

2. (a) Given that $y = x \arcsin x$, $0 \leq x \leq 1$, find

(i) an expression for $\frac{dy}{dx}$,

(ii) the exact value of $\frac{dy}{dx}$ when $x = \frac{1}{2}$.

(3)

(b) Given that $y = \arctan(3e^{2x})$, show that

$$\frac{dy}{dx} = \frac{3}{5 \cosh 2x + 4 \sinh 2x}$$

(5)



4.

$$I_n = \int_1^e x^2 (\ln x)^n dx, \quad n \geq 0$$

(a) Prove that, for $n \geq 1$,

$$I_n = \frac{e^3}{3} - \frac{n}{3} I_{n-1} \quad (4)$$

(b) Find the exact value of I_3 . (4)



5. The curve C_1 has equation $y = 3\sinh 2x$, and the curve C_2 has equation $y = 13 - 3e^{2x}$.
- (a) Sketch the graph of the curves C_1 and C_2 on one set of axes, giving the equation of any asymptote and the coordinates of points where the curves cross the axes. **(4)**
- (b) Solve the equation $3\sinh 2x = 13 - 3e^{2x}$, giving your answer in the form $\frac{1}{2}\ln k$, where k is an integer. **(5)**



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Question 5 continued

Lined area for writing the answer to Question 5.



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7. The matrix \mathbf{M} is given by

$$\mathbf{M} = \begin{pmatrix} k & -1 & 1 \\ 1 & 0 & -1 \\ 3 & -2 & 1 \end{pmatrix}, \quad k \neq 1$$

(a) Show that $\det \mathbf{M} = 2 - 2k$.

(2)

(b) Find \mathbf{M}^{-1} , in terms of k .

(5)

The straight line l_1 is mapped onto the straight line l_2 by the transformation represented

by the matrix $\begin{pmatrix} 2 & -1 & 1 \\ 1 & 0 & -1 \\ 3 & -2 & 1 \end{pmatrix}$.

The equation of l_2 is $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = 0$, where $\mathbf{a} = 4\mathbf{i} + \mathbf{j} + 7\mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} + \mathbf{j} + 3\mathbf{k}$.

(c) Find a vector equation for the line l_1 .

(5)



8. The hyperbola H has equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

(a) Use calculus to show that the equation of the tangent to H at the point $(a \cosh \theta, b \sinh \theta)$ may be written in the form

$$xb \cosh \theta - ya \sinh \theta = ab \tag{4}$$

The line l_1 is the tangent to H at the point $(a \cosh \theta, b \sinh \theta)$, $\theta \neq 0$. Given that l_1 meets the x -axis at the point P ,

(b) find, in terms of a and θ , the coordinates of P . **(2)**

The line l_2 is the tangent to H at the point $(a, 0)$. Given that l_1 and l_2 meet at the point Q ,

(c) find, in terms of a, b and θ , the coordinates of Q . **(2)**

(d) Show that, as θ varies, the locus of the mid-point of PQ has equation

$$x(4y^2 + b^2) = ab^2 \tag{6}$$
