A-LEVEL
Mathematics

Further Pure3 – MFP3
Mark scheme

6360
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Version/Stage: Final Mark Scheme V1

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
	DO NOT ALLOW ANY MISREADS IN THIS QUESTION			
(a)	$y(2.05) = y(2) + 0.05\left(\frac{2+5^2}{2}\right)$	**M1**		
	$= 5 + 0.05 \times 13.5$ $= 5.675$	**A1**	**2**	OE
(b)	$y(2.1) = y(2) + 2 \times 0.05\,\mathrm{f}[2.05, y(2.05)]$	**M1**		
	$= 5 + 2 \times 0.05 \times \left(\frac{2.05 + 5.675^2}{2.05}\right)$	**A1F**		PI Ft on c's **(a)** answer.
	$= 6.67$ to 3 sf	**A1**	**3**	CAO Must be 6.67
	Total		**5**	
	(b) For the PI if line missing, check to see if evaluation matches $5.1 + \frac{2}{41} \times [\text{answer (a)}]^2$ to at least 3sf			

Q2	Solution	Mark	Total	Comment
	I.F. $\mathrm{e}^{\int \tan x\,\mathrm{d}x}$	**M1**		
	$= \mathrm{e}^{\ln \sec x}$	**A1**		OE eg $\mathrm{e}^{-\ln \cos x}$
	$= \sec x$	**A1F**		OE Only ft sign error in integrating tan x.
	$\sec x \frac{\mathrm{d}y}{\mathrm{d}x} + \sec x(\tan x)y = \tan^3 x \sec^2 x$			
	$\frac{\mathrm{d}}{\mathrm{d}x}[y \sec x] = \tan^3 x \sec^2 x$	**M1**		LHS as $\frac{\mathrm{d}}{\mathrm{d}x}[y \times \text{candidate's IF}]$ PI
	$y \sec x = \int \tan^3 x \sec^2 x\ (\mathrm{d}x)$	**A1**		
	$y \sec x = \int t^3\ \mathrm{d}t$	**m1**		PI OE eg $y \sec x = \int \left(\frac{1}{u^3} - \frac{1}{u^5}\right)\mathrm{d}u$, where $u = \cos x$
	$y \sec x = \frac{1}{4}\tan^4 x\ (+c)$	**A1**		
	$2\sec\frac{\pi}{3} = \frac{1}{4}\tan^4\frac{\pi}{3} + c;\ \ 4 = \frac{9}{4} + c$	**m1**		Dep on prev MMm. Correct boundary condition applied to obtain an eqn in c with correct exact value for either $\sec\frac{\pi}{3}$ or $\tan^4\frac{\pi}{3}$ used
	$y \sec x = \frac{1}{4}\tan^4 x + \frac{7}{4}$			
	$y = \frac{\cos x}{4}(7 + \tan^4 x)$	**A1**	**9**	ACF
	Total		**9**	
	Condone answer left in a 'correct' form different to y= f(x), eg $4y \sec x = \tan^4 x + 7$.			

Q3	Solution	Mark	Total	Comment
(a)(i)	$\ln(1+2x)=2x-\frac{(2x)^2}{2}+\frac{(2x)^3}{3}-\frac{(2x)^4}{4}\ldots$ $=2x-2x^2+\frac{8}{3}x^3-4x^4\ldots$	B1	1	ACF Condone correct unsimplified
(a)(ii)	$\ln[(1+2x)(1-2x)]=\ln(1+2x)+\ln(1-2x)$	M1		$\ln(1+2x)+\ln(1-2x)$ PI {or $\ln(1-4x^2)=-4x^2-\frac{(-4x^2)^2}{2}\ldots$} PI
	$=-4x^2-8x^4\ldots\ldots$	A1		CSO Must be simplified
	Expansion valid for $-\frac{1}{2}<x<\frac{1}{2}$	B1	3	Condone $\lvert x\rvert<\frac{1}{2}$
(b)	$x\sqrt{9+x}=3x\left[1+\frac{x}{18}+O(x^2)\right]$	B1		Correct first two terms in expn. of $\sqrt{9+x}$
	$\left[\frac{3x-x\sqrt{9+x}}{\ln[(1+2x)(1-2x)]}\right]=\left[\frac{3x-3x-\frac{3x^2}{18}\ldots}{-4x^2-8x^4\ldots}\right]$	M1		Series expansions used in both numerator and denominator.
	$\lim_{x\to 0}\left[\frac{3x-x\sqrt{9+x}}{\ln[(1+2x)(1-2x)]}\right]$ $=\lim_{x\to 0}\left[\frac{-\frac{1}{6}+O(x)}{-4+O(x^2)}\right]$	m1		Dividing numerator and denominator by x^2 to get constant term in each, leading to a finite limit. Must be at least a total of 3 'terms' divided by x^2
	$=\frac{1}{24}$	A1	4	$=\frac{1}{24}$ NOT $\to\frac{1}{24}$
	Total		**8**	

Q4	Solution	Mark	Total	Comment
(a)	The interval of integration is infinite	E1	1	OE
(b)	$\int (x-2)\mathrm{e}^{-2x}\,\mathrm{d}x$			
	$u = x-2\,,\ \frac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{e}^{-2x}\,,\ \frac{\mathrm{d}u}{\mathrm{d}x} = 1\,,\ v = -0.5\mathrm{e}^{-2x}$	M1		$\frac{\mathrm{d}u}{\mathrm{d}x} = 1,\ \ v = k\,\mathrm{e}^{-2x}$ with $k = \pm 0.5\,, \pm 2$
	….. = $-\frac{1}{2}(x-2)\mathrm{e}^{-2x} - \int -\frac{1}{2}\mathrm{e}^{-2x}\,\mathrm{d}x$	A1		$-\frac{1}{2}(x-2)\mathrm{e}^{-2x} - \int -\frac{1}{2}\mathrm{e}^{-2x}\,(\mathrm{d}x)$ OE
	$= -\frac{1}{2}(x-2)\mathrm{e}^{-2x} - \frac{1}{4}\mathrm{e}^{-2x}\ (+c)$	A1		
	$\int_2^{\infty}(x-2)\,\mathrm{e}^{-2x}\,\mathrm{d}x = \lim_{a\to\infty}\int_2^{a}(x-2)\,\mathrm{e}^{-2x}\,\mathrm{d}x$	M1		Evidence of limit ∞ having been replaced by a (OE) at any stage and $\lim_{a\to\infty}$ seen or taken at any stage with no remaining lim relating to 2.
	$\lim_{a\to\infty}\left[-\frac{1}{2}(a-2)\mathrm{e}^{-2a} - \frac{1}{4}\mathrm{e}^{-2a}\right] - \left(-\frac{1}{4}\mathrm{e}^{-4}\right)$			
	Now $\lim_{a\to\infty} a^{p}\,\mathrm{e}^{-2a} = 0\,,\ \ (p>0)$	E1		General statement or specific statement with $p = 1$ stated explicitly. Each must include the 2 in the exponential.
	$\int_2^{\infty}(x-2)\mathrm{e}^{-2x}\,\mathrm{d}x = \frac{1}{4}\mathrm{e}^{-4}$	A1	6	No errors seen in $\mathrm{F}(a) - \mathrm{F}(2)$. (M1E0A1 is possible)
	Total		**7**	

Q5	Solution	Mark	Total	Comment
(a)	Aux eqn $m^2+6m+9=0$ $(m+3)^2=0$	M1		Factorising or using quadratic formula OE on correct aux eqn. PI by correct value of 'm' seen/used.
	$(y_{CF}=)\ (Ax+B)e^{-3x}$	A1		
	Try $(y_{PI}=)\ a\sin 3x+b\cos 3x$ $(y'_{PI}=)\ 3a\cos 3x-3b\sin 3x$ $(y''_{PI}=)\ -9a\sin 3x-9b\cos 3x$	M1		$a\sin 3x+b\cos 3x$ or Altn. $k\cos 3x$
	$-9a\sin 3x-9b\cos 3x+6(3a\cos 3x-3b\sin 3x)$ $+9(a\sin 3x+b\cos 3x)=36\sin 3x$	m1		Substitution into DE, dep on previous M and differentiations being in form $p\cos 3x+q\sin 3x$ or Altn. $-3k\sin 3x$ and $-9k\cos 3x$
	$-18b=36 \qquad 18a=0$	A1		Seen or used
	$y_{PI}=-2\cos 3x$	A1		Correct y_{PI} seen or used
	$(y_{GS}=)\ (Ax+B)e^{-3x}-2\cos 3x$	B1F	7	$(y_{GS}=)$ c's CF + c's PI, must have exactly two arbitrary constants
(b)(i)	$f''(0)+6f'(0)+9f(0)=36\sin 0$ $f''(0)+6(0)+9(0)=0 \quad \Rightarrow f''(0)=0$	E1	1	AG Convincingly shown with no errors.
(b)(ii)	$f'''(0)=108\cos 0-0-0=108$ $f^{(iv)}(0)=0-6f'''(0)-0=-648$	B1		$f'''(0)=108$ and $f^{(iv)}(0)=-648$ seen or used
	$f(x)\approx 0+x(0)+\frac{x^2}{2}(0)+\frac{x^3}{3!}f'''(0)+\frac{x^4}{4!}f^{(iv)}(0)\ldots$ $f(x)\approx \frac{x^3}{3!}(108)+\frac{x^4}{4!}(-648)\ldots$	M1		$f(x)\approx \frac{x^3}{3!}f'''(0)+\frac{x^4}{4!}f^{(iv)}(0)$ used with c's non-zero values for $f'''(0)$ and $f^{(iv)}(0)$
	$=18x^3-27x^4$	A1	3	$18x^3-27x^4$ Ignore any extra higher powers of x terms
	<u>Altn</u>: Use of answer to part **(a)** $f(x)=(6x+2)e^{-3x}-2\cos 3x$	[B1]		
	$=$	[M1]		Correct series for e^{-3x} (at least from x^2 terms up to x^4 terms inclusive) and cos3x (at least x^2 terms and x^4 terms) substituted and also product of $(px+q)$ term with e^{-3x} series attempted where p and q are numbers.
	$=(2-2)+(6-6)x+(9-18+9)x^2+(27-9)x^3+$ $+(6.75-27-6.75)x^4$ $=18x^3-27x^4$	[A1]	[3]	
	Total		**11**	
	If using **(a)** to answer **(b)(i)**, for guidance, $f''(x)=54xe^{-3x}-18e^{-3x}+18\cos 3x$			

Q6	Solution	Mark	Total	Comment
(a)	$\frac{\mathrm{d}x}{\mathrm{d}t}\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}$	M1		OE Relevant chain rule eg $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}t}{\mathrm{d}x}\frac{\mathrm{d}y}{\mathrm{d}t}$
	$2\mathrm{e}^{2t}\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\Rightarrow\ 2x\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}$	A1		OE eg $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}\mathrm{e}^{-2t}\frac{\mathrm{d}y}{\mathrm{d}t}$
	$\frac{\mathrm{d}}{\mathrm{d}t}\left(2x\frac{\mathrm{d}y}{\mathrm{d}x}\right)=\frac{\mathrm{d}^2y}{\mathrm{d}t^2};\ \frac{\mathrm{d}x}{\mathrm{d}t}\frac{\mathrm{d}}{\mathrm{d}x}\left(2x\frac{\mathrm{d}y}{\mathrm{d}x}\right)=\frac{\mathrm{d}^2y}{\mathrm{d}t^2}$	M1		OE.Valid 1st stage to differentiate $xy'(x)$ oe wrt t or to differentiate $x^{-1}y'(t)$ oe wrt x.
	$\frac{\mathrm{d}x}{\mathrm{d}t}\left(2\frac{\mathrm{d}y}{\mathrm{d}x}+2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)=\frac{\mathrm{d}^2y}{\mathrm{d}t^2}$	m1		Product rule OE (dep on MM) to obtain an eqn involving both second derivatives
	$4x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+4x\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}^2y}{\mathrm{d}t^2}$	A1		OE eg $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{1}{2}\mathrm{e}^{-2t}\left[-\mathrm{e}^{-2t}\frac{\mathrm{d}y}{\mathrm{d}t}+\frac{1}{2}\mathrm{e}^{-2t}\frac{\mathrm{d}^2y}{\mathrm{d}t^2}\right]$ {Note: e^{-t} could be replaced by $\frac{1}{\sqrt{x}}$ }
	$4\sqrt{x^5}\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\sqrt{x}\,y=\sqrt{x}\,(\ln x)^2+5$ becomes $\frac{\mathrm{d}^2y}{\mathrm{d}t^2}-4x\frac{\mathrm{d}y}{\mathrm{d}x}+2y=(\ln x)^2+\frac{5}{\sqrt{x}}$	A1		Or better
	$\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}t^2}-2\frac{\mathrm{d}y}{\mathrm{d}t}+2y=(2t)^2+\frac{5}{\mathrm{e}^t}$ $\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}t^2}-2\frac{\mathrm{d}y}{\mathrm{d}t}+2y=4t^2+5\mathrm{e}^{-t}$	A1	7	AG Be convinced
(b)	Auxl eqn $m^2-2m+2=0 \qquad (m-1)^2+1=0$	M1		$(m-1)^2+k$ or using quadratic formula on correct aux eqn. PI by correct values of 'm' seen/used.
	$m=1\pm\mathrm{i}$	A1		
	CF: $(y_C=)\ \ \mathrm{e}^t(A\cos t+B\sin t)$	B1F		Ft on $m=p\pm q\mathrm{i}$, $p, q\neq0$ and 2 arb. constants in CF. Condone x for t here
	P.Int. Try $(y_P=)\ a+bt+ct^2+d\mathrm{e}^{-t}$	M1		
	$(y'(t)=)\ b+2ct-d\mathrm{e}^{-t}$; $(y''(t)=)\ 2c+d\mathrm{e}^{-t}$ Substitute into DE gives $2c+d\mathrm{e}^{-t}-2(b+2ct-d\mathrm{e}^{-t})+$ $+2(a+bt+ct^2+d\mathrm{e}^{-t})=4t^2+5\mathrm{e}^{-t}$	M1		Substitution and comparing coeffs at least once
	$d=1;\ \ c=2$	B1		Need both
	$2b-4c=0$ and $2c-2b+2a=0$	A1		OE PI by c's b=2×c's c and c's a=c's c provided c's $c\neq0$
	$b=4$ and $a=2$	A1		Need both
	GS (y=) $\mathrm{e}^t(A\cos t+B\sin t)+2+4t+2t^2+\mathrm{e}^{-t}$	B1F		Ft on c's CF + PI, provided PI is non-zero and CF has two arbitrary constants and RHS is fn of t only
	$y=\sqrt{x}\left[A\cos\left(\ln\sqrt{x}\right)+B\sin\left(\ln\sqrt{x}\right)\right]+2+$ $+2\ln x+\frac{1}{2}(\ln x)^2+\frac{1}{\sqrt{x}}$	A1	10	y=f(x) with ACF for f(x)
	Total		17	

Q7	Solution	Mark	Total	Comment
(a)	Area = $\frac{1}{2}\int_{(-\frac{\pi}{2})}^{(\frac{\pi}{2})}(1+\cos 2\theta)^2\,(\mathrm{d}\theta)$	M1		Use of $\frac{1}{2}\int r^2\,(\mathrm{d}\theta)$ or $\int_0^{\frac{\pi}{2}} r^2\,(\mathrm{d}\theta)$
	$=\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(1+2\cos 2\theta+\cos^2 2\theta)\mathrm{d}\theta$	B1		Correct expn of $[1+\cos 2\theta]^2$ and correct limits
	$=\frac{1}{2}\int(1+2\cos 2\theta+0.5+0.5\cos 4\theta)\,\mathrm{d}\theta$	M1		$2\cos^2 2\theta=\pm1\pm\cos 4\theta$ used with $k\int r^2\,\mathrm{d}\theta$
	$=\frac{1}{2}\left[\theta+\sin 2\theta+0.5\theta+\frac{1}{8}\sin 4\theta\right]_{-\pi/2}^{\pi/2}$	A1F		Correct integration ft wrong coefficients
	$=\frac{3}{4}\pi$	A1	5	CSO
(b)(i)	$1+\sin\theta=1+\cos 2\theta$ $1+\sin\theta=1+1-2\sin^2\theta$ $(2\sin\theta-1)(\sin\theta+1)$ (=0) $\sin\theta=-1$ gives the pole, O	M1 A1 E1		Equating rs (or equating $\sin\theta$s) followed (or preceded) by $\cos 2\theta=\pm(1\pm 2\sin^2\theta)$ Or $r(2r-3)$ (=0), each PI by correct 2 roots Or $r=0$ gives the pt O. OE eg finds 2nd pair of coords (0, - π/2) and chooses (3/2, π/6)
	At A, $\sin\theta=0.5$ so $A\left(\frac{3}{2},\frac{\pi}{6}\right)$	A1	4	$r=1.5$, $\theta=\frac{\pi}{6}$
(b)(ii)	Eqn of line thro' A parallel to initial line is $r\sin\theta=\frac{3}{4}$	B1F		PI Ft on $r\sin\theta=r_A\sin\theta_A$
	At B, $r=2-2\sin^2\theta=2-2\left(\frac{9}{16r^2}\right)$	M1		Solving $r\sin\theta=k$ and $r=1+\cos 2\theta$ to reach a cubic eqn in r or in $\sin\theta$
	$16r^3=32r^2-18$	A1		Correct cubic eqn in r (or in $\sin\theta$ eg $8\sin^3\theta=8\sin\theta-3$)
	$(2r-3)(4r^2-2r-3)=0$	A1		Or $(2\sin\theta-1)(4\sin^2\theta+2\sin\theta-3)=0$
	Since $r_A=1.5$ and $r_B>0$, $OB=r_B=\frac{2+\sqrt{4+48}}{8}=\frac{1}{4}(\sqrt{13}+1)$	A2,1,0	6	A.G. Note: A2 requires correct surd for OB and also correct justifications for ignoring the other two roots of the cubic eqn. Max of A1 if justification absent
(b)(iii)	$AB=\pm(r_A\cos\theta_A-r_B\cos\theta_B)$	M1		OE method to find AB or AB^2. eg $AB=\frac{OB\sin(\theta_B-\theta_A)}{\sin\theta_A}$ OE single 'eqn' or $AB^2=r_A{}^2+OB^2-2r_A OB\cos(\theta_B-\theta_A)$ or $OB^2=r_A{}^2+AB^2-2r_A AB\cos\theta_A$
	$\cos\theta_B=\sqrt{\frac{r_B}{2}}\left(=\sqrt{\frac{\sqrt{13}+1}{8}}\right)$=(0.758(7..))	m1		OE eg solving correct quadratic eg $\sin\theta_B=\frac{3}{\sqrt{13}+1}$ or $\theta_B=0.709(41...)$
	$AB=0.425$ (to 3sf)	A1	3	0.425 Condone >3sf (0.425428….)
	Total		**18**	
	TOTAL		**75**	
(b)(ii)	$(2\sin\theta-1)(4\sin^2\theta+2\sin\theta-3)=0$ $\sin\theta=0.5$ (pt A), eg $\sin\theta<-1$ impossible, so $\sin\theta=\frac{-2+\sqrt{52}}{8}$			