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General Certificate of Education (A-level)
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Mathematics MFP3

(Specification 6360)

Further Pure 3

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1	$k_1 = 0.2\times(2-1)\sqrt{2+1}$ $(= 0.2\sqrt{3})$	M1		PI. May be seen within given formula.
	$= 0.346(410\ldots)$ $(= *)$			Accept 3dp or better as evidence of the M1 line.
	$k_2 = 0.2\times \mathrm{f}(2.2, 1+*\ldots)$			
	$= 0.2\times(2.2-1.346\ldots)\sqrt{2.2+1.346\ldots}$	M1		$0.2\times(2.2-1-\text{c's } k_1)\sqrt{(2.2+1+\text{c's } k_1)}$ PI May be seen within given formula.
	…. = 0.321(4946….)	A1		3dp or better. PI by later work
	$y(2.2) = y(2) + \frac{1}{2}[k_1+k_2]$ $= 1 + 0.5\times[0.3464\ldots + 0.3214\ldots]$	m1		Dep on previous two Ms but ft on c's numerical values for k_1 and k_2 following evaluation of these.
	$= 1 + 0.5\times0.667904\ldots$			
	(= 1.33395…) = 1.334 to 3dp	A1	5	CAO Must be 1.334 **SC** Any consistent use of a MR/MC of printed f(x,y) expression in applying IEF, mark as SC2 for a correct ft final 3dp value otherwise SC0.
	Total		**5**	
2	$(x+8)^2 + (y-6)^2 = 100$			
	$x^2 + y^2 + 16x - 12y + 64 + 36$ $(= 100)$	B1		OE If polar form before expn of brackets award the B1 for correct expansions of both $(r\cos\theta - m)^2$ and $(r\sin\theta - n)^2$ where $(m,n) = (-8, 6)$ or $(m,n) = (6, -8)$
	$r^2 + 16r\cos\theta - 12r\sin\theta = 0$	M1M1		1st M1 for replacement using any one of $\{[x^2 + y^2 = r^2,\ x = r\cos\theta,\ y = r\sin\theta](*)\}$ 2nd M1 for use of (*) to convert the form $x^2+y^2+ax+by=0$ correctly to the form $r^2+ar\cos\theta+br\sin\theta=0$ or better
	$\{r=0$, origin$\}$ Circle: $r = 12\sin\theta - 16\cos\theta$	A1	4	
	Total		**4**	

Q	Solution	Marks	Total	Comments
3(a)	$\frac{d^2y}{dx^2}+2\frac{dy}{dx}-3y=3x-8e^{-3x}$			
	P. Integral : $y_{PI}=a+bx+cxe^{-3x}$			
	$y'_{PI}=b+ce^{-3x}-3cxe^{-3x}$	M1		Product rule used at least once giving terms in the form $\pm pe^{-3x}\pm qxe^{-3x}$
	$y''_{PI}=-6ce^{-3x}+9cxe^{-3x}$			
	$-6ce^{-3x}+9cxe^{-3x}+2b+2ce^{-3x}-6cxe^{-3x}$ $-3a-3bx-3cxe^{-3x}=3x-8e^{-3x}$	M1		Substitution into LHS of DE
	$-3b=3$; $2b-3a=0$; $-4c=-8$	m1		Dep on 2nd M only Equating coeffs to obtain at least two of these correct eqns; PI by correct values for at least two constants
	$b=-1$; $c=2$; $a=-\frac{2}{3}$	A2,1,0	5	Dep on M1M1m1 all awarded. A1 if any two correct; A2 if all three correct but do not award the 2nd A mark if terms in xe^{-3x} were incorrect in the M1 line
	[$y_{PI}=-\frac{2}{3}-x+2xe^{-3x}$]			
(b)	Aux. eqn. $m^2+2m-3=0$			
	$(m+3)(m-1)=0$	M1		Factorising or using quadratic formula OE PI by correct two values of 'm' seen/used
	$(y_{CF}=)Ae^{-3x}+Be^{x}$	A1		
	$(y_{GS}=)Ae^{-3x}+Be^{x}-\frac{2}{3}-x+2xe^{-3x}$	B1F	3	c's CF + c's PI with 2 arbitrary constants, non-zero values for a,b and c and no trig or ln terms in c's CF
(c)	$x=0,\ y=1\ \Rightarrow 1=A+B-\frac{2}{3}$	B1F		Only ft if previous B1F has been awarded
	$\frac{dy}{dx}=-3Ae^{-3x}+Be^{x}-1+2e^{-3x}-6xe^{-3x}$			
	As $x\to\infty$, ($e^{-3x}\to 0$ and) $xe^{-3x}\to 0$	E1		Must treat xe^{-3x} separately
	(As $x\to\infty$, $\frac{dy}{dx}\to -1$ so) $B=0$	B1		B=0, where B is the coefficient of e^x.
	When $B=0$, $1=A-\frac{2}{3}\Rightarrow A=\frac{5}{3}$			
	$y=\frac{5}{3}e^{-3x}-\frac{2}{3}-x+2xe^{-3x}$	A1	4	
	Total		**12**	

Q	Solution	Marks	Total	Comments
4	$\int \left(\frac{2x}{x^2+4} - \frac{4}{2x+3}\right) dx = \ln(x^2+4)$	B1		
	$-2\ln(2x+3)\ \{+c\}$	B1		OE
	(I =) $\lim_{a\to\infty} \int_0^a \left(\frac{2x}{x^2+4} - \frac{4}{2x+3}\right) dx$	M1		∞ replaced by a (OE) and $\lim_{a\to\infty}$ seen or taken at any stage
	$= \lim_{a\to\infty} \left[\ln(x^2+4) - 2\ln(2x+3)\right]_0^a$			**Remaining marks are dep** on getting $p\ln(x^2+4)+q\ln(2x+3)$ after integration, where p and q are non-zero constants
	$= \lim_{a\to\infty} \left[\ln(a^2+4) - 2\ln(2a+3)\right] - (\ln 4 - 2\ln 3)$ $= \lim_{a\to\infty} \left[\ln\left(\frac{a^2+4}{(2a+3)^2}\right)\right] - (\ln 4 - \ln 9)$	M1		Dealing with the 0 limit correctly and using $\ln P - \ln Q = \ln(P/Q)$ at least once at any stage either before or after using F( )−F(0). OE
	$= \lim_{a\to\infty} \left[\ln\left(\frac{1+\frac{4}{a^2}}{4+\frac{12}{a}+\frac{9}{a^2}}\right)\right] - (\ln 4 - \ln 9)$	M1		Writing F(a) OE in a suitable form when considering $a \to \infty$. OE
	$I = \int_0^\infty \left(\frac{2x}{x^2+4} - \frac{4}{2x+3}\right) dx = \ln\frac{1}{4} - \ln\frac{4}{9} = \ln\frac{9}{16}$	A1	6	CSO
	Total		**6**	

Q	Solution	Marks	Total	Comments
5(a)	$\frac{\mathrm{d}}{\mathrm{d}x}[\ln(\ln x)] = \frac{1}{\ln x}\times\frac{1}{x}$	B1	1	ACF
(b)(i)	$\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{1}{x\ln x}y=9x^2$			
	An IF is exp $\{\int[1/(x\ln x)]\ (\mathrm{d}x)\}$	M1		…. and with integration attempted
	$= \mathrm{e}^{\ln(\ln x)} = \ln x$	A1	2	AG Must see $\mathrm{e}^{\ln(\ln x)}$ before $\ln x$
(ii)	$\ln x\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{1}{x}y=9x^2\ln x$			
	$\frac{\mathrm{d}}{\mathrm{d}x}[y\ln x] = 9x^2\ \ln x$	M1		LHS as differential of $y\times\ln x$ PI
	$y\ln x = \int 9x^2\ln x\,\mathrm{d}x$	A1		
	$\Rightarrow y\ln x = \int \ln x\ \ \mathrm{d}[3x^3]$			
	$=3x^3\ln x-\int 3x^3\left(\frac{1}{x}\right)\mathrm{d}x$	m1		$\int kx^2\ln x\,(\mathrm{d}x) = px^3\ln x-\int px^3\left(\frac{1}{x}\right)(\mathrm{d}x)$ or better
	$y\ln x = 3x^3\ln x - x^3\ (+c)$	A1		ACF Condone missing '$+c$'
	When $x=\mathrm{e}$, $y=4\mathrm{e}^3$, $4\mathrm{e}^3=3\mathrm{e}^3-\mathrm{e}^3+c$	m1		Dep on previous M1m1. Boundary condition used in attempt to find value of 'c' after integration is completed
	$c=2\mathrm{e}^3$			
	$\Rightarrow y\ln x = 3x^3\ln x - x^3 + 2\mathrm{e}^3$ $y=3x^3-\frac{(x^3-2\mathrm{e}^3)}{\ln x}$	A1	6	ACF
	Total		**9**	

Q	Solution	Marks	Total	Comments
6(a)	$y=(4+\sin x)^{1/2}$ so $y^2 = 4+\sin x$			
	$2y\dfrac{dy}{dx} = \cos x$	M1		$\dfrac{d}{dx}\left(y^2\right)=2y\dfrac{dy}{dx}$
	$y\dfrac{dy}{dx}=\dfrac{1}{2}\cos x$	A1	2	
(a)	**Altn** $\dfrac{dy}{dx}=\dfrac{1}{2}(4+\sin x)^{-1/2}(\cos x)$	(M1)		Chain rule
	$y\dfrac{dy}{dx}=\dfrac{1}{2}\cos x$	(A1)	(2)	
(b)	$y\dfrac{d^2y}{dx^2}+\left(\dfrac{dy}{dx}\right)^2 = -\dfrac{1}{2}\sin x$	M1		Correct differentiation of $y\dfrac{dy}{dx}$
	When $x = 0$, $y = 2$, $\dfrac{dy}{dx}=\dfrac{1}{4}$, $2\dfrac{d^2y}{dx^2}+\left(\dfrac{1}{4}\right)^2=0$	A1F		Ft on RHS of M1 line as $k\sin x$
	$y\dfrac{d^3y}{dx^3}+\dfrac{dy}{dx}\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}\dfrac{d^2y}{dx^2} = -\dfrac{1}{2}\cos x$	m1 A1		Correct LHS
	When x=0, $2\dfrac{d^3y}{dx^3}+3\left(\dfrac{1}{4}\right)\left(-\dfrac{1}{32}\right)=-\dfrac{1}{2} \Rightarrow \dfrac{d^3y}{dx^3}=-\dfrac{61}{256}$	A1	5	CSO
(b)	**Altn** $\dfrac{d^2y}{dx^2}=-\dfrac{1}{4}(4+\sin x)^{-3/2}(\cos^2 x)+\dfrac{1}{2}(4+\sin x)^{-1/2}(-\sin x)$	(M1)		Sign and numerical coeffs errors only.
		(A1)		ACF
	$\dfrac{d^3y}{dx^3}=\dfrac{3}{8}(4+\sin x)^{-2.5}(\cos^3 x)-\dfrac{1}{4}(4+\sin x)^{-1.5}(-2\cos x\sin x)$	(m1)		Sign and numerical coeffs errors only.
	$-\dfrac{1}{4}(4+\sin x)^{-1.5}(\cos x)(-\sin x)-\dfrac{1}{2}(4+\sin x)^{-0.5}\cos x$	(A1)		ACF
	When $x = 0$, $\dfrac{d^3y}{dx^3}=\dfrac{3}{8}\times\dfrac{1}{32}-\dfrac{1}{2}\times\left(\dfrac{1}{2}\right)=-\dfrac{61}{256}$	(A1)	(5)	CSO
(c)	McC. Thm: $y(0) + x\,y'(0) + \dfrac{x^2}{2}\,y''(0) + \dfrac{x^3}{3!}\,y'''(0)$	M1		Maclaurin's theorem used with c's numerical values for $y(0)$, $y'(0)$, $y''(0)$ and $y'''(0)$, all found with at least three being non-zero.
	$(4+\sin x)^{1/2} \approx 2+\dfrac{1}{4}x-\dfrac{1}{64}x^2-\dfrac{61}{1536}x^3$ …..	A1	2	CSO Previous 6 marks must have been scored
	Total		**9**	

Q	Solution	Marks	Total	Comments
7(a)	$\sin^2 x\dfrac{d^2y}{dx^2} - 2\sin x\cos x\dfrac{dy}{dx} + 2y = 2\sin^4 x\cos x$			
	$y = u\sin x$			
	$\dfrac{dy}{dx} = \dfrac{du}{dx}\sin x + u\cos x$	M1		Both derivatives attempted and product rule used at least twice.
	$\dfrac{d^2y}{dx^2} = \dfrac{d^2u}{dx^2}\sin x + \dfrac{du}{dx}\cos x + \dfrac{du}{dx}\cos x - u\sin x$	A1		Both correct
	$\dfrac{d^2u}{dx^2}\sin^3 x + 2\dfrac{du}{dx}\cos x\sin^2 x - u\sin^3 x - 2\dfrac{du}{dx}\sin^2 x\cos x$ $- 2u\sin x\cos^2 x + 2u\sin x = 2\sin^4 x\cos x$	m1		Substitution into original DE
	$\dfrac{d^2u}{dx^2}\sin^3 x + u\sin x\,[-\sin^2 x - 2\cos^2 x + 2] = 2\sin^4 x\cos x$			
	$\dfrac{d^2u}{dx^2}\sin^3 x + u\sin x\,[-\sin^2 x + 2\sin^2 x] = 2\sin^4 x\cos x$	A1		Need to see clear use of the trig identity
	(Divide throughout by $\sin^3 x$,) $\dfrac{d^2u}{dx^2} + u = 2\sin x\cos x$			
	$\Rightarrow \dfrac{d^2u}{dx^2} + u = \sin 2x$	A1	5	AG Completion, be convinced
(b)	For $\dfrac{d^2u}{dx^2} + u = \sin 2x$, aux eqn, $m^2+1=0 \Rightarrow m = \pm i$	M1		PI
	CF: $(u =)\ A\sin x + B\cos x$	A1		OE
	For PI try $(u=)\ p\sin 2x$	M1		Condone extra terms provided their coefficients are shown to be zero
	$-4p\sin 2x + p\sin 2x = \sin 2x \Rightarrow p = -\dfrac{1}{3}$	A1		Correct Particular integral
	GS for $u = A\sin x + B\cos x - \dfrac{1}{3}\sin 2x$	B1F		u=g(x), where g(x)= c's (CF+PI) with two arb. constants, PI≠0 and all real. Can be implied by next line.
	GS: $y = A\sin^2 x + B\sin x\cos x - \dfrac{1}{3}\sin 2x\sin x$	A1	6	y=f(x) with ACF for f(x)
	Total		**11**	

Q	Solution	Marks	Total	Comments
8(a)	At intersections of r=2 and $r=3+2\sin\theta$ $2=3+2\sin\theta$	M1		Elimation of r
	$\sin\theta=-\frac{1}{2}$, $\Rightarrow$ $\theta=\frac{7}{6}\pi$, $\theta=\frac{11}{6}\pi$	A1		Any one correct solution of $\sin\theta=-\frac{1}{2}$
	(P=) $\left(2,\frac{7\pi}{6}\right)$, ($Q$=) $\left(2,\frac{11\pi}{6}\right)$	A1	3	$\left(2,\frac{7\pi}{6}\right)$ and $\left(2,\frac{11\pi}{6}\right)$
(b)(i)	Angle between OA and initial line = $\frac{\pi}{6}$	B1F		If not correct, ft on $\theta_P-\pi$
	When $\theta=\frac{\pi}{6}$, $r=3+2\sin\frac{\pi}{6}=4$; $A\left(4,\frac{\pi}{6}\right)$	B1	2	
(ii)	$OA=4$, $OQ=2$ Angle AOQ = $\pi-(\theta_Q-\theta_P)=\frac{\pi}{3}$	B1F		If not correct, ft on $\pi-(\theta_Q-\theta_P)$. OE eg Cartesian coords of A and Q both attempted and at least one correct ft.
	$AQ^2=4^2+2^2-2(4)(2)\cos AOQ$ (=12)	M1		Valid method to find AQ (or AQ^2). Ft on c's r_A for OA
	$AQ=\sqrt{12}$	A1	3	ACF but must be exact surd form.
(iii)	Since $4^2=2^2+(\sqrt{12})^2$ so 90°	E1		Justifying why (angle OQA=) 90° OE
	angle OQA=90° $\Rightarrow$ AQ is a tangent	E1	2	Must have convincingly shown that OQA = 90°
(c)	Area of minor sector OPQ of circle $=\frac{1}{2}(2)^2[\theta_Q-\theta_P]$	M1		$\frac{1}{2}(2)^2[\theta_Q-\theta_P]$
	$=\frac{4\pi}{3}$	A1		PI by combined $-\frac{7\pi}{3}$ OE term later.
	Area of minor region OPQ of curve = $\frac{1}{2}\int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}}(4\sin^2\theta+12\sin\theta+9)\,d\theta$	M1		Use of $\frac{1}{2}\int r^2\,d\theta$ or use of $\int_{\theta_P}^{3\pi/2} r^2\,d\theta$ OE
		B1		$r^2=4\sin^2\theta+12\sin\theta+9$
	$=\frac{1}{2}\int(2-2\cos2\theta+12\sin\theta+9)\,d\theta$	M1		Use of $\cos2\theta=\pm1\pm2\sin^2\theta$ with $k\int r^2\,(d\theta)$
	$=\frac{1}{2}[2\theta-\sin2\theta-12\cos\theta+9\theta]$ =	A1F		Ft wrong non zero coefficients, ie for correct integration of $a+b\cos2\theta+c\sin\theta$
	$\left[\frac{121\pi}{12}+\frac{\sqrt{3}}{4}-\frac{6\sqrt{3}}{2}\right]-\left[\frac{77\pi}{12}-\frac{\sqrt{3}}{4}+\frac{6\sqrt{3}}{2}\right]$ {$=\frac{11\pi}{3}-\frac{11\sqrt{3}}{2}$ }	A1		OE eg $\left[\frac{33\pi}{2}\right]-\left[\frac{77\pi}{6}-\frac{\sqrt{3}}{2}+6\sqrt{3}\right]$ eg $\left[\frac{121\pi}{6}+\frac{\sqrt{3}}{2}-6\sqrt{3}\right]-\left[\frac{33\pi}{2}\right]$
	Area of shaded region = $\frac{4\pi}{3}-\{\frac{11\pi}{3}-\frac{11\sqrt{3}}{2}\}$	M1		$\frac{1}{2}(2)^2[\theta_Q-\theta_P]-\frac{1}{2}\int_{\theta_P}^{\theta_Q}(3+2\sin\theta)^2\,d\theta$
	$=\frac{11}{2}\sqrt{3}-\frac{7}{3}\pi=\frac{1}{6}(33\sqrt{3}-14\pi)$	A1	9	CSO $\frac{1}{6}(33\sqrt{3}-14\pi)$. ($m$ = 33, n = −14)
	Total		**19**	
	TOTAL		**75**	