A-LEVEL
MATHEMATICS
Further Pure 3 – MFP3
Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Mark	Total	Comment
1	**DO NOT ALLOW ANY MISREADS IN THIS QUESTION**			
	$k_1 = 0.4\left[\frac{\ln(6+3)}{\ln 3}\right]$ (=0.8)	M1		PI. May be seen within given formula
	$k_2 = 0.4\times f(6.4, 3+k_1)$ $= 0.4\times\frac{\ln(6.4+3.8)}{\ln 3.8}$	M1		$0.4\times\frac{\ln(6+0.4+3+\text{c's } k_1)}{\ln(3+\text{c's } k_1)}$ PI. May be seen within given formula
	$k_2 = 0.4\times 1.7396\ldots = 0.6958(459\ldots)$	A1		0.696 or better. PI by later work
	$y(6.4) = y(6) + \frac{1}{2}[k_1 + k_2]$ $= 3+\frac{1}{2}[0.8+0.6958(459...)]$	m1		$3+\frac{1}{2}[\text{c's } k_1 + \text{c's } k_2]$ but dependent on previous two Ms scored. PI by 3.748 or 3.7479….
	$(= 3.747922975\ldots)$ $= 3.748$ (to 3dp)	A1	**5**	CAO Must be 3.748
	Total		**5**	

Q	Solution	Mark	Total	Comment
2(a)	$y = a + b\sin 2x + c\cos 2x$			
	$\frac{dy}{dx} = 2b\cos 2x - 2c\sin 2x$	B1		Correct expression for $\frac{dy}{dx}$
	$2b\cos 2x - 2c\sin 2x + 4(a + b\sin 2x + c\cos 2x)$ $(= 20 - 20\cos 2x)$	M1		Differentiation and substitution into LHS of DE
	$4a = 20$; $4b - 2c = 0$; $2b + 4c = -20$	m1		Equating coefficients OE to form 3 equations at least two correct. PI by next line
	$a = 5$, $b = -2$, $c = -4$	A1	**4**	
(b)	Aux. eqn. $m + 4 = 0$	M1		PI Or solving $y'(x)+4y=0$ as far as $y=Ae^{\pm 4x}$ OE
	$(y_{CF} =)$ Ae^{-4x}	A1		OE
	$(y_{GS} =)$ $Ae^{-4x} + 5 - 2\sin 2x - 4\cos 2x$	B1F		c's CF + c's PI with exactly one arbitrary constant
	When $x=0$, $y=4$ $\Rightarrow A = 3$ $y = 3e^{-4x} + 5 - 2\sin 2x - 4\cos 2x$	A1	**4**	$y = 3e^{-4x} + 5 - 2\sin 2x - 4\cos 2x$ ACF
	Total		**8**	

Q	Solution	Mark	Total	Comment
3	$4r-3x=4$	M1		$x=r\cos\theta$ used
	$4r=3x+4$	A1		$4r=3x+4$
	$16r^2=(3x+4)^2$			
	$16(x^2+y^2)=(3x+4)^2$	M1		$x^2+y^2=r^2$ used
	$y^2=\dfrac{16+24x-7x^2}{16}$	A1	4	Must be in form $y^2=\mathrm{f}(x)$ but accept ACF for $\mathrm{f}(x)$ eg $y^2=\dfrac{(4+7x)(4-x)}{16}$
	Total		**4**	
	Accept $y^2=\dfrac{(3x+4)^2-16x^2}{16}$ and apply ISW if incorrect simplification after seeing this form.			

Q	Solution	Mark	Total	Comment
4	Aux eqn $m^2-2m-3=0$			
	$(m-3)(m+1)=0$	M1		Correctly factorising or using quadratic formula OE for relevant Aux eqn. PI by correct two values of 'm' seen/used.
	$(y_{CF}=)\ Ae^{-x}+Be^{3x}$	A1		
	Try $(y_{PI}=)\ axe^{-x}$	M1		
	$(y'_{PI}=)\ ae^{-x}-axe^{-x}$ $(y''_{PI}=)\ -2ae^{-x}+axe^{-x}$	M1		Product rule OE used to differentiate xe^{-x} in at least one derivative, giving terms in the form $\pm e^{-x}\pm xe^{-x}$
	$-2ae^{-x}+axe^{-x}-2(ae^{-x}-axe^{-x})-3axe^{-x}$ $(=2e^{-x})$	m1		Subst. into LHS of DE
	$\Rightarrow -4a=2 \quad \Rightarrow a=-\dfrac{1}{2}$	A1		A0 if terms in xe^{-x} were incorrect in m1 line
	$(y_{GS}=)\ Ae^{-x}+Be^{3x}-\dfrac{1}{2}xe^{-x}$	B1F		$(y_{GS}=)$ c's CF + c's PI, must have exactly two arbitrary constants
	As $x\to\infty$, $xe^{-x}\to 0$ (and $e^{-x}\to 0$)	E1		As $x\to\infty$, $xe^{-x}\to 0$ OE. Must be treating xe^{-x} term separately
	$y\to 0$ so $B=0$ $(y'(x)=-Ae^{-x}-0.5e^{-x}+0.5xe^{-x})$ $(y'(0)=-3\Rightarrow -3=-A-0.5\Rightarrow A=2.5)$	B1		$B=0$, where B is the coefficient of e^{3x}
	$y=\dfrac{5}{2}e^{-x}-\dfrac{1}{2}xe^{-x}$	B1	10	$y=\dfrac{5}{2}e^{-x}-\dfrac{1}{2}xe^{-x}$ OE
	Total		**10**	

Q	Solution	Mark	Total	Comment
5(a)	$\ldots = x\left(\frac{1}{8}\sin 8x\right) - \int \frac{1}{8}\sin 8x\,(\mathrm{d}x)$	M1		$kx\sin 8x - \int k\sin 8x\,(\mathrm{d}x)$, with $k = 1, -1, 8, -8, 1/8$ or $-1/8$
		A1		$x\left(\frac{1}{8}\sin 8x\right) - \int \frac{1}{8}\sin 8x\,(\mathrm{d}x)$
	$= x\left(\frac{1}{8}\sin 8x\right) + \frac{1}{64}\cos 8x\ (+c)$	A1	3	
(b)	$\left[\frac{1}{x}\sin 2x\right] = \frac{2x + O(x^3)}{x}$	M1		$\sin 2x \approx 2x$ Ignore higher powers of x. PI by answer 2.
	$\ldots = \lim_{x\to 0}\left[2 + O(x^2)\right] = 2$	A1	2	CSO Must see correct intermediate step
(c)	$2\cot 2x$ and $1/x$ are not defined at $x=0$	E1	1	Only need to use one of the two terms. Condone 'Integrand not defined at lower limit' OE
(d)	$\left(\int \left(2\cot 2x - x^{-1} + x\cos 8x\right)\,\mathrm{d}x =\right)$			
	$\ln\sin 2x - \ln x + x\left(\frac{1}{8}\sin 8x\right) + \frac{1}{64}\cos 8x$	B1F		Ft c's answer to part (a) ie $\ln\sin 2x - \ln x$ + c's answer to part (a)
	$\int_0^{\frac{\pi}{4}}(\ldots)\,\mathrm{d}x = \lim_{a\to 0}\int_a^{\frac{\pi}{4}}(\ldots.)\,\mathrm{d}x$	M1		Limit 0 replaced by a (OE) and $\lim_{a\to 0}$ seen or taken at any stage with no remaining lim relating to $\pi/4$.
	$\int_0^{\frac{\pi}{4}}(\ldots)\,\mathrm{d}x = \left[\frac{x\sin 8x}{8} + \frac{\cos 8x}{64}\right]_0^{\pi/4} + \ln 1 - \ln(\pi/4) - \lim_{a\to 0}\left[\ln\left(\frac{\sin 2a}{a}\right)\right]$			$\lim_{a\to 0}\left[\ln\left(\frac{\sin 2a}{a}\right)\right]$
	$= \frac{1}{64} - \frac{1}{64} - \ln\left(\frac{\pi}{4}\right) - \lim_{a\to 0}\left[\ln\left(\frac{\sin 2a}{a}\right)\right]$	M1		$F(\pi/4) - F(0)$, with $\ln[(\sin 2x)/x]$ a term in $F(x)$, and at least all non ln terms evaluated
	$= -\ln\left(\frac{\pi}{4}\right) - \ln 2 = -\ln\left(\frac{\pi}{2}\right)$	A1	4	OE single term in exact form, eg $\ln\left(\frac{2}{\pi}\right)$.
	Total		**10**	
(a)	Example: $u=x$, $v'=\cos 8x$; $u'=1$, $v = \frac{1}{8}\sin 8x$ and $\ldots = uv - \int v\,u'$ all seen and substitution into $uv - \int v\,u'$ with no more than one miscopy, award the M1			

Q	Solution	Mark	Total	Comment
6(a)	IF is $e^{\int -\frac{2x}{x^2+4}\,dx}$	M1		PI With or without the negative sign
	$= e^{-\ln(x^2+4)\ (+c)} = e^{\ln(x^2+4)^{-1}\ (+c)}$	A1		Either O.E. Condone missing '+c'
	$= (A)(x^2+4)^{-1}$	A1F		Ft on earlier $e^{\lambda\ln(x^2+4)}$, condone missing A
	$\frac{1}{(x^2+4)}\frac{du}{dx} - \frac{2x}{(x^2+4)^2}u = 3$			
	$\frac{d}{dx}[(x^2+4)^{-1}u] = 3$	M1		LHS as d/dx(u×c's IF) PI
	$(x^2+4)^{-1}u = 3x\ (+C)$	A1		Condone missing '+C' here.
	(GS): $u = (3x+C)(x^2+4)$	A1	6	Must be in the form u = f(x), where f(x) is ACF
(b)	$u = x^2\frac{dy}{dx}$ so $\frac{du}{dx} = x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx}$	M1		$\frac{du}{dx} = \pm x^2\frac{d^2y}{dx^2} \pm px\frac{dy}{dx}$, $p\neq 0$
		A1		
	$x^2(x^2+4)\frac{d^2y}{dx^2} + 8x\frac{dy}{dx} =$			
	$= (x^2+4)[\frac{du}{dx} - 2x\frac{dy}{dx}] + 8x\frac{dy}{dx}$			
	$= (x^2+4)\frac{du}{dx} - 2x^3\frac{dy}{dx}$			
	$= (x^2+4)\frac{du}{dx} - 2xu$	m1		Substitution into LHS of DE and correct ft simplification as far as no y's present.
	Given DE becomes:			
	$(x^2+4)\frac{du}{dx} - 2xu = 3(x^2+4)^2$			
	$\Rightarrow \frac{du}{dx} - \frac{2x}{x^2+4}u = 3(x^2+4)$	A1	4	CSO AG
(c)	From (**a**), $u = (3x+C)(x^2+4)$			
	So $\frac{dy}{dx} = \frac{(3x+C)(x^2+4)}{x^2}$	M1		$\frac{dy}{dx} = \frac{\text{c's f}(x)\text{ answer to part (a)}}{x^2}$ stated or used
	$\frac{dy}{dx} = \frac{12}{x} + \frac{4C}{x^2} + 3x + C$			
	$y = 12\ln x - \frac{4C}{x} + \frac{3x^2}{2} + Cx + D$	A1	2	OE
	Total		**12**	
(b)	Altn: $\frac{d^2y}{dx^2} = \frac{\pm x^2\frac{du}{dx} \pm pxu}{(x^2)^2}$, $p\neq 0$ (M1) $\frac{d^2y}{dx^2} = \frac{x^2\frac{du}{dx} - 2xu}{(x^2)^2}$ (A1)			

Q	Solution	Mark	Total	Comment
7(a)(i)	$y = \ln(\cos x + \sin x)$, $\frac{dy}{dx} = \frac{-\sin x + \cos x}{\cos x + \sin x}$	M1 A1		Chain rule OE (sign errors only) ACF eg $e^y y'(x) = \cos x - \sin x$
	$y'' = \frac{-(\cos x + \sin x)^2 - (-\sin x + \cos x)^2}{(\cos x + \sin x)^2}$	m1		Quotient rule (sign errors only) OE eg $e^y [y']^2 + e^y y'' = \pm\cos x \pm \sin x$
	$= \frac{-2(\cos^2 x + \sin^2 x)}{(\cos x + \sin x)^2} = \frac{-2}{1 + 2\cos x \sin x}$			
	$\frac{d^2 y}{dx^2} = -\frac{2}{1 + \sin 2x}$	A1	**4**	CSO AG Completion must be convincing
(a)(ii)	$\frac{d^3 y}{dx^3} = 4(1 + \sin 2x)^{-2} \cos 2x$	B1	**1**	ACF for $\frac{d^3 y}{dx^3}$
(b)(i)	$y(0) = 0$; $y'(0) = 1$; $y''(0) = -2$; $y'''(0) = 4$	B1F		Ft only for $y'(0)$ and $y'''(0)$
	$y(x) \approx y(0) + xy'(0) + \frac{x^2}{2} y''(0) + \frac{x^3}{3!} y'''(0)$	M1		Maclaurin's theorem applied with numerical vals. for $y'(0)$, $y''(0)$ and $y'''(0)$. M0 if cand is missing an expression OE for the 1st or 3rd derivatives
	$y(x) \approx x - \frac{2}{2}x^2 + \frac{4}{6}x^3 = x - x^2 + \frac{2}{3}x^3$	A1	**3**	CSO AG Dep on all previous 7 marks awarded with no errors seen.
(b)(ii)	$\ln(\cos x - \sin x) \approx -x - x^2 - \frac{2}{3}x^3$	B1	**1**	$-x - x^2 - \frac{2}{3}x^3$
(c)	$\ln\left(\frac{\cos 2x}{e^{3x-1}}\right) = \ln\cos 2x - (3x - 1)$	B1		
	$\ln(\cos 2x) = \ln[(\cos x + \sin x)(\cos x - \sin x)]$ $= \ln(\cos x + \sin x) + \ln(\cos x - \sin x)$	B1		
	$\ln\left(\frac{\cos 2x}{e^{3x-1}}\right) \approx$ $\approx x - x^2 + \frac{2}{3}x^3 - x - x^2 - \frac{2}{3}x^3 - 3x + 1$	M1		
	$\approx 1 - 3x - 2x^2$	A1	**4**	CSO Must have used 'Hence'.
	Total		**13**	
(a)(i)	For guidance, working towards AG may include $y'' = -1 - [y']^2$			

Q	Solution	Mark	Total	Comment
8(a)	(Area=) $\frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(1-\tan^2\theta\right)^2 \sec^2\theta\,(\mathrm{d}\theta)$	M1		Use of $\frac{1}{2}\int r^2\,(\mathrm{d}\theta)$ or use of $\int_0^{\frac{\pi}{4}} r^2(\mathrm{d}\theta)$ OE
	(or) $\int_0^{\frac{\pi}{4}} \left(1-\tan^2\theta\right)^2 \sec^2\theta\,(\mathrm{d}\theta)$	B1		Correct limits
	Let $u=\tan\theta$ so (Area)= $\int_{(0)}^{(1)} \left(1-u^2\right)^2\,\mathrm{d}u$	M1		Valid method to integrate $\tan^n\theta\sec^2\theta$, n=2 or 4, could be by inspection.
	(Area) = $\left[u-\frac{2u^3}{3}+\frac{u^5}{5}\right]_0^1$	A1		Correct integration of $k(1-\tan^2\theta)^2\sec^2\theta$ OE; ignore limits at this stage
	$=\left(1-\frac{2}{3}+\frac{1}{5}\right)\quad(-0)\ =\frac{8}{15}$	A1	5	CSO AG
(b) (i)	$\left(1-\tan^2\theta\right)\sec\theta=\frac{1}{2}\sec^3\theta$	M1		Elimination of r or θ. $[\,r=2(2r)^{\frac{1}{3}}-2r\,]$
	$1-\tan^2\theta=\frac{1}{2}\left(1+\tan^2\theta\right)$	m1		Using $1+\tan^2\theta=\sec^2\theta$ OE to reach a correct equation in one 'unknown'.
	$\tan^2\theta=\frac{1}{3};\quad \theta=\pm\frac{\pi}{6};\quad r=\frac{4}{3\sqrt{3}}$			
	Coordinates $\left(\frac{4}{3\sqrt{3}},\ \frac{\pi}{6}\right)\ \left(\frac{4}{3\sqrt{3}},\ -\frac{\pi}{6}\right)$	A1	3	
(b) (ii)	$\frac{4}{3\sqrt{3}}\sin\alpha=(1)\sin\left(\pi-\frac{\pi}{6}-\alpha\right)$ OE	B1F		OE eg $AP=\sqrt{\frac{7}{27}}$ or eg $\sin\alpha=\sqrt{\frac{27}{28}}$.
	$\frac{4}{3\sqrt{3}}\sin\alpha=\sin\frac{\pi}{6}\cos\alpha+\cos\frac{\pi}{6}\sin\alpha$	B1		Or $\cos\alpha=-\frac{1}{\sqrt{28}}\ \left(=-\frac{\sqrt{7}}{14}\right)$
	$\tan\alpha=\dfrac{-1/2}{\frac{\sqrt{3}}{2}-\frac{4}{3\sqrt{3}}}$	M1		OE Valid method to reach an exact numerical expression for $\tan\alpha$.
	$\tan\alpha=-3\sqrt{3}\quad(k=-3)$	A1	4	
	Altn for the two B marks			
	$ON=\frac{4}{3\sqrt{3}}\cos\frac{\pi}{6}$; $AN=\frac{4}{3\sqrt{3}}\sin\frac{\pi}{6}$; OP=1	(B1F)		OE Any two correct ft. PI eg NP=1/3 (N is foot of perp from A or B to OP)
	$\tan OPA=\frac{2}{\sqrt{3}}$	(B1)		$\tan OPA=\frac{2}{\sqrt{3}}$ OE or $\tan PAN=\frac{\sqrt{3}}{2}$ OE [Then (M1)(A1) as above]
(b)(iii)	Since $\tan\alpha$ is negative, α is obtuse so point A lies inside the circle. (If A was on the circle α would be a right angle.)	E1F	1	Ft c's sign of k.
	Total		**13**	
	TOTAL		**75**	
Altn (a)	Converts to Cartesian eqn. $y^2=x^2(1-x)$ (M1A1); sets up a correct integral with correct limits for the area using the sym of the curve (B1); valid method to integrate $x(1-x)^{\frac{1}{2}}$ (M1); 8/15 obtained convincingly (A1)			
(b)(ii) alt	Altn expressions for M1: $\tan\alpha=-\tan\left(\frac{\pi}{6}+OPA\right)=\dfrac{-\frac{1}{\sqrt{3}}-\frac{2}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}\frac{2}{\sqrt{3}}}$; $\tan\alpha=\tan\left(\frac{\pi}{3}+PAN\right)=\dfrac{\sqrt{3}+\frac{\sqrt{3}}{2}}{1-\sqrt{3}\frac{\sqrt{3}}{2}}$			