Mark Scheme (Results)

Summer 2017

Pearson Edexcel GCE
In Further Pure Mathematics FP3 (6669/01)

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Summer 2017
Publications Code 6669_01_1706_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper or ag- answer given
- ⎣ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q),\ \text{where}\ |pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q),\ \text{where}\ |pq| = |c|\ \text{and}\ |mn| = |a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0,\quad q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Question Number	Scheme	Notes	Marks
1	$y=\text{arsinh}(\tanh x)$		
Way 1	$\sinh y=\tanh x$		B1
	$\cosh y\dfrac{dy}{dx}=\text{sech}^2x$ or $\cosh y=\text{sech}^2x\dfrac{dx}{dy}$	M1: $\pm\cosh y$ or $\pm\text{sech}^2x$ A1: All correct	M1A1
	$\dfrac{dy}{dx}=\dfrac{\text{sech}^2x}{\cosh y}$		
	$\dfrac{dy}{dx}=\dfrac{\text{sech}^2x}{\sqrt{1+\sinh^2 y}}=f(x)$	Uses a **correct identity** to express $\dfrac{dy}{dx}$ in terms of x only	M1
	$=\dfrac{\text{sech}^2x}{\sqrt{1+\tanh^2x}}$ *	cso. There must be no errors such as incorrect or missing or inconsistent variables and no missing h's.	A1*
			Total 5
Way 2	$t=\tanh x\Rightarrow y=\text{arsinh}t$	Replaces tanh$x$ by e,g. t	B1
	$\dfrac{dt}{dx}=\text{sech}^2x,\ \dfrac{dy}{dt}=\dfrac{1}{\sqrt{1+t^2}}$	M1: $\dfrac{dt}{dx}=\pm\text{sech}^2x,\ \dfrac{dy}{dt}=\pm\dfrac{1}{\sqrt{1+t^2}}$ A1: Correct $\dfrac{dt}{dx}$ and $\dfrac{dy}{dt}$ and correctly labelled	M1A1
	$\dfrac{dy}{dx}=\dfrac{dy}{dt}\dfrac{dt}{dx}=\dfrac{\text{sech}^2x}{\sqrt{1+t^2}}=f(x)$	Uses **correct form of the chain rule** for their variables to express $\dfrac{dy}{dx}$ in terms of x only	M1
	$=\dfrac{\text{sech}^2x}{\sqrt{1+\tanh^2x}}$ *	Cso. There must be no errors such as incorrect or missing or inconsistent variables and no missing h's.	A1*
			Total 5
Way 3	$u=\tanh x\Rightarrow\dfrac{du}{dx}=\text{sech}^2 x$	Correct derivative	B1
	$\displaystyle\int\frac{\text{sech}^2x}{\sqrt{1+\tanh^2x}}dx=\int\frac{\text{sech}^2 x}{\sqrt{1+u^2}}\frac{1}{\text{sech}^2 x}du$	M1: Complete substitution including the "dx" A1: Fully correct substitution	M1A1
	$\displaystyle=\int\frac{1}{\sqrt{1+u^2}}du=\text{arsinh}u(+c)$	Reaches arsinh$u$	M1
	$y=\text{arsinh}(\tanh x)(+c)$	Reaches $y=\text{arsinh}(\tanh x)$ with or without + c and no errors such as incorrect or missing or inconsistent variables or missing h's.	A1*
			Total 5

	Special Case:	
	$y = \text{arsinh}\left(\tanh x\right) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sqrt{1+\tanh^2 x}}\left(\times\right)\text{sech}^2 x$ $= \frac{\text{sech}^2 x}{\sqrt{1+\tanh^2 x}}$ **Note that the sech^2*x* needs to appear separate from the fraction as above <u>and not just the printed answer written down</u>.** **<u>To score more than 2 marks using a chain rule method, a third variable must be introduced</u>**	M1A1

Question Number	Scheme	Notes	Marks
2(a)	$\frac{2x}{36}+\frac{2y}{25}\frac{dy}{dx}=0\Rightarrow\frac{dy}{dx}=-\frac{25x}{36y}=\frac{5\cos\theta}{-6\sin\theta}$ or $x=6\cos\theta, y=5\sin\theta\Rightarrow\frac{dy}{dx}=\frac{5\cos\theta}{-6\sin\theta}$ or $\frac{y^2}{25}=1-\frac{x^2}{36}\Rightarrow y=5\sqrt{1-\frac{x^2}{36}}\Rightarrow\frac{dy}{dx}=-\frac{5x}{36}\left(1-\frac{x^2}{36}\right)^{-\frac{1}{2}}=-\frac{5\cos\theta}{6\sin\theta}$ M1: Correct attempt at $\frac{dy}{dx}$ using implicit or parametric or explicit differentiation $\left(ax+by\frac{dy}{dx}=0\Rightarrow\frac{dy}{dx}=...,\frac{dy}{dx}=\pm\frac{a\cos\theta}{b\sin\theta},\frac{dy}{dx}=ax\left(1-bx^2\right)^{-\frac{1}{2}}(oe)\Rightarrow\frac{dy}{dx}=...\right)$		M1
	$=-\frac{5\cos\theta}{6\sin\theta}$	A1: Correct tangent gradient in terms of θ. May be implied in their attempt the normal gradient.	A1
	$m_N=\frac{6\sin\theta}{5\cos\theta}$	Correct perpendicular gradient rule. May be awarded if working in terms of x and y.	M1
	$y-5\sin\theta=Their\ m_N\left(x-6\cos\theta\right)$	Correct straight line method for the normal using a "changed" $\frac{dy}{dx}$ in terms of θ which must have come from calculus. If using $y = mx + c$, must reach as far as $c = \ldots$	M1
	$6x\sin\theta-5y\cos\theta=11\sin\theta\cos\theta$*	Correct completion to printed answer with no errors.	A1*
	Note that if the candidate uses e.g $y-5\sin\theta=-\frac{36y}{25x}\left(x-6\cos\theta\right)$ before introducing θ, the final mark can be withheld.		
			(5)
(b)	$b^2=a^2\left(1-e^2\right)\Rightarrow 25=36\left(1-e^2\right)\Rightarrow e^2=\frac{11}{36}$ or $e=\sqrt{\frac{11}{36}}$	Uses the correct eccentricity formula to obtain a value for e or e^2. Ignore ± values for e.	M1
	$y=0\Rightarrow x=\frac{11\cos\theta}{6}$ or $\frac{11\sin\theta\cos\theta}{6\sin\theta}$	Correct x coordinate for Q	B1
	$\left(\frac{OQ}{OR}=\right)\frac{11\cos\theta}{6}\times\frac{1}{6\cos\theta}$	Attempts $\frac{\text{their } OQ}{\text{their } OR}$. May be implied by their ratio.	M1
	$=\frac{11}{36}$	Correct completion with no errors to obtain $\frac{11}{36}$ both times.	A1
	Ignore any references to the foci or directrices but the final mark can be withheld if there are any incorrect statements such as e.g. using cos θ = 1 in their ratio.		
			(4)
			Total 9

Question Number	Scheme	Notes	Marks
3	$\cosh 2x \equiv 2\cosh^2 x - 1$		
	Note that exponentials must be used in (a)		
(a) Way 1	$\text{rhs} = 2\cosh^2 x - 1 = 2\left(\frac{e^x + e^{-x}}{2}\right)^2 - 1$	Substitutes the **correct** exponential form into the rhs	M1
	$= 2\left(\frac{e^{2x} + 2 + e^{-2x}}{4}\right) - 1$	**Squares** correctly to obtain an expression in e^{2x} and e^{-2x}. **Dependent on the previous mark.**	**d**M1
	$= \frac{e^{2x} + e^{-2x}}{2} + 1 - 1$		
	$= \frac{e^{2x} + e^{-2x}}{2} = \cosh 2x = \text{lhs}$*	Complete proof with no errors	A1*
			(3)
	(a) Way 2		
	$\text{lhs} = \cosh 2x = \frac{e^{2x} + e^{-2x}}{2}$	Substitutes the correct exponential form	M1
	$= 2\left(\frac{\left(e^x + e^{-x}\right)^2 - 2}{4}\right)$	**Completes the square** correctly to obtain an expression in e^x and e^{-x} **Dependent on the previous mark.**	**d**M1
	$2\left(\frac{e^x + e^{-x}}{2}\right)^2 - 1 = 2\cosh^2 x - 1 = \text{rhs}$*	Complete proof with no errors	A1*

(b) **Way 1**	$29\cosh x - 3\left(2\cosh^2 x - 1\right) = 38$	Substitutes the result from part (a)	M1
	$6\cosh^2 x - 29\cosh x + 35 = 0 \Rightarrow \cosh x = ...$	Forms a 3-term quadratic and attempt to solve for cosh x. You can apply the General Principles for solving a 3TQ if necessary.	M1
	$\cosh x = \frac{7}{3}$ or $\cosh x = \frac{5}{2}$	Both correct (or equivalent values)	A1
	$\cosh x = \alpha \Rightarrow x = \ln\left(\alpha + \sqrt{\alpha^2 - 1}\right)$ or $\cosh x = \alpha \Rightarrow x = \ln\left(\alpha - \sqrt{\alpha^2 - 1}\right)$ or $\frac{e^x + e^{-x}}{2} = \alpha \Rightarrow x = ...$	Uses the correct ln form for arcosh to find at least one value for x for $\alpha > 1$ or uses the correct exponential form for cosh and solves the resulting 3TQ in e^x to find at least one value for x for $\alpha > 1$	M1
	$x = \ln\left(\frac{7}{3} \pm \sqrt{\frac{40}{9}}\right)$ and $x = \ln\left(\frac{5}{2} \pm \sqrt{\frac{21}{4}}\right)$ Or equivalent exact forms e.g. $x = \ln\frac{7 \pm 2\sqrt{10}}{3}$ and $x = \ln\frac{5 \pm \sqrt{21}}{2}$ $x = \pm\ln\left(\frac{7 + 2\sqrt{10}}{3}\right)$ and $x = \pm\ln\left(\frac{5 + \sqrt{21}}{2}\right)$ $x = \ln\left(7 \pm 2\sqrt{10}\right) - \ln 3$ and $x = \ln\left(5 \pm \sqrt{21}\right) - \ln 2$ **A1: Any 2 of these 4 solutions. Penalise lack of brackets once where necessary, the first time it occurs and penalise lack of simplification once, the first time it occurs** e.g. $\ln\frac{5}{2} \pm \frac{\sqrt{21}}{2}$, $\ln\left(\frac{5}{2} \pm \sqrt{\left(\frac{5}{2}\right)^2 - 1}\right)$ **A1: All 4 correct**		A1A1
	Note that the decimal answers are, ±1.49…, ±1.56…,		
			(6)
			Total 9

	(b) Way 2		
	$29\left(\frac{e^{x}+e^{-x}}{2}\right)-3\left(\frac{e^{2x}+e^{-2x}}{2}\right)=38$ or $6\left(\frac{e^{x}+e^{-x}}{2}\right)^{2}-29\left(\frac{e^{x}+e^{-x}}{2}\right)+35=0$	Substitutes the correct exponential forms	M1
	$3e^{4x}-29e^{3x}+76e^{2x}-29e^{x}+3=0$	M1: Multiplies by e^{2x} or e^{-2x} to obtain a quartic in e^{x} or e^{-x} A1: Correct quartic in any form (not necessarily all on one side)	M1A1
	$\left(3e^{2x}-14e^{x}+3\right)\left(e^{2x}-5e^{x}+1\right)=0\Rightarrow x=...$	Solves their quartic to find at least one value for x	M1
	$x=\ln\left(\frac{7}{3}\pm\sqrt{\frac{40}{9}}\right)$ and $x=\ln\left(\frac{5}{2}\pm\sqrt{\frac{21}{4}}\right)$ Or equivalent exact forms e.g. $x=\ln\frac{7\pm2\sqrt{10}}{3}$ and $x=\ln\frac{5\pm\sqrt{21}}{2}$ $x=\pm\ln\left(\frac{7+2\sqrt{10}}{3}\right)$ and $x=\pm\ln\left(\frac{5+\sqrt{21}}{2}\right)$ $x=\ln\left(7\pm2\sqrt{10}\right)-\ln3$ and $x=\ln\left(5\pm\sqrt{21}\right)-\ln2$ e.g. $\ln\frac{5}{2}\pm\frac{\sqrt{21}}{2}$, $\ln\left(\frac{5}{2}\pm\sqrt{\left(\frac{5}{2}\right)^{2}-1}\right)$ **A1: All 4 correct**		A1A1

Question Number	Scheme	Notes	Marks
4	$\frac{\mathrm{d}x}{\mathrm{d}u}=2u$ or $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2}(x+2)^{-\frac{1}{2}}$	Or equivalent correct derivative **in any form**. May be implied by their substitution.	B1
	$\int\frac{(x+2)^{\frac{1}{2}}}{x+5}\,\mathrm{d}x=\int\frac{(u^2)^{\frac{1}{2}}}{u^2-2+5}2u(\mathrm{d}u)$ or $\int\frac{(x+2)^{\frac{1}{2}}}{x+5}\,\mathrm{d}x=\int\frac{(x+2)^{\frac{1}{2}}}{u^2-2+5}\times\frac{2}{(x+2)^{\frac{1}{2}}}(\mathrm{d}u)$	Complete substitution including their "dx". Allow the omission of "du" if it is implied by later work.	M1
	$=2\int\frac{u^2}{u^2+3}(\mathrm{d}u)$ or $\int\frac{2u^2}{u^2+3}(\mathrm{d}u)$	Correct integral	A1
	$(2)\int\frac{u^2}{u^2+3}\,\mathrm{d}u=(2)\int\left(1-\frac{3}{u^2+3}\right)\mathrm{d}u$	Splits the fraction into $A+\frac{B}{u^2+3}$	M1
	$=(2)\left[u-\frac{3}{\sqrt{3}}\arctan\frac{u}{\sqrt{3}}\right]$	A1: u A1: $-\frac{3}{\sqrt{3}}\arctan\frac{u}{\sqrt{3}}$	A1 A1
	$x=-1\Rightarrow u=1,\quad x=7\Rightarrow u=3$	Correct limits.	B1
	$=2\left[\left(3-\frac{3}{\sqrt{3}}\frac{\pi}{3}\right)-\left(1-\frac{3}{\sqrt{3}}\frac{\pi}{6}\right)\right]$	Substitutes u limits correctly into an expression of the form $\pm\alpha u\pm\beta\arctan(ku),\ \alpha,\beta\neq0$ and subtracts the right way round.	M1
	$=4-\frac{\sqrt{3}}{3}\pi$	Cao (oe)	A1
			(9)
			Total 9
	Alternative using substitution again for last 6 marks:		
	$u=\sqrt{3}\tan\theta\Rightarrow(2)\int\frac{u^2}{u^2+3}\,\mathrm{d}u=(2)\int\frac{3\tan^2\theta}{3\tan^2\theta+3}\sqrt{3}\sec^2\theta\,\mathrm{d}\theta$ Use of $u=\sqrt{3}\tan\theta$ and a complete substitution.		M1
	$=(2\sqrt{3})\int\tan^2\theta\,\mathrm{d}\theta=(2\sqrt{3})\int(\sec^2\theta-1)\,\mathrm{d}\theta$ $=(2\sqrt{3})[\tan\theta-\theta]$	A1: θ A1: $\tan\theta$	A1A1
	$u=1\Rightarrow\theta=\frac{\pi}{6},\ u=3\Rightarrow\theta=\frac{\pi}{3}$	Correct limits	B1
	$=2\sqrt{3}\left[\left(\sqrt{3}-\frac{\pi}{3}\right)-\left(\frac{1}{\sqrt{3}}-\frac{\pi}{6}\right)\right]$	Substitutes θ limits correctly into an expression of the form $\pm\alpha\tan\theta\pm\beta\theta,\ \alpha,\beta\neq0$ and subtracts the right way round.	M1
	$=4-\frac{\sqrt{3}}{3}\pi$	cao	A1

Question Number	Scheme	Notes	Marks
5.	$\Pi_1: x-2y-3z=5,\quad \Pi_2: 6x+y-4z=7$		
(a) Way 1	$\begin{pmatrix}1\\-2\\-3\end{pmatrix}\bullet\begin{pmatrix}6\\1\\-4\end{pmatrix}=6-2+12$	Attempts scalar product of normal vectors allowing one slip. May be implied by a value of 16.	M1
	$16=\sqrt{1^2+2^2+3^2}\sqrt{6^2+1^2+4^2}\cos\theta$ $\Rightarrow\cos\theta=...$	Complete attempt to find cos θ	M1
	$\cos\theta=\dfrac{16}{\sqrt{14}\sqrt{53}}\Rightarrow\theta=54^{\text{o}}$	Cao and do **not** isw. E.g. if they subsequently find 90 – 54 or 180 – 54, score A0. **Do not allow 54.0.**	A1
(a) Way 2	$\begin{pmatrix}1\\-2\\-3\end{pmatrix}\times\begin{pmatrix}6\\1\\-4\end{pmatrix}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&-2&-3\\6&1&-4\end{vmatrix}=\begin{pmatrix}11\\-14\\13\end{pmatrix}$	Attempts cross product of normal vectors. 2 components should be correct if there is no working.	M1
	$\sqrt{11^2+14^2+13^2}=\sqrt{1^2+2^2+3^2}\sqrt{6^2+1^2+4^2}\sin\theta$ $\Rightarrow\sin\theta=...$	Complete attempt to find sin θ	M1
	$\sin\theta=\dfrac{9\sqrt{6}}{\sqrt{14}\sqrt{53}}\Rightarrow\theta=54^{\text{o}}$	Cao and do **not** isw. E.g. if they subsequently find 90 – 54 or 180 – 54, score A0. **Do not allow 54.0.**	A1
			(3)
(b)	$\mathbf{PQ}=\begin{pmatrix}2\\3\\-1\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\-3\end{pmatrix}$ or $\begin{pmatrix}2+\lambda\\3-2\lambda\\-1-3\lambda\end{pmatrix}$	Attempt parametric form of **PQ** by using the point P and the normal to Π_1	M1
	$6(2+\lambda)+(3-2\lambda)-4(-1-3\lambda)=7$ $\Rightarrow\lambda=...$	Substitutes parametric form of **PQ** into the equation of Π_2 and solves for λ	M1
	$\lambda=-\dfrac{3}{4}\Rightarrow Q\ is\left(\dfrac{5}{4},\dfrac{9}{2},\dfrac{5}{4}\right)$	M1: Uses their value of λ in their **PQ** equation A1: Correct coordinates or vector.	M1A1
			(4)

(c)	$\begin{pmatrix}1\\-2\\-3\end{pmatrix}\times\begin{pmatrix}6\\1\\-4\end{pmatrix}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\1 & -2 & -3\\6 & 1 & -4\end{vmatrix}=\begin{pmatrix}11\\-14\\13\end{pmatrix}$	M1: Attempt cross product between normals A1: Correct normal vector (any multiple)	M1A1
	Alternative: $x-2y-3z=0,\quad 6x+y-4z=0: x=1 \Rightarrow y=-\frac{14}{11}, z=\frac{13}{11}$ $\Rightarrow \mathbf{n}=\begin{pmatrix}11\\-14\\13\end{pmatrix}$ M1: Solves $x-2y-3z=0,\quad 6x+y-4z=0$ to obtain values for x, y and z A1: Correct vector (or values)		
	$\begin{pmatrix}11\\-14\\13\end{pmatrix}\cdot\begin{pmatrix}\frac{5}{4}\\\frac{9}{2}\\\frac{5}{4}\end{pmatrix}=\ldots$ or $\begin{pmatrix}11\\-14\\13\end{pmatrix}\cdot\begin{pmatrix}2\\3\\-1\end{pmatrix}=\ldots$	Attempt scalar product between their normal and their **OQ** or **OP**. Must obtain a value.	M1
	$\mathbf{r}\cdot\begin{pmatrix}11\\-14\\13\end{pmatrix}=-33$	Any multiple e.g. $\mathbf{r}\cdot\begin{pmatrix}11k\\-14k\\13k\end{pmatrix}=-33k\quad(k\neq 0)$	A1
	Note that if they use the intersection with Π_1 $\left(\frac{17}{7},\frac{15}{7},\frac{-16}{7}\right)$ for Q allow all the marks to score in (c).		
			(4)
			Total 11

Question Number	Scheme	Notes	Marks
6(a)	$\det \mathbf{M} = 1\times(2-1)-k(-2+4)(+0)=1-2k$ * or e.g. $\det \mathbf{M} = (0)-1(1+4k)-1(-2-2k)=1-2k$ * or rule of Sarrus: $\det \mathbf{M} = 2-4k-1+2k=1-2k$ * Or e.g. $(1)\begin{vmatrix}-2 & 1\\ 1 & -1\end{vmatrix}-k\begin{vmatrix}2 & 1\\ -4 & -1\end{vmatrix}+0\begin{vmatrix}2 & -2\\ -4 & 1\end{vmatrix}$	M1: Correct attempt at determinant (at least 2 'elements' correct). May need to check as they might use a different row/column. A1: Obtains printed answer with no errors. If they use determinant notation as in the last example, then you must see at least one intermediate step before the printed answer e.g. minimally 1 - 2k + 0.	M1A1*
			(2)
(b)	$(\mathbf{M}^{\mathrm{T}})$ $\begin{pmatrix}1 & 2 & -4\\ k & -2 & 1\\ 0 & 1 & -1\end{pmatrix}$ or (minors) $\begin{pmatrix}1 & 2 & -6\\ -k & -1 & 1+4k\\ k & 1 & -2-2k\end{pmatrix}$ or (cofactors) $\begin{pmatrix}1 & -2 & -6\\ k & -1 & -1-4k\\ k & -1 & -2-2k\end{pmatrix}$		B1
	$\mathbf{M}^{-1}=\dfrac{1}{1-2k}\begin{pmatrix}1 & k & k\\ -2 & -1 & -1\\ -6 & -1-4k & -2-2k\end{pmatrix}$	M1: Full attempt at inverse ignoring determinant. Need to see all stages but allow numerical slips. A1: 2 correct rows or 2 correct columns including reciprocal of determinant A1: All correct including reciprocal of determinant	M1A1A1
			(4)
(c)	$l_2:(1+5\lambda)\mathbf{i}+(-2+2\lambda)\mathbf{j}+(3+\lambda)\mathbf{k}$	M1: Attempt l_2 in parametric form A1: Correct parametric form	M1A1
	$\frac{1}{1}\begin{pmatrix}1 & 0 & 0\\ -2 & -1 & -1\\ -6 & -1 & -2\end{pmatrix}\begin{pmatrix}1+5\lambda\\ -2+2\lambda\\ 3+\lambda\end{pmatrix}=\begin{pmatrix}1+5\lambda\\ -3-13\lambda\\ -10-34\lambda\end{pmatrix}$ or e.g. $\frac{1}{1}\begin{pmatrix}1 & 0 & 0\\ -2 & -1 & -1\\ -6 & -1 & -2\end{pmatrix}\begin{pmatrix}1 & 5\\ -2 & 2\\ 3 & 1\end{pmatrix}=\begin{pmatrix}1 & 5\\ -3 & -13\\ -10 & -34\end{pmatrix}$	M1: Puts k = 0 in their $\mathbf{M}^{-1}$ and multiplies this by their parametric form correctly. Or starts again to find the inverse and multiplies. A1: Correct parametric form for l_1 or correct matrix.	M1A1
	$\dfrac{x-1}{5}=\dfrac{y+3}{-13}=\dfrac{z+10}{-34}$ oe $\begin{matrix}a_1+b_1\lambda\\ a_2+b_2\lambda\\ a_3+b_3\lambda\end{matrix}\rightarrow\dfrac{x-a_1}{b_1}=\dfrac{y-a_2}{b_2}=\dfrac{z-a_3}{b_3}$	M1: Attempts cartesian form from their parametric l_1 **correctly**. **Dependent on both previous M's.** A1: A complete correct equation	**d**M1A1
	If their $\mathbf{M}^{-1}$ is incorrect in terms of k but by substituting k = 0, a correct answer is obtained in (c) allow a full recovery.		
			(6)
			Total 12

	(c) Way 2		
	$\mathbf{i}-2\mathbf{j}+3\mathbf{k}$ and $6\mathbf{i}+4\mathbf{k}$ are on l_2		
	$\mathbf{M}^{-1}(\mathbf{i}-2\mathbf{j}+3\mathbf{k})=\mathbf{i}-3\mathbf{j}-10\mathbf{k}$ $\mathbf{M}^{-1}(6\mathbf{i}+4\mathbf{k})=6\mathbf{i}-16\mathbf{j}-44\mathbf{k}$	M1: Attempt two points on l_1 A1: Two correct points on l_1	M1A1
	$\begin{pmatrix}6+5\lambda\\-16-13\lambda\\-44-34\lambda\end{pmatrix}$	M1: Uses their points to obtain parametric form for l_1 A1: Correct parametric form for l_1 or correct position and direction.	M1A1
	$\frac{x-6}{5}=\frac{y+16}{-13}=\frac{z+44}{-34}$ oe $\begin{matrix}a_1+b_1\lambda\\a_2+b_2\lambda\\a_3+b_3\lambda\end{matrix}\rightarrow\frac{x-a_1}{b_1}=\frac{y-a_2}{b_2}=\frac{z-a_3}{b_3}$	M1: Attempts cartesian form from their parametric l_1 **correctly**. **Dependent on both previous M's.** A1: A complete correct equation	**d**M1A1
	(c) Way 3		
	$\begin{pmatrix}1&0&0\\2&-2&1\\-4&1&-1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\-2\\3\end{pmatrix}\Rightarrow\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\-3\\-10\end{pmatrix}$	M1: Solves $\mathbf{M}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\-2\\3\end{pmatrix}\Rightarrow\begin{pmatrix}x\\y\\z\end{pmatrix}=...$ A1: **i** – 3**j** – 10**k.** Correct vector or values for x, y and z	M1A1
	$\begin{pmatrix}1&0&0\\2&-2&1\\-4&1&-1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}5\\2\\1\end{pmatrix}\Rightarrow\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}5\\-13\\-34\end{pmatrix}$	M1: Solves $\mathbf{M}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}5\\2\\1\end{pmatrix}\Rightarrow\begin{pmatrix}x\\y\\z\end{pmatrix}=...$ A1: 5**i** – 13**j** – 34**k.** Correct vector or values for x, y and z	M1A1
	$\frac{x-1}{5}=\frac{y+3}{-13}=\frac{z+10}{-34}$	M1: Attempts Cartesian form from their values **correctly**. **Dependent on both previous M's.** A1: A complete correct equation	**d**M1A1
	(c) Way 4		
	$l_2:(1+5\lambda)\mathbf{i}+(-2+2\lambda)\mathbf{j}+(3+\lambda)\mathbf{k}$	M1: Attempt l_2 in parametric form correctly A1: Correct	M1A1
	$\begin{pmatrix}1&0&0\\2&-2&1\\-4&1&-1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1+5\lambda\\-2+2\lambda\\3+\lambda\end{pmatrix}\Rightarrow\begin{matrix}x=1+5\lambda\\y=-3-13\lambda\\z=-10-34\lambda\end{matrix}$ M1: Uses $\mathbf{Mx}=l_2$ in parametric form A1: Correct expressions for x, y and z		M1A1
	$\frac{x-1}{5}=\frac{y+3}{-13}=\frac{z+10}{-34}$	M1: Attempts Cartesian form from their values **correctly**. **Dependent on both previous M's.** A1: A complete correct equation	**d**M1A1

Question Number	Scheme	Notes	Marks
7	$I_n = \int_0^{\ln 2} \cosh^n x \, dx$		
(a)	$I_n = \int \cosh^{n-1} x \cosh x \, dx$		
	$I_n = \int \cosh^{n-1} x \cosh x \, dx = \sinh x \cosh^{n-1} x - \int (n-1)\cosh^{n-2} x \sinh^2 x \, dx$ M1: Integration by parts in the correct direction. If the formula is quoted it must be correct otherwise look for an expression of the form $\pm \sinh x \cosh^{n-1} x \pm k \int \cosh^{n-2} x \sinh^2 x \, dx$ A1: Correct expression		M1A1
	$= \sinh x \cosh^{n-1} x - \int (n-1)\cosh^{n-2} x \left(\cosh^2 x - 1\right) dx$	Replaces $\sinh^2 x$ with $\pm\cosh^2 x \pm 1$ on the "integration part" to obtain an expression in cosh x only. **Dependent on the first method mark.**	**d**M1
	$= \sinh x \cosh^{n-1} x - (n-1)\int \cosh^n x \, dx + (n-1)\int \cosh^{n-2} x \, dx$		
	$= \sinh x \cosh^{n-1} x - (n-1) I_n + (n-1) I_{n-2}$	Introduces I_n and I_{n-2}. **Dependent on both previous method marks.**	**dd**M1
	$\left[\sinh x \cosh^{n-1} x\right]_0^{\ln 2} = \sinh(\ln 2)\cosh^{n-1}(\ln 2)(-0)$ $\left(= \left(\frac{3}{4}\right)\left(\frac{5}{4}\right)^{n-1}\right)$	Use of given limits on their $\sinh x \cosh^{n-1} x$. Does not need to be evaluated but note that $\cosh(\ln 2) = \frac{5}{4}, \sinh(\ln 2) = \frac{3}{4}$	M1
	$I_n = \frac{3 \times 5^{n-1}}{n \times 4^n} + \frac{(n-1)}{n} I_{n-2}$ *	cao	A1*
			(6)

	(a) Way 2		
	$I_n = \int \cosh^{n-2}x\cosh^2 x\,dx = \int \cosh^{n-2}x\,dx + \int \cosh^{n-2}x\sinh^2 x\,dx$ Writes $\cosh^n x$ as $\cosh^{n-2}x\cosh^2 x$ and uses $\sinh^2 x = \pm\cosh^2 x \pm 1$		M1
	$\int \cosh^{n-2}x\sinh^2 x\,dx = \left[\frac{\sinh x\cosh^{n-1}x}{n-1}\right] - \frac{1}{n-1}\int \cosh^n x\,dx$ M1: Integration by parts in the correct direction. If the formula is quoted it must be correct otherwise look for an expression of the form $p\sinh x\cosh^{n-1}x \pm q\int \cosh^n x\,dx$ A1: Correct expression		dM1A1
	$(n-1)I_n = (n-1)I_{n-2} + \left[\sinh x\cosh^{n-1}x\right] - I_n$	Introduces I_n and I_{n-2}. **Dependent on both previous method marks.**	**dd**M1
	$\left[\sinh x\cosh^{n-1}x\right]_0^{\ln 2} = \sinh(\ln 2)\cosh^{n-1}(\ln 2)(-0)$ $\left(= \left(\frac{3}{4}\right)\left(\frac{5}{4}\right)^{n-1}\right)$	Use of given limits on their $\sinh x\cosh^{n-1}x$. Does not need to be evaluated but note that $\cosh(\ln 2) = \frac{5}{4}, \sinh(\ln 2) = \frac{3}{4}$	M1
	$I_n = \frac{3\times 5^{n-1}}{n\times 4^n} + \frac{(n-1)}{n}I_{n-2}$ *	cao	A1*
	You can condone the occasional missing *x*, d*x* and limits along the way and "invisible" brackets may be recovered. **Do not allow e.g. an obvious sign error that gets "corrected" later – withhold the final A1 in such cases.**		

(b)	$I_4 = \frac{3\times5^3}{4\times4^4} + \frac{3}{4}I_2$ or $\frac{3\times a^3}{4\times b^4} + \frac{3}{4}I_2$	Correct first application of **their or the given** reduction formula	M1
	$= \frac{3\times5^3}{4\times4^4} + \frac{3}{4}\left(\frac{3\times5}{2\times4^2} + \frac{1}{2}I_0\right)$ or $\frac{3\times a^3}{4\times b^4} + \frac{3}{4}\left(\frac{3\times a}{2\times b^2} + \frac{1}{2}I_0\right)$ Correct second application of **their or the given** reduction formula **that is consistent with the formula used in the first application** to obtain I_4 in terms of I_0		M1
	$I_0 = \ln 2$		B1
	$I_4 = \frac{735}{1024} + \frac{3}{8}\ln 2$	Cao (Allow equivalent exact forms e.g. may be factorised but fractions must be collected)	A1
	Note that candidates may work from the "other end" e.g. $I_0 = \ln 2$ B1 $I_2 = \frac{3\times5}{2\times4^2} + \frac{1}{2}I_0$ M1 I_2 *in terms of* I_0 $I_4 = \frac{3\times5^3}{4\times4^4} + \frac{3}{4}\left(\frac{3\times5}{2\times4^2} + \frac{1}{2}I_0\right)$ M1 I_4 *in terms of* I_0 $I_4 = \frac{735}{1024} + \frac{3}{8}\ln 2$ A1 Cao (Allow equivalent exact forms e.g. may be factorised but fractions must be collected)		
			(4)
(b) Way 2	$I_4 = \frac{3\times5^3}{4\times4^4} + \frac{3}{4}I_2$	Correct application of their reduction formula	M1
	$I_2 = \int_0^{\ln 2} \cosh^2 x\,dx = \int_0^{\ln 2}\left(\frac{1}{2} + \frac{1}{2}\cosh 2x\right)dx$		
	$\int\left(\frac{1}{2} + \frac{1}{2}\cosh 2x\right)dx = \frac{x}{2} + \frac{1}{4}\sinh 2x$	Correct integration	B1
	$I_2 = \left[\frac{x}{2} + \frac{1}{4}\sinh 2x\right]_0^{\ln 2} = \frac{1}{2}\ln 2 + \frac{15}{32}$	Correct use of limits on an expression of the form $\alpha x + \beta\sinh 2x$	M1
	$I_4 = \frac{3\times5^3}{4\times4^4} + \frac{3}{4}\left(\frac{1}{2}\ln 2 + \frac{15}{32}\right)$		
	$I_4 = \frac{735}{1024} + \frac{3}{8}\ln 2$	Cao (Allow equivalent exact forms e.g. may be factorised but fractions must be collected)	A1

(b) Way 3	$I_4 = \int_0^{\ln 2} \cosh^4 x \, dx = \int_0^{\ln 2} \left(\frac{1}{2}+\frac{1}{2}\cosh 2x\right)^2 dx$		
	$\int_0^{\ln 2} \left(\frac{1}{4}+\frac{1}{2}\cosh 2x+\frac{1}{4}\cosh^2 2x\right) dx$	$\cosh^4 x = \frac{1}{4}+\frac{1}{2}\cosh 2x+\frac{1}{4}\cosh^2 2x$	B1
	$\frac{1}{4}\int_0^{\ln 2} \left(1+2\cosh 2x+\frac{1}{2}(1+\cosh 4x)\right) dx$	$\cosh^2 2x = \pm\frac{1}{2}\pm\frac{1}{2}\cosh 4x$ and attempt to integrate	M1
	$\frac{1}{4}\left[\frac{3x}{2}+\sinh 2x+\frac{1}{8}\sinh 4x\right]_0^{\ln 2}$	Correct use of correct limits	M1
	$I_4 = \frac{735}{1024}+\frac{3}{8}\ln 2$	Cao (Allow equivalent exact forms e.g. may be factorised but fractions must be collected)	A1

(b) Way 4	$I_4 = \int_0^{\ln 2} \cosh^4 x \, dx = \int_0^{\ln 2} \left(\frac{e^x+e^{-x}}{2}\right)^4 dx$		
	$= \int_0^{\ln 2} \left(\frac{e^x+e^{-x}}{2}\right)^4 dx = \left(\frac{1}{16}\right)\int_0^{\ln 2} \left(e^{4x}+4e^{2x}+6+4e^{-2x}+e^{-4x}\right) dx$ Correct expansion		B1
	$= \left(\frac{1}{16}\right)\left[\frac{e^{4x}}{4}+2e^{2x}+6x-2e^{-2x}-\frac{e^{-4x}}{4}\right]_0^{\ln 2}$	Attempts to integrate their expansion	M1
	$\left(\frac{1}{16}\right)\left[\left(4+8+6\ln 2-\frac{1}{2}-\frac{1}{64}\right)-(0)\right]_0^{\ln 2}$	Correct use of correct limits	M1
	$I_4 = \frac{735}{1024}+\frac{3}{8}\ln 2$	Cao (Allow equivalent exact forms e.g. may be factorised but fractions must be collected)	A1
			Total 10

Question Number	Scheme	Notes	Marks
8(a) Way 1	$y=\ln\left(\frac{e^x+1}{e^x-1}\right)=\ln\left(e^x+1\right)-\ln\left(e^x-1\right)\Rightarrow\frac{dy}{dx}=\frac{e^x}{e^x+1}-\frac{e^x}{e^x-1}$ M1: Uses correct log rule and attempts derivative using chain rule A1: Correct Derivative		M1A1
	$=\frac{e^{2x}-e^x-e^{2x}-e^x}{e^{2x}-1}=\frac{-2e^x}{e^{2x}-1}$ *	dM1: Attempt single fraction and uses $(e^x-1)(e^x+1)=e^{2x}-1$. **Dependent on the first method mark.** A1: Completes correctly with no errors	**d**M1A1*
			(4)
	(a) Way 2		
	$\frac{dy}{dx}=\frac{e^x-1}{e^x+1}\left(\frac{e^x\left(e^x-1\right)-e^x\left(e^x+1\right)}{\left(e^x-1\right)^2}\right)$ Or $\frac{dy}{dx}=\frac{e^x-1}{e^x+1}\left(e^x\left(e^x-1\right)^{-1}-e^x\left(e^x+1\right)\left(e^x-1\right)^{-2}\right)$	M1: Uses chain and quotient or product rules A1: Correct derivative	M1A1
	$=\frac{1}{e^x+1}\left(-\frac{2e^x}{e^x-1}\right)=-\frac{2e^x}{e^{2x}-1}$ *	dM1: Cancels e^x-1 and uses $(e^x-1)(e^x+1)=e^{2x}-1$. **Dependent on the first method mark.** A1: Completes correctly with no errors	**d**M1A1*
	(a) Way 3		
	$y=\ln\left(\frac{e^x+1}{e^x-1}\right)\Rightarrow e^y=\frac{e^x+1}{e^x-1}\Rightarrow e^y\frac{dy}{dx}=\frac{e^x\left(e^x-1\right)-e^x\left(e^x+1\right)}{\left(e^x-1\right)^2}$ M1: Removes logs correctly and differentiates implicitly using chain and quotient rules A1: Correct differentiation		M1A1
	$\frac{dy}{dx}=-\frac{2e^x}{\left(e^x-1\right)^2}\times\frac{e^x-1}{e^x+1}=-\frac{2e^x}{e^{2x}-1}$ *	dM1: Divides by e^y in terms of *x*. **Dependent on the first method mark.** A1: Completes correctly with no errors	**d**M1A1

	(a) Way 4		
	$y=\ln\left(\frac{e^x+1}{e^x-1}\right)=\ln\left(\coth\frac{1}{2}x\right)\Rightarrow\frac{dy}{dx}=\frac{1}{\coth\frac{1}{2}x}\times-\frac{1}{2}\text{cosech}^2\frac{1}{2}x$ M1: Writes as $\ln\left(\coth\frac{1}{2}x\right)$ and differentiates using the chain rule A1: Correct differentiation		M1A1
	$=\left(\frac{e^x-1}{e^x+1}\right)\times\frac{-2e^x}{\left(e^x-1\right)^2}=-\frac{2e^x}{e^{2x}-1}$	dM1: Substitutes the correct exponential forms. **Dependent on the first method mark.** A1: Completes correctly with no errors	**d**M1A1

	(a) Way 5		
	$\text{artanh}x=\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)\Rightarrow y=2\text{artanh}\left(e^{-x}\right)$ $\frac{dy}{dx}=\frac{2}{1-\left(e^{-x}\right)^2}\times-e^{-x}$ M1: Writes y correctly in terms of artanh and attempts to differentiate using the chain rule A1: Correct differentiation		M1A1
	$\frac{dy}{dx}=\frac{-2e^{-x}}{1-e^{-2x}}=\frac{-2e^x}{e^{2x}-1}$ *	dM1: Multiplies numerator and denominator by e^{2x}. **Dependent on the first method mark.** A1: Completes correctly with no errors	**d**M1A1

	(a) Way 6		
	$y=\ln\left(1+\frac{2}{e^x-1}\right)\Rightarrow\frac{dy}{dx}=\frac{1}{1+2\left(e^x-1\right)^{-1}}\times-2e^x\left(e^x-1\right)^{-2}$ M1: Writes $\frac{e^x+1}{e^x-1}$ as $1+\frac{2}{e^x-1}$ and differentiates using the chain rule A1: Correct differentiation		M1A1
	$=\frac{-2e^x}{\left(e^x-1\right)^2+2\left(e^x-1\right)}=\frac{-2e^x}{e^{2x}-1}$	dM1: Multiplies denominator by $\left(e^x-1\right)^2$. **Dependent on the first method mark.** A1: Completes correctly with no errors	**d**M1A1

(b)	$L=\int\sqrt{\left(1+\left(\pm\frac{2e^x}{e^{2x}-1}\right)^2\right)}\,dx$	Uses the correct arc length formula with ± the result from part (a). **Note that we condone the omission of the minus sign on the fraction)**	M1
	$=\int\sqrt{\left(\frac{e^{4x}-2e^{2x}+1+4e^{2x}}{\left(e^{2x}-1\right)^2}\right)}\,dx$	Attempt single fraction. **Dependent on the first method mark**.	**d**M1
	Note that, for the first 2 marks, the candidate may just work on the integrand e.g. $\sqrt{\left(1+\left(\pm\frac{2e^x}{e^{2x}-1}\right)^2\right)}=\sqrt{\left(\frac{e^{4x}-2e^{2x}+1+4e^{2x}}{\left(e^{2x}-1\right)^2}\right)}$ **Would score the first 2 marks.**		
	$L=\int\sqrt{\frac{\left(e^{2x}+1\right)^2}{\left(e^{2x}-1\right)^2}}\,dx=\int\frac{\left(e^{2x}+1\right)}{\left(e^{2x}-1\right)}\,dx$	Correct integral with square root removed. No limits required.	A1
	$=\int\coth x\,dx,\ \int\frac{e^x+e^{-x}}{e^x-e^{-x}}\,dx,\ \int 1+\frac{2e^{-2x}}{1-e^{-2x}}\,dx,\ \frac{1}{2}\int\frac{2}{u}-\frac{1}{u+1}\,du\left(u=e^{2x}-1\right)$ $\frac{1}{2}\int\frac{2}{u-1}-\frac{1}{u}\,du\left(u=e^{2x}\right),\ \int\frac{1}{u+1}+\frac{1}{u-1}-\frac{1}{u}\,du\left(u=e^x\right),$, $\frac{1}{2}\int\frac{2}{u-2}-\frac{1}{u-1}\,du\left(u=e^{2x}+1\right)$		
	$=\left[\ln\sinh x\right],\left[\ln\left(e^x-e^{-x}\right)\right],\left[x+\ln\left(1-e^{-2x}\right)\right],\left[\ln u-\ln\sqrt{(1+u)}\right],$ $\left[\ln(u-1)-\ln\sqrt{u}\right],\left[\ln\frac{\left(u^2-1\right)}{u}\right],\left[\ln(u-2)-\ln\sqrt{u-1}\right]$ Correct integration		A1
	$=\ln\sinh(\ln 3)-\ln\sinh(\ln 2)$ $\left(=\ln\frac{4}{3}-\ln\frac{3}{4}\right)$	Correct use of limits e.g. ln3 and ln2 for x and e.g. 3 and 8 if $u=e^{2x}-1$. They must be the correct limits for their method if they use substitution. **Dependent on both previous method marks.**	**dd**M1
	$=\ln\frac{16}{9}$	cao	A1
			(6)
			Total 10

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