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Fp3 Moch Kerime 2

$$1. \quad 4 \tanh^2 x - 2 \operatorname{sech}^2 x = 3$$

$$\therefore 4 - 4 \operatorname{sech}^2 x = 2 \operatorname{sech}^2 x = 3$$

$$\therefore 4 - 6 \operatorname{sech}^2 x = 3$$

$$6 \operatorname{sech}^2 x = 1$$

$$\Rightarrow \operatorname{sech}^2 x = 1/6$$

$$\Rightarrow \cosh^2 x = 6$$

$\cosh x > 0$

$$\Rightarrow \cosh x = \sqrt{6}$$

$$\Rightarrow \operatorname{arcosh} \sqrt{6} = \ln \left(\sqrt{6} + \sqrt{6+1} \right)$$

$$\cosh(x) \equiv \cosh(\pm x) \Rightarrow x = \pm \ln \left(\sqrt{6} + \sqrt{5} \right)$$

$$2. \quad y = 2 \cosh\left(\frac{1}{2}x\right)$$

$$c^2 - s^2 = 1$$

$$\frac{\partial y}{\partial x} = \sinh\left(\frac{1}{2}x\right)$$

$$\text{Surface Area} = 2\pi \int_{-\ln 2}^{\ln 2} 2 \cosh\left(\frac{x}{2}\right) \sqrt{1 + \sinh^2\left(\frac{x}{2}\right)} \, dx$$

$$= 4\pi \int_{-\ln 2}^{\ln 2} \cosh^2\left(\frac{x}{2}\right) \, dx$$

$$\begin{aligned} \cosh 2x &= c^2 + s^2 \\ &= c^2 + c^2 - 1 \\ \cosh 2x &= 2c^2 - 1 \end{aligned}$$

$$\cosh^2 x = \frac{(\cosh 2x + 1)}{2}$$

$$= \frac{4\pi}{2} \int_{-\ln 2}^{\ln 2} \cosh x + 1 \, dx$$

$$\therefore \cosh^2\left(\frac{x}{2}\right) = \frac{(\cosh x + 1)}{2}$$

$$= 2\pi \left[\sinh x + x \right]_{-\ln 2}^{\ln 2}$$

$$= 2\pi \left(\frac{e^{\ln 2} - e^{-\ln 2}}{2} + \ln 2 - \frac{e^{-\ln 2} - e^{\ln 2}}{2} + \ln 2 \right)$$

$$= 2\pi \left(\frac{3}{4} + 2\ln 2 - \frac{3}{4} \right) = 2\pi \left(\frac{3}{2} + 2\ln 2 \right)$$

$$= 3\pi + 4\pi \ln 2$$

$$3. \quad x = \frac{3}{\sinh \theta} = 3(\sinh \theta)^{-1}$$

$$\frac{\partial x}{\partial \theta} = -3(\sinh \theta)^{-2} \cosh \theta$$

$$= -\frac{3 \cosh \theta}{\sinh^2 \theta}$$

$$\therefore \quad \partial x = -\frac{3 \cosh \theta}{\sinh^2 \theta} \partial \theta$$

$$\frac{1}{x \sqrt{x^2 + 9}} = \frac{1}{\frac{3}{\sinh \theta} \sqrt{\frac{9}{\sinh^2 \theta} + 9}}$$

$$= \frac{1}{\frac{9}{\sinh \theta} \sqrt{\frac{1}{\sinh^2 \theta} + 1}}$$

$$\therefore \quad \frac{1}{x \sqrt{x^2 + 9}} = \frac{\sinh \theta}{9 \sqrt{\frac{1}{\sinh^2 \theta} + 1}}$$

$$\therefore \quad 3\sqrt{3} = \frac{3}{\sinh \theta} \Rightarrow \sinh \theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = \sqrt{3} \quad \theta = \ln \sqrt{3}$$

$$4 = \frac{3}{\sinh \theta} \Rightarrow \sinh \theta = \frac{3}{4} \Rightarrow \theta = \ln 2$$

$$\therefore \int_{\ln 2}^{\ln \sqrt{3}} \frac{\sinh \theta}{9 \sqrt{\frac{1}{\sinh^2 \theta} + 1}} \cdot -\frac{3 \cosh \theta}{\sinh^2 \theta} d\theta$$

$$= - \int_{\ln 2}^{\ln \sqrt{3}} \frac{3 \cosh \theta}{9 \sinh \theta \sqrt{\frac{1}{\sinh^2 \theta} + 1}} d\theta$$

$$= - \int_{\ln 2}^{\ln \sqrt{3}} \frac{3 \cosh \theta}{9 \sqrt{1 + \sinh^2 \theta}} d\theta$$

$$= - \int_{\ln 2}^{\ln \sqrt{3}} \frac{3}{9} \frac{\cosh \theta}{\sqrt{\cosh^2 \theta}} d\theta$$

$$= - \int_{\ln 2}^{\ln \sqrt{3}} \frac{1}{3} d\theta = - \left[\frac{1}{3} \theta \right]_{\ln 2}^{\ln \sqrt{3}}$$

$$= - \left(\frac{1}{3} \ln \sqrt{3} - \frac{1}{3} \ln 2 \right)$$

$$= \frac{1}{3} \ln 2 - \frac{1}{3} \ln \sqrt{3} = \frac{1}{3} \ln \left(\frac{2}{\sqrt{3}} \right)$$

$$= \frac{1}{3} \ln \left(\frac{2}{\sqrt{3}} \right) = \frac{1}{3} \ln \sqrt{\frac{4}{3}}$$

$$= \frac{1}{3} \ln \sqrt{\frac{4}{3}} = \frac{1}{6} \ln \left(\frac{4}{3} \right)$$

$$4(a) \quad y = \arctan \sqrt{x}$$

$$\begin{aligned} s^2 + c^2 &= 1 \\ t^2 + 1 &= \sec^2 \end{aligned}$$

$$\therefore \tan y = \sqrt{x}$$

$$\therefore \frac{\partial y}{\partial x} \sec^2 y = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}} \quad \tan^2 y + 1 = x + 1$$

$$\therefore \frac{\partial y}{\partial x} (\tan^2 y + 1) = \frac{1}{2\sqrt{x}}$$

$$\therefore \frac{\partial y}{\partial x} = \frac{1}{2\sqrt{x}} \cdot \frac{1}{x+1}$$

$$\frac{\partial y}{\partial x} = \frac{1}{2\sqrt{x}(x+1)}$$

$$\therefore \left(\frac{\partial y}{\partial x} \right)_{x=1/4} = \frac{1}{2\sqrt{1/4} \left(\frac{1}{4} + 1 \right)} = \frac{4}{5}$$

$$(b) \quad \frac{\partial y}{\partial x} = (2\sqrt{x}(x+1))^{-1} \quad \frac{1}{2} x^{-1/2}$$

$$\therefore \frac{\partial^2 y}{\partial x^2} = - (2\sqrt{x}(x+1))^{-2} \left[2\sqrt{x} + (x+1) \left(2 \cdot \frac{1}{2} x^{-1/2} \right) \right]$$

$$= - (2\sqrt{x}(x+1))^{-2} \left[2\sqrt{x} + \frac{(x+1)}{\sqrt{x}} \right]$$

$$= - \frac{2\sqrt{x} + \frac{(x+1)}{\sqrt{x}}}{(2\sqrt{x}(x+1))^2} = - \frac{3x+1}{4\sqrt{x} \cdot x(x+1)^2}$$

$$\frac{\partial^2 y}{\partial n^2} = - \frac{3n+1}{4n^{3/2}(n+1)^2}$$

$$\therefore 2n(1+n) \frac{\partial^2 y}{\partial n^2} = - \frac{3n+1}{4n^{3/2}(n+1)^2} \cdot 2n(1+n)$$

$$= - \frac{2n(3n+1)}{4n^{3/2}(n+1)} = - \frac{2(3n+1)}{4\sqrt{n}(n+1)}$$

$$= - \frac{3n+1}{2\sqrt{n}(n+1)}$$

$$+ (1+3n) \frac{\partial y}{\partial n} = \frac{1}{2\sqrt{n}(n+1)}$$

$$\therefore (1+3n) \frac{\partial y}{\partial n} = \frac{1+3n}{2\sqrt{n}(n+1)}$$

$$\therefore 2n(1+n) \frac{\partial^2 y}{\partial n^2} + (1+3n) \frac{\partial y}{\partial n} = - \frac{2(3n+1)}{4\sqrt{n}(n+1)} + \frac{1+3n}{2\sqrt{n}(n+1)}$$

$$= - \frac{(3n+1)}{2\sqrt{n}(n+1)} + \frac{1+3n}{2\sqrt{n}(n+1)}$$

$$= \frac{-(3n+1) + 1+3n}{2\sqrt{n}(n+1)} = \frac{0}{2\sqrt{n}(n+1)}$$

$$= 0$$

as required.

$$5 \quad 6). \quad I_n = \int_0^{\pi/2} \sin^n x \, dx$$

$$\begin{aligned} u &= \sin^n x & u' &= n \sin^{n-1} x \cos x \\ v' &= 1 & v &= x \end{aligned}$$

$$5 \quad 6). \quad I_n = \int_0^{\pi/2} \sin^n x \, dx = \int_0^{\pi/2} \sin^{n-1} x \sin x \, dx$$

$$u = \sin^{n-1} x \quad u' = (n-1) \sin^{n-2} x \cos x$$

$$v' = \sin x \quad v = -\cos x$$

$$\therefore I_n = \left[-\cos x \sin^{n-1} x \right]_0^{\pi/2} + (n-1) \int_0^{\pi/2} \sin^{n-2} x \cos^2 x \, dx$$

$$\therefore I_n = (n-1) \int_0^{\pi/2} \sin^{n-2} x (1 - \sin^2 x) \, dx$$

$$\therefore I_n = (n-1) \int_0^{\pi/2} \sin^{n-2} x - \sin^n x \, dx$$

$$\therefore I_n = (n-1) \int_0^{\pi/2} \sin^{n-2} x - (n-1) \int_0^{\pi/2} \sin^n x \, dx$$

$$\therefore I_n = (n-1) I_{n-2} - (n-1) I_n$$

$$\therefore I_n + (n-1) I_n = (n-1) I_{n-2}$$

$$\therefore n I_n = (n-1) I_{n-2}$$

$$\therefore I_{n-2} = \frac{n-1}{n} I_{n-2} \quad \text{as required.}$$

$$(6) \int_0^{\pi/2} x \sin^5 x \cos x \, dx$$

$$\text{Let } u = x \quad u' = 1$$

$$v' = \cos x (\sin x)^5 \quad v = \frac{\sin^6 x}{6}$$

$$\therefore \int_0^{\pi/2} x \sin^5 x \cos x \, dx = \left[\frac{x \sin^6 x}{6} \right]_0^{\pi/2} - \frac{1}{6} I_6$$

$$\frac{1 - \cos 2x}{2}$$

$$1 - 2s^2 = \cos 2x$$

$$c^2 - s^2 = \cos 2x$$

$$s^2 = \frac{c^2 - 1}{2c^2 - 1}$$

$$= \frac{\pi}{12} - \frac{1}{6} I_6$$

$$= \frac{\pi}{12} - \frac{1}{6} \cdot \frac{5}{6} I_4$$

$$= \frac{\pi}{12} - \frac{5}{36} \cdot \frac{3}{4} I_2 = \frac{\pi}{12} - \frac{5}{48} I_2$$

$$= \frac{\pi}{12} - \frac{5}{48} \cdot \frac{1}{2} I_0 = \frac{\pi}{12} - \frac{5}{96} \left[x - \frac{1}{2} \sin 2x \right]_0^{\pi/2}$$

$$= \frac{\pi}{12} - \frac{5}{96} \left(\frac{\pi}{2} \right)$$

$$= \frac{\pi}{12} - \frac{5}{96} \int_0^{\pi/2} 1 \, dx$$

$$= \frac{\pi}{12} - \frac{5}{96} [x]_0^{\pi/2} = \frac{\pi}{12} - \frac{5}{96} \left(\frac{\pi}{2} \right)$$

$$= \frac{\pi}{12} - \frac{5}{96} \left(\frac{6\pi}{12} \right)$$

$$= \frac{\pi}{12} - \frac{5\pi}{128}$$

$$= \frac{11}{192} \pi$$

$$= \frac{11\pi}{192}$$

$$6(a). \overrightarrow{PQ} = \begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix} - \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ -4 \end{pmatrix}$$

$$\overrightarrow{QR} = \begin{pmatrix} 0 \\ 3 \\ -5 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$$

$$\overrightarrow{PQ} \times \overrightarrow{QR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 4 & -4 \\ 2 & 2 & -2 \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 4 & -4 \\ 2 & 2 & -2 \end{vmatrix}$$

$$= \begin{pmatrix} 0 \\ -14 \\ -14 \end{pmatrix}$$

$$(b) \quad \hat{n} = \begin{pmatrix} 0 \\ -14 \\ -14 \end{pmatrix}$$

$$\underline{r} \cdot \hat{n} = \underline{p} \cdot \hat{n}$$

$$\therefore -14y - 14z = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -14 \\ -14 \end{pmatrix} = 42 - 14$$

$$\therefore 14y + 14z = -28$$

$$\underline{y + z = -2}$$

$$(C). \quad \begin{cases} \vec{r} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} = 6 \\ \vec{r} \cdot \begin{pmatrix} 0 \\ -1 \\ 4 \end{pmatrix} = 28 \end{cases} \Rightarrow \begin{cases} x+y-z=6 \\ y+z=-2 \end{cases}$$

$$\begin{aligned} \therefore y &= -2-z \\ x + (-2-z) - z &= 6 \Rightarrow \\ x - 2 - 2z &= 6 \\ x &= 8 + 2z \end{aligned}$$

$$\begin{aligned} \begin{pmatrix} x \\ y \\ z \end{pmatrix} &= \begin{pmatrix} 8+2z \\ -2-z \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ -2 \\ 0 \end{pmatrix} + z \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \\ z &= \lambda \\ \Rightarrow \vec{r} &= \begin{pmatrix} 8 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \end{aligned}$$

$$\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$$

~~$$\vec{r} \times \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 8 \\ -2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$~~

$$\vec{a} = \begin{pmatrix} 8 \\ -2 \\ 0 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$$\therefore \left(\vec{r} - \begin{pmatrix} 8 \\ -2 \\ 0 \end{pmatrix} \right) \times \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = 0$$

$$7(a). \quad Ax = \begin{pmatrix} 2 & k & 0 \\ 1 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} 9 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 18+3k \\ 12 \\ -8 \end{pmatrix} \quad \text{---}$$

$$= \begin{pmatrix} 3(6+k) \\ 3(4) \\ 3(-2) \end{pmatrix} \quad \text{---} \quad = \begin{pmatrix} 9 \\ 3 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 3(6+k) \\ 3(4) \\ 3(-2) \end{pmatrix} = \begin{pmatrix} 4\left(\frac{9}{2} + \frac{3}{4}k\right) \\ 4(3) \\ 4(-2) \end{pmatrix} = 4 \begin{pmatrix} \frac{9}{2} + \frac{3}{4}k \\ 3 \\ -2 \end{pmatrix}$$

$$\Rightarrow \frac{9}{2} + \frac{3}{4}k = 9 \Rightarrow k = \left(9 - \frac{9}{2}\right) \times \frac{4}{3}$$

$\therefore k = 6$ as required.

(b) $\lambda = 4$ is an eigen value

$$A - \lambda I = \begin{pmatrix} 2-\lambda & 6 & 0 \\ 1 & 1-\lambda & 0 \\ 0 & -2 & 1-\lambda \end{pmatrix}$$

$$\det(A - \lambda I) = (2-\lambda)(1-\lambda)^2 - 6(1-\lambda)$$

$$= (1-\lambda)(2-\lambda)(1-\lambda) - 6(1-\lambda)$$

$$= (1-\lambda)(2 - 3\lambda + \lambda^2 - 6)$$

$$= (1-\lambda)(\lambda^2 - 3\lambda - 4)$$

$$= (1-\lambda)(\lambda - 4)(\lambda + 1)$$

$\therefore$ $\lambda = 1$ $\lambda = 4$ are eigen-values
 $\lambda = -1$

$$\textcircled{C}) \quad A\rho = \begin{pmatrix} 2 & 6 & 0 \\ 1 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} t-2 \\ t \\ 2t \end{pmatrix}$$

$$= \begin{pmatrix} 8t-4 \\ 2t-2 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ 0 \end{pmatrix} + t \begin{pmatrix} 8 \\ 2 \\ 0 \end{pmatrix}$$

$\therefore$ the line represented has equation

$$\underline{r} = \begin{pmatrix} -4 \\ -2 \\ 0 \end{pmatrix} + t \begin{pmatrix} 8 \\ 2 \\ 0 \end{pmatrix}$$

$$\Rightarrow x = 8t - 4 = \cancel{2t-2}$$

$$y = 2t - 2$$

$$z = 0$$

$$\Rightarrow y + 2 = 2t$$

$$\cancel{2y+4=}$$

$$4y + 8 = 8t$$

$$\therefore x = 8t - 4 = 4y + 8 - 4 = 4y + 4$$

$$\therefore x = 4y + 4$$

$$\Rightarrow x - 4y - 4 = 0$$

$$8(a). \quad \frac{x^2}{25} - \frac{y^2}{9} = 1$$

$$\frac{2}{25}x - \frac{2}{9}y \frac{\partial y}{\partial x} = 0$$

$$\Rightarrow \frac{\partial y}{\partial x} = \frac{1}{25} \frac{x}{y} \quad \frac{\partial y}{\partial x} = \frac{9}{25} \frac{x}{y}$$

$$\therefore \left(\frac{\partial y}{\partial x} \right) \text{ at } p = \frac{9}{25} \cdot \frac{5}{3} \frac{\sec u}{\tan u} = \frac{3}{5} \operatorname{cosec} u$$

$$\therefore y - 3 \tan u = \frac{3}{5} (x - 5 \sec u) \operatorname{cosec} u$$

$$\therefore 5y - 15 \tan u = 3 \operatorname{cosec} u (x - 5 \sec u)$$

$$\therefore 5y - 15 \tan u = 3 \operatorname{cosec}(u) x - 15 \operatorname{cosec}(u) \sec(u)$$

$$\cancel{\frac{3}{5} \operatorname{cosec} u} \quad x \sin u \quad 5y \sin u - \frac{15 \sin^2 u}{\cos u} = 3x - \frac{15}{\cos u}$$

$$\therefore \cancel{5y \sin u} \quad 3x = 5y \sin u - \frac{15 \sin^2 u}{\cos u} + \frac{15}{\cos u}$$

$$\therefore 3x = 5y \sin u + \frac{15(1 - \sin^2 u)}{\cos u}$$

$$\text{since } 1 - \sin^2 u = \cos^2 u$$

$$\therefore 3x = 5y \sin u + 15 \cos u \quad \text{as required}$$

(b) at R $y = \frac{3}{5}x$

$$\Rightarrow 3x = 5 \left(\frac{3}{5}x \right) \sin u + 5 \cos u$$

$$\therefore 3x = 3x \sin u + 5 \cos u$$

$$\therefore \cancel{x = x} (1 - \sin u)(3x) = 5 \cos u$$

$$\therefore x = \frac{5 \cos u}{1 - \sin u} \quad \& \quad y = \frac{3}{5} \left(\frac{5 \cos u}{1 - \sin u} \right) = \frac{3 \cos u}{1 - \sin u}$$

$$\therefore R \left(\frac{5 \cos u}{1 - \sin u}, \frac{3 \cos u}{1 - \sin u} \right)$$

at S, $y = -\frac{3}{5}x$

$$\therefore \cancel{3x} \quad 3x = 5 \left(-\frac{3}{5}x \right) \sin u + 5 \cos u$$

$$\therefore 3x = -3x \sin u + 5 \cos u$$

$$\cancel{y = -\frac{3}{5} \left(\frac{5 \cos u}{1 + \sin u} \right)}$$

$$\therefore (1 + \sin u)3x = 5 \cos u$$

$$\therefore x = \frac{5 \cos u}{1 + \sin u}$$

$$y = -\frac{3 \cos u}{1 + \sin u}$$

$$\therefore \text{ at } \int \left(\frac{5 \cos u}{1 + \sin u}, \frac{-3 \cos u}{1 + \sin u} \right)$$

~~$\therefore P$~~ x coordinate of

$$x_p = \frac{1}{2} \left(\frac{5 \cos u}{1 + \sin u} + \frac{5 \cos u}{1 - \sin u} \right)$$

$$x_p = \frac{1}{2} \left(\frac{5 \cos u (1 - \sin u) + 5 \cos u (1 + \sin u)}{1 - \sin^2 u} \right)$$

$$\therefore x_p = \frac{1}{2} \left(\frac{5 \cos u (1 - \sin u) + 5 \cos u (1 + \sin u)}{\cos^2 u} \right)$$

$$\therefore x_p = \frac{1}{2} \left(\frac{5 \cos u (2)}{\cos^2 u} \right) = \frac{5 \cos u}{\cos^2 u} = \frac{5}{\cos u}$$

$$\therefore x_p = \frac{5}{\cos u} = 5 \sec u //$$

$$(b) \text{ at } R \quad y = \frac{3}{5}x$$

$$\Rightarrow 3x = 5 \left(\frac{3}{5}x \right) \sin u + 15 \cos u$$

$$\therefore 3x = 3x \sin u + 15 \cos u$$

$$\therefore \cancel{x = x} \quad (1 - \sin u)(3x) = 15 \cos u$$

$$\therefore \quad x = \frac{5 \cos u}{1 - \sin u} \quad \& \quad y = \frac{3}{5} \left(\frac{5 \cos u}{1 - \sin u} \right) = \frac{3 \cos u}{1 - \sin u}$$

$$\therefore R \left(\frac{5 \cos u}{1 - \sin u}, \frac{3 \cos u}{1 - \sin u} \right)$$

$$\text{at } S, \quad y = -\frac{3}{5}x$$

$$\therefore \cancel{3x} \quad 3x = 5 \left(-\frac{3}{5}x \right) \sin u + 15 \cos u$$

$$\therefore 3x = -3x \sin u + 15 \cos u$$

$$\cancel{y = -\frac{3}{5} \left(\frac{5 \cos u}{1 + \sin u} \right)}$$

$$\therefore (1 + \sin u)3x = 15 \cos u$$

$$\therefore x = \frac{5 \cos u}{1 + \sin u} \quad y = -\frac{3 \cos u}{1 + \sin u}$$

$$y_p = \frac{1}{2} \left(\frac{3 \cos u}{1 - \sin u} - \frac{3 \cos u}{1 + \sin u} \right)$$

$$= \frac{1}{2} \left(\frac{3 \cos u (1 + \sin u)}{1 - \sin^2 u} - \frac{3 \cos u (1 - \sin u)}{1 - \sin^2 u} \right)$$

$$= \frac{1}{2} \left(\frac{3 \cos u (1 + \sin u - (1 - \sin u))}{\cos^2 u} \right)$$

$$= \frac{1}{2} \left(\frac{3 (2 \sin u)}{\cos u} \right) = \frac{3 \sin u}{\cos u}$$

$$= \underline{\underline{3 \tan u}} \quad \text{as required}$$

$$x_p = 5 \sec u \quad y_p = 3 \tan u$$

$\therefore$ P is midpoint of RS.