that the area of the surface generated is

$$\pi(a\sqrt{5} + b)$$

where *a* and *b* are constants to be found.

(6)

(*b*) Show that the length of the curve *C* is given by

$$k\int_0^4 \sqrt{(t^2 + 4)} \, \mathrm{d}t$$

where *k* is a constant to be found.

(1)

(*c*) Use the substitution $t = 2\sinh \theta$ to show that the exact value of the length of the curve *C* is

$$24\sqrt{5} + 12\ln(2 + \sqrt{5})$$

(6)

---

8. The line l has equation

$$\mathbf{r} = (2\mathbf{i} + \mathbf{j} - 2\mathbf{k}) + \lambda(3\mathbf{i} + 2\mathbf{j} + \mathbf{k}), \text{ where } \lambda \text{ is a scalar parameter,}$$

and the plane Π has equation

$$\mathbf{r}.(\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 19$$

(*a*) Find the coordinates of the point of intersection of l and Π.

(4)

The perpendicular to Π from the point A (2, 1, –2) meets Π at the point B.

(*b*) Verify that the coordinates of B are (4, 3, –6).

(3)

The point A (2, 1, –2) is reflected in the plane Π to give the image point A'.

(*c*) Find the coordinates of the point A'.

(2)

(*d*) Find an equation for the line obtained by reflecting the line l in the plane Π, giving your answer in the form

$$\mathbf{r} \times \mathbf{a} = \mathbf{b},$$

where **a** and **b** are vectors to be found.

(4)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6669/01R

Edexcel GCE

Further Pure Mathematics FP3 (R)

Advanced/Advanced Subsidiary

Monday 23 June 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P44513A

1. Solve the equation

$$5 \tanh x + 7 = 5 \operatorname{sech} x$$

Give each answer in the form $\ln k$ where k is a rational number.

(5)

2.

$$9x^2 + 6x + 5 \equiv a(x + b)^2 + c$$

(*a*) Find the values of the constants a, b and c.

(3)

Hence, or otherwise, find

(*b*) $\displaystyle\int \frac{1}{9x^2+6x+5}\,dx$

(2)

(*c*) $\displaystyle\int \frac{1}{\sqrt{9x^2+6x+5}}\,dx$

(2)

---

3. The curve C has equation

$$y = \frac{1}{2}\ln(\coth x), \qquad x > 0$$

(*a*) Show that

$$\frac{dy}{dx} = -\operatorname{cosech} 2x$$

(3)

The points A and B lie on C.

The x coordinates of A and B are $\ln 2$ and $\ln 3$ respectively.

(*b*) Find the length of the arc AB, giving your answer in the form $p \ln q$, where p and q are rational numbers.

(6)

4.

$$I_n = \int_0^{\sqrt{3}} \left(3 - x^2\right)^n dx, \qquad n \geq 0$$

(*a*) Show that, for $n \geq 1$,

$$I_n = \frac{6n}{2n+1} I_{n-1}$$

(6)

(*b*) Hence find the exact value of I_4, giving your answer in the form $k\sqrt{3}$ where k is a rational number to be found.

(5)

---

5. The ellipse E has equation

$$x^2 + 9y^2 = 9$$

The point $P(a \cos \theta, b \sin \theta)$ is a general point on the ellipse E.

(*a*) Write down the value of a and the value of b.

(1)

The line L is a tangent to E at the point P.

(*b*) Show that an equation of the line L is given by

$$3y \sin \theta + x \cos \theta = 3$$

(3)

The line L meets the x-axis at the point Q and meets the y-axis at the point R.

(*c*) Show that the area of the triangle OQR, where O is the origin, is given by

$$k \operatorname{cosec} 2\theta$$

where k is a constant to be found.

(3)

The point M is the midpoint of QR.

(*d*) Find a cartesian equation of the locus of M, giving your answer in the form $y^2 = f(x)$.

(4)

PMT

6. The symmetric matrix $\mathbf{M}$ has eigenvectors $\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

with eigenvalues 5, 2 and –1 respectively.

(*a*) Find an orthogonal matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that

$$\mathbf{P}^{\mathrm{T}}\mathbf{M}\mathbf{P} = \mathbf{D}$$

(4)

Given that $\mathbf{P}^{-1} = \mathbf{P}^{\mathrm{T}}$

(*b*) show that

$$\mathbf{M} = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$$

(2)

(*c*) Hence find the matrix $\mathbf{M}$.

(5)

---

7. The curve C has equation

$$y = \mathrm{e}^{-x}, \qquad x \in \mathbb{R}$$

The part of the curve C between $x = 0$ and $x = \ln 3$ is rotated through 2π radians about the x-axis.

(*a*) Show that the area S of the curved surface generated is given by

$$S = 2\pi \int_0^{\ln 3} e^{-x}\sqrt{1+e^{-2x}}\,\mathrm{d}x$$

(3)

(*b*) Use the substitution $\mathrm{e}^{-x} = \sinh u$ to show that

$$S = 2\pi \int_{\operatorname{arsinh}\alpha}^{\operatorname{arsinh}\beta} \cosh^2 u \,\mathrm{d}u$$

where α and β are constants to be determined.

(5)

(*c*) Show that

$$2\int \cosh^2 u\,\mathrm{d}u = \frac{1}{2}\sinh 2u + u + k$$

where k is an arbitrary constant.

(2)

(*d*) Hence find the value of S, giving your answer to 3 decimal places.

(2)

---

8. The plane Π_1 has vector equation $\mathbf{r}.\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = 5$.

The plane Π_2 has vector equation $\mathbf{r}.\begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix} = 7$.

(*a*) Find a vector equation for the line of intersection of Π_1 and Π_2, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors and λ is a scalar parameter.

(6)

The plane Π_3 has cartesian equation

$$x - y + 2z = 31$$

(*b*) Using your answer to part (*a*), or otherwise, find the coordinates of the point of intersection of the planes Π_1, Π_2 and Π_3.

(3)

TOTAL FOR PAPER: 75 MARKS

END

PMT

Paper Reference(s)
6669/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced/Advanced Subsidiary

Monday 23 June 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

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Check that you have the correct question paper.
Answer ALL the questions.
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Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. The line l passes through the point $P(2, 1, 3)$ and is perpendicular to the plane Π whose vector equation is

$$\mathbf{r}.(\mathbf{i} - 2\mathbf{j} - \mathbf{k}) = 3$$

Find

(*a*) a vector equation of the line l,
(2)

(*b*) the position vector of the point where l meets Π.
(4)

(*c*) Hence find the perpendicular distance of P from Π.
(2)

2.

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 4 & 1 \\ 0 & 5 & 0 \end{pmatrix}$$

(*a*) Show that matrix **M** is not orthogonal.
(2)

(*b*) Using algebra, show that 1 is an eigenvalue of **M** and find the other two eigenvalues of **M**.
(5)

(*c*) Find an eigenvector of **M** which corresponds to the eigenvalue 1.
(2)

The transformation $M : \mathbb{R}^3 \to \mathbb{R}^3$ is represented by the matrix **M**.

(*d*) Find a cartesian equation of the image, under this transformation, of the line

$$x = \frac{y}{2} = \frac{z}{-1}$$

(4)

PMT

3. Using calculus, find the exact value of

(*a*) $\int_1^2 \frac{1}{\sqrt{(x^2 - 2x + 3)}}\,dx$

(4)

(*b*) $\int_0^1 e^{2x} \sinh x \, dx$

(4)

---

4. Using the definitions of hyperbolic functions in terms of exponentials,

(*a*) show that

$$\text{sech}^2 x = 1 - \tanh^2 x$$

(3)

(*b*) solve the equation

$$4\sinh x - 3\cosh x = 3$$

(4)

---

5. Given that $y = \text{artanh}\frac{x}{\sqrt{(1+x^2)}}$

show that $\frac{dy}{dx} = \frac{1}{\sqrt{(1+x^2)}}$

(4)

---

6. [*In this question you may use the appropriate trigonometric identities on page 6 of the pink Mathematical Formulae and Statistical Tables.*]

The points $P(3\cos\alpha, 2\sin\alpha)$ and $Q(3\cos\beta, 2\sin\beta)$, where $\alpha \neq \beta$, lie on the ellipse with equation

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

(*a*) Show the equation of the chord *PQ* is

$$\frac{x}{3}\cos\frac{(\alpha+\beta)}{2} + \frac{y}{2}\sin\frac{(\alpha+\beta)}{2} = \cos\frac{(\alpha-\beta)}{2}$$

(4)

(*b*) Write down the coordinates of the mid-point of *PQ*.

(1)

Given that the gradient, *m*, of the chord *PQ* is a constant,

(*c*) show that the centre of the chord lies on a line

$$y = -kx$$

expressing *k* in terms of *m*.

(5)

---

7. A circle *C* with centre *O* and radius *r* has cartesian equation $x^2 + y^2 = r^2$ where *r* is a constant.

(*a*) Show that $1 + \left(\frac{dy}{dx}\right)^2 = \frac{r^2}{r^2 - x^2}$.

(3)

(*b*) Show that the surface area of the sphere generated by rotating *C* through π radians about the *x*-axis is $4\pi r^2$.

(5)

(*c*) Write down the length of the arc of the curve $y = \sqrt{(1 - x^2)}$ from $x = 0$ to $x = 1$.

(1)

---

8. The position vectors of the points A, B and C from a fixed origin O are

$$\mathbf{a} = \mathbf{i} - \mathbf{j}, \quad \mathbf{b} = \mathbf{i} + \mathbf{j} + \mathbf{k}, \quad \mathbf{c} = 2\mathbf{j} + \mathbf{k}$$

respectively.

(*a*) Using vector products, find the area of the triangle ABC.

(4)

(*b*) Show that $\frac{1}{6}\mathbf{a}.(\mathbf{b} \times \mathbf{c}) = 0$.

(3)

(*c*) Hence or otherwise, state what can be deduced about the vectors **a**, **b** and **c**.

(1)

9.

$$I_n = \int \left(x^2 + 1\right)^{-n} \mathrm{d}x, \quad n > 0$$

(*a*) Show that, for $n > 0$,

$$I_{n+1} = \frac{x\left(x^2+1\right)^{-n}}{2n} + \frac{2n-1}{2n} I_n$$

(5)

(*b*) Find I_2.

(3)

TOTAL FOR PAPER: 75 MARKS

END