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June 2009
6669 Further Pure Mathematics FP3 (new)
Mark Scheme

Question Number	Scheme	Marks
Q1	$\frac{7}{\cosh x} - \frac{\sinh x}{\cosh x} = 5 \Rightarrow \frac{14}{e^x + e^{-x}} - \frac{(e^x - e^{-x})}{e^x + e^{-x}} = 5$	M1
	$\therefore 14 - (e^x - e^{-x}) = 5(e^x + e^{-x}) \Rightarrow 6e^x - 14 + 4e^{-x} = 0$	A1
	$\therefore 3e^{2x} - 7e^x + 2 = 0 \Rightarrow (3e^x - 1)(e^x - 2) = 0$	M1
	$\therefore e^x = \frac{1}{3}$ or 2	A1
	$x = \ln(\frac{1}{3})$ or $\ln 2$	B1ft **[5]**
Alternative (i)	Write 7 – sinh$x$ = 5cosh$x$, then use exponential substitution	M1
	$7 - \frac{1}{2}(e^x - e^{-x}) = \frac{5}{2}(e^x + e^{-x})$	
	Then proceed as method above.	
Alternative (ii)	$(7\text{sech}x - 5)^2 = \tanh^2 x = 1 - \text{sech}^2 x$	M1
	$50\text{sech}^2 x - 70\text{sech}x + 24 = 0$	A1
	$2(5\text{sech}x - 3)(5\text{sech}x - 4) = 0$	M1
	sech$x$= $\frac{3}{5}$ or sech$x$= $\frac{4}{5}$	A1
	$x = \ln(\frac{1}{3})$ or $\ln 2$	B1ft
Q2 (a)	$\mathbf{b} \times \mathbf{c} = 0\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}$	M1 A1 A1 (3)
(b)	$\mathbf{a}.(\mathbf{b} \times \mathbf{c}) = 0 + 5 = 5$	M1 A1 ft (2)
(c)	Area of triangle OBC = $\frac{1}{2}\lvert 5\mathbf{j} + 5\mathbf{k}\rvert = \frac{5}{2}\sqrt{2}$	M1 A1 (2)
(d)	Volume of tetrahedron = $\frac{1}{6} \times 5 = \frac{5}{6}$	B1 ft (1) **[8]**

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Q3 (a)	$\begin{vmatrix} 6-\lambda & 1 & -1 \\ 0 & 7-\lambda & 0 \\ 3 & -1 & 2-\lambda \end{vmatrix} = 0 \quad \therefore (6-\lambda)(7-\lambda)(2-\lambda)+3(7-\lambda)=0$	M1
	$(7-\lambda)=0$ verifies $\lambda=7$ is an eigenvalue (can be seen anywhere)	M1
	$\therefore (7-\lambda)\{12-8\lambda+\lambda^2+3\}=0 \quad \therefore (7-\lambda)\{\lambda^2-8\lambda+15\}=0$	A1
	$\therefore (7-\lambda)(\lambda-5)(\lambda-3)=0$ and 3 and 5 are the other two eigenvalues	M1 A1 (5)
(b)	Set $\begin{pmatrix} 6 & 1 & -1 \\ 0 & 7 & 0 \\ 3 & -1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 7\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ or $\begin{pmatrix} -1 & 1 & -1 \\ 0 & 0 & 0 \\ 3 & -1 & -5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	M1
	Solve $-x+y-z=0$ and $3x-y-5z=0$ to obtain $x = 3z$ or $y = 4z$ and a second equation which can contain 3 variables	M1 A1
	Obtain eigenvector as 3**i** + 4**j** + **k** (or multiple)	A1 (4) **[9]**

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Q4 (a)	$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} \times \frac{1}{\sqrt{1+(\sqrt{x})^2}}$	B1, M1
	$\therefore \frac{dy}{dx} = \frac{\frac{1}{2}x^{-\frac{1}{2}}}{\sqrt{1+x}} \quad \left(= \frac{1}{2\sqrt{x(1+x)}}\right)$	A1 (3)
(b)	$\therefore \int_{\frac{1}{4}}^{4} \frac{1}{\sqrt{x(x+1)}}dx = \left[2\text{ar}\sinh\sqrt{x}\right]_{\frac{1}{4}}^{4}$	M1
	$= \left[2\text{ar}\sinh 2 - 2\text{ar}\sinh(\frac{1}{2})\right]$	M1
	$= \left[2\ln(2+\sqrt{5})\right] - \left[2\ln(\frac{1}{2}+\sqrt{\frac{5}{4}})\right]$	M1
	$2\ln\frac{(2+\sqrt{5})}{(\frac{1}{2}+\sqrt{(\frac{5}{4})})} = 2\ln\frac{2(2+\sqrt{5})}{(1+\sqrt{5})} = 2\ln\frac{2(\sqrt{5}+2)(\sqrt{5}-1)}{(\sqrt{5}+1)(\sqrt{5}-1)} = 2\ln\frac{(3+\sqrt{5})}{2}$	M1
	$= \ln\frac{(3+\sqrt{5})(3+\sqrt{5})}{4} = \ln\frac{14+6\sqrt{5}}{4} = \ln\left(\frac{7}{2}+\frac{3\sqrt{5}}{2}\right)$	A1 A1 (6) **[9]**
Alternative (i) for part (a)	Use $\sinh y = \sqrt{x}$ and state $\cosh y\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$	B1
	$\therefore \frac{dy}{dx} = \frac{\frac{1}{2}x^{-\frac{1}{2}}}{\sqrt{1+\sinh^2 y}} = \frac{\frac{1}{2}x^{-\frac{1}{2}}}{\sqrt{\left(1+\left(\sqrt{x}\right)^2\right)}}$	M1
	$\therefore \frac{dy}{dx} = \frac{\frac{1}{2}x^{-\frac{1}{2}}}{\sqrt{1+x}} \quad \left(= \frac{1}{2\sqrt{x(1+x)}}\right)$	A1 (3)
Alternative (i) for part (b)	Use $x = \tan^2\theta$, $\frac{dx}{d\theta} = 2\tan\theta\sec^2\theta$ to give $2\int \sec\theta d\theta = \left[2\ln(\sec\theta+\tan\theta)\right]$	M1
	$= \left[2\ln(\sec\theta+\tan\theta)\right]_{\tan\theta=\frac{1}{2}}^{\tan\theta=2}$ i.e. use of limits	M1
	then proceed as before from line 3 of scheme	
Alternative (ii) for part (b)	Use $\int \frac{1}{\sqrt{[(x+\frac{1}{2})^2-\frac{1}{4}]}}dx = \text{arcosh}\frac{x+\frac{1}{2}}{\frac{1}{2}}$	M1
	$= \left[\text{arcosh}\, 9 - \text{arcosh}(\frac{3}{2})\right]$	M1
	$= \left[\ln(9+\sqrt{80})\right] - \left[\ln(\frac{3}{2}+\frac{1}{2}\sqrt{5})\right]$	M1
	$= \ln\frac{(9+\sqrt{80})}{(\frac{3}{2}+\frac{1}{2}\sqrt{5})} = \ln\frac{2(9+\sqrt{80})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})},$	M1
	$= \ln\frac{2(9+4\sqrt{5})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})} = \ln\left(\frac{7}{2}+\frac{3\sqrt{5}}{2}\right)$	A1 A1 (6) **[9]**

Question Number	Scheme	Marks
Q5 (a)	$-(25-x^2)^{\frac{1}{2}}$ (+c)	M1A1 (2)
(b)	$I_n = \int x^{n-1}.\frac{x}{\sqrt{(25-x^2)}}\mathrm{d}x = -x^{n-1}\sqrt{25-x^2} + \int (n-1)x^{n-2}\sqrt{(25-x^2}\mathrm{d}x$	M1 A1ft
	$I_n = \left[-x^{n-1}\sqrt{25-x^2}\right]_0^5 + \int_0^5 \frac{(n-1)x^{n-2}(25-x^2)}{\sqrt{(25-x^2)}}\mathrm{d}x$	M1
	$I_n = 0 + 25(n\text{-}1)\ I_{n-2}\ \text{-}\ (n\text{-}1)\ I_n$	M1
	$\therefore nI_n = 25(n-1)I_{n-2}$ and so $I_n = \frac{25(n-1)}{n}I_{n-2}$ ✳	A1 (5)
(c)	$I_0 = \int_0^5 \frac{1}{\sqrt{(25-x^2}}\mathrm{d}x = \left[\arcsin(\tfrac{x}{5})\right]_0^5 = \frac{\pi}{2}$	M1 A1
	$I_4 = \frac{25\times3}{4}\times\frac{25\times1}{2}I_0 = \frac{1875}{16}\pi$	M1 A1 (4)
		[11]
Alternative for (b)	Using substitution $x = 5\sin\theta$	
	$I_n = 5^n\int_0^{\frac{\pi}{2}}\sin^n\theta\mathrm{d}\theta = \left[-5^n\sin^{n-1}\theta\cos\theta\right]_0^{\frac{\pi}{2}} + 5^n(n-1)\int_0^{\frac{\pi}{2}}\sin^{n-2}\theta\cos^2\theta\mathrm{d}\theta$	M1A1
	$= \left[-5^n\sin^{n-1}\theta\cos\theta\right]_0^{\frac{\pi}{2}} + 5^n(n-1)\int_0^{\frac{\pi}{2}}\sin^{n-2}\theta(1-\sin^2\theta)\mathrm{d}\theta$	M1
	$I_n = 0 + 25(n\text{-}1)\ I_{n-2}\ \text{-}\ (n\text{-}1)\ I_n$	M1
	$\therefore nI_n = 25(n-1)I_{n-2}$ and so $I_n = \frac{25(n-1)}{n}I_{n-2}$ ✳	A1
	(need to see that $I_{n-2} = 5^{n-2}\int_0^{\frac{\pi}{2}}\sin^{n-2}\theta\mathrm{d}\theta$ for final A1)	(5)

Question Number	Scheme	Marks
Q6 (a)	$\frac{x^2}{a^2}-\frac{(mx+c)^2}{b^2}=1$ and so $b^2x^2-a^2(mx+c)^2=a^2b^2$	M1
	$\therefore\ (b^2-a^2m^2)x^2-2a^2mcx-a^2(c^2+b^2)=0$ Or $(a^2m^2-b^2)x^2+2a^2mcx+a^2(c^2+b^2)=0$ ✳	A1 (2)
(b)	$(2a^2mc)^2=4(a^2m^2-b^2)\times a^2(c^2+b^2)$	M1
	$4a^4m^2c^2=-4a^2(b^2c^2+b^4-a^2m^2c^2-a^2m^2b^2)$	
	$c^2=a^2m^2-b^2$ or $a^2m^2=b^2+c^2$ ✳	A1 (2)
(c)	Substitute (1, 4) into $y = mx+c$ to give $4 = m + c$ and	
	Substitute a = 5 and b = 4 into $c^2=a^2m^2-b^2$ to give $c^2=25m^2-16$	B1
	Solve simultaneous equations to eliminate m or c : $(4-m)^2=25m^2-16$	M1
	To obtain $24m^2+8m-32=0$	A1
	Solve to obtain $8(3m+4)(m-1)=0......m=..$or...	M1
	$m=1$ or $-\frac{4}{3}$	A1
	Substitute to get $c=3$ or $\frac{16}{3}$	M1
	Lines are $y=x+3$ and $3y+4x=16$	A1 (7) **[11]**

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Q7 (a)	If the lines meet, $-1+3\lambda=-4+3\mu$ and $2+4\lambda=2\mu$	M1
	Solve to give $\lambda=0$ $(\mu=1$ but this need not be seen$)$.	M1 A1
	Also $1-\lambda=\alpha$ and so $\alpha=1$.	B1 (4)
(b)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 3 & 4 \\ 0 & 3 & 2 \end{vmatrix}$ = -6**i**+2**j**-3**k** is perpendicular to both lines and hence to the plane	M1 A1
	The plane has equation **r.n**=**a.n ,** which is –6x + 2y - 3z = -14,	M1
	i.e. –6x + 2y - 3z +14 = 0.	A1 o.a.e. (4)
OR (b)	**Alternative scheme**	
	Use (1, -1,2) and (α ,-4,0) in equation $ax+by+cz+d=0$	M1
	And third point so three equations, and attempt to solve	M1
	Obtain $6x-2y+3z=$	A1
	$(6x-2y+3z)-14=0$	A1 o.a.e. (4)
(c)	$(\mathbf{a}_1 - \mathbf{a}_2) = \mathbf{i} - 3\mathbf{j} - 2\mathbf{k}$	M1
	Use formula $\dfrac{(\mathbf{a}_1-\mathbf{a}_2)\bullet\mathbf{n}}{\lvert\mathbf{n}\rvert} = \dfrac{(\mathbf{i}-3\mathbf{j}-2\mathbf{k})\bullet(-6\mathbf{i}+2\mathbf{j}-3\mathbf{k})}{\sqrt{(36+4+9)}} = \left(\dfrac{-6}{7}\right)$	M1
	Distance is $\dfrac{6}{7}$	A1 (3) **[11]**

Question Number	Scheme	Marks
Q8 (a)	$\frac{dx}{d\theta} = -3\sin\theta,\ \ \frac{dy}{d\theta} = 5\cos\theta$	B1
	so S = $2\pi\int 5\sin\theta\sqrt{(-3\sin\theta)^2+(5\cos\theta)^2}\,d\theta$	M1
	$\therefore S = \quad 2\pi\int 5\sin\theta\sqrt{9-9\cos^2\theta+25\cos^2\theta}\,d\theta$	M1
	Let $c = \cos\theta,\ \frac{dc}{d\theta} = -\sin\theta,$ limits 0 and $\frac{\pi}{2}$ become 1 and 0	M1
	So $S = k\pi\int_0^{\alpha}\sqrt{16c^2+9}\,dc$, where k = 10, and α is 1	A1, A1 (6)
(b)	Let $c = \frac{3}{4}\sinh u$. Then $\frac{dc}{du} = \frac{3}{4}\cosh u$	M1
	So $S = k\pi\int_?^?\sqrt{9\sinh^2 u+9}\ \frac{3}{4}\cosh u\,du$	A1
	$= k\pi\int_?^?\ \frac{9}{4}\cosh^2 u\,du = k\pi\int_?^?\ \frac{9}{8}(\cosh 2u+1)du$	M1
	$= k\pi\left[\frac{9}{16}\sinh 2u+\frac{9}{8}u\right]_0^d$	A1
	$= \frac{45\pi}{4}\left[\frac{20}{9}+\ln 3\right]$ i.e. $\underline{117}$	B1 (5)
		[11]