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General Certificate of Education (A-level)
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Mathematics MFP3

(Specification 6360)

Further Pure 3

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP3

Q	Solution	Marks	Total	Comments
1	$k_1 = 0.2\times[2+\ln(1+1)]$	M1		PI. May be seen within given formula
	$= 0.5386(29\ldots)$ $(= *)$			Accept 3sf rounded or truncated or better as evidence of the M1 line
	$k_2 = 0.2\times \mathrm{f}(2.2,\ 1+*\ldots)$			
	$\ldots = 0.2\times[2.2 + \ln(1+1.5386\ldots)]$	M1		$0.2\times[2.2+\ln(1+1+\text{c's } k_1)]$. PI May be seen within given formula
	$\ldots. = 0.6263(248\ldots.)$	A1		4dp or better. PI by later work
	$y(2.2) = y(2)+\frac{1}{2}[k_1+k_2]$ $= 1 + 0.5\times[0.5386\ldots + 0.6263\ldots]$	m1		Dep on previous two Ms but ft on c's numerical values (or numerical expressions) for k's following evaluation of these.
	$= 1+ 0.5\times1.16495\ldots.$			
	$(= 1.582477\ldots) = 1.5825$ to 4dp	A1	5	CAO Must be 1.5825
				SC For those scoring M1M0 who have k_2=0.5261(78..), and final answer 1.5324 (ie 4 dp) for y(2.2) award a total of 2 marks [M1B1]
	Total		**5**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
2(a)	PI: $y_{PI} = p + qx\mathrm{e}^{-2x}$			
	$y'_{PI} = q\mathrm{e}^{-2x} - 2qx\mathrm{e}^{-2x}$	M1		Product rule used
	$y''_{PI} = -4q\mathrm{e}^{-2x} + 4qx\mathrm{e}^{-2x}$			
	$-4q\mathrm{e}^{-2x} + 4qx\mathrm{e}^{-2x} + q\mathrm{e}^{-2x} - 2qx\mathrm{e}^{-2x}$	M1		Subst. into DE
	$-2p - 2qx\mathrm{e}^{-2x} = 4 - 9\mathrm{e}^{-2x}$			
	$-3q = -9$ and $-2p = 4$	m1		Equating coefficients
	$-3q = -9$ so $q = 3$;	A1		
	$-2p = 4$ so $p = -2$;	B1	5	
	[$y_{PI} = 3x\mathrm{e}^{-2x} - 2$]			
(b)	Aux. eqn. $m^2 + m - 2 = 0$			
	$(m-1)(m+2) = 0$	M1		Factorising or using quadratic formula OE PI by correct two values of 'm' seen/used
	$y_{CF} = A\mathrm{e}^{x} + B\mathrm{e}^{-2x}$	A1		
	$y_{GS} = A\mathrm{e}^{x} + B\mathrm{e}^{-2x} + 3x\mathrm{e}^{-2x} - 2$	B1F	3	(y_{GS}) = c's CF + c's PI, provided 2 arbitrary constants
(c)	$x = 0,\ y = 4 \Rightarrow 4 = A + B - 2$	B1F		Only ft if exponentials in GS
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = A\mathrm{e}^{x} - 2B\mathrm{e}^{-2x} + 3\mathrm{e}^{-2x} - 6x\mathrm{e}^{-2x}$			
	As $x \to \infty$, ($\mathrm{e}^{-2x} \to 0$ and) $x\mathrm{e}^{-2x} \to 0$	E1		
	As $x \to \infty$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} \to 0$ so $A = 0$	B1		
	When $A = 0$, $4 = 0 + B - 2 \Rightarrow B = 6$			
	$y = 6\mathrm{e}^{-2x} + 3x\mathrm{e}^{-2x} - 2$	B1	4	$y = 6\mathrm{e}^{-2x} + 3x\mathrm{e}^{-2x} - 2$ OE
	Total		**12**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\int x^2 \ln x \, dx = \frac{x^3}{3}\ln x - \int \frac{x^3}{3}\left(\frac{1}{x}\right) dx$	M1		$... = kx^3 \ln x \pm \int f(x)$, with f(x) not involving the 'original' $\ln x$
		A1		
	$...... = \frac{x^3}{3}\ln x - \frac{x^3}{9}$ (+ c)	A1	3	Condone absence of '+ c'
(b)	Integrand is not defined at $x = 0$	E1	1	OE
(c)	$\int_0^e x^2 \ln x \, dx = \left\{\lim_{a\to 0} \int_a^e x^2 \ln x \, dx\right\}$			OE
	$= \left(\frac{e^3}{3}\ln e - \frac{e^3}{9}\right) - \lim_{a\to 0}\left[\frac{a^3}{3}\ln a - \frac{a^3}{9}\right]$	M1		F(e) – $\lim_{a\to 0}$ [F(a)]
	But $\lim_{a\to 0} a^3 \ln a = 0$	E1		Accept a general form eg $\lim_{x\to 0} x^k \ln x = 0$
	So $\int_0^e x^2 \ln x \, dx = \frac{2e^3}{9}$	A1	3	CSO
	Total		**7**	
4	$\frac{dy}{dx} + (\cot x)y = \sin 2x$			
	IF is exp ($\int \cot x \, dx$)	M1		and with integration attempted
	$= e^{\ln(\sin x)\ (+c)}$	A1		OE Condone missing '+c'
	$= (k) \sin x$	A1		'IF = $\sin x$' scores M1A1A1
	$\sin x \frac{dy}{dx} + (\cos x)y = \sin 2x \sin x$			
	$\frac{d}{dx}[y \sin x] = \sin 2x \sin x$	M1		LHS as differential of $y\times$IF PI
	$y \sin x = \int \sin 2x \sin x \, dx$	A1F		Ft on c's IF provided no exp. or logs
	$\Rightarrow y \sin x = \int 2\sin^2 x \cos x \, dx$	B1		$\sin 2x = 2\sin x \cos x$ used
	$\Rightarrow y \sin x = \int 2\sin^2 x \, d(\sin x)$	m1		dep on both Ms Use of relevant substitution to stage $\int 2s^2 ds$ or further or by inspection to $k\sin^3 x$
	$y \sin x = \frac{2}{3}\sin^3 x$ (+c)	A1		ACF
	$\frac{1}{2}\sin\frac{\pi}{6} = \frac{2}{3}\sin^3\frac{\pi}{6} + c$	m1		dep on both Ms Boundary condition used in attempt to find value of c after integration
	$c = \frac{1}{6}$ so $y \sin x = \frac{2}{3}\sin^3 x + \frac{1}{6}$	A1	10	CSO – no errors seen – accept equivalent forms
	Total		**10**	

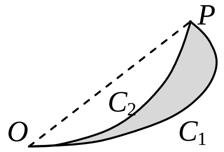
MFP3 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\frac{dy}{dx} = \frac{2\sec^2 x}{1+2\tan x}$	M1 A1		Chain rule ACF for $y'(x)$
	$\frac{d^2y}{dx^2} = \frac{(1+2\tan x)(4\sec^2 x\tan x) - 2\sec^2 x(2\sec^2 x)}{(1+2\tan x)^2}$	M1 A1	 4	Quotient rule OE in which both u and v are not const. or applied to a correct form of y' ACF for $y''(x)$
(b)	McC. Thm: $y(0) + x\,y'(0) + \frac{x^2}{2}\,y''(0)$ $(y(0) = 0)$; $y'(0) = 2$; $y''(0) = -4$	M1		Attempt to evaluate at least $y'(0)$ and $y''(0)$. PI
	$\ln(1+2\tan x) \approx 2x - 2x^2$	A1	2	Dep on previous 5 marks
(c)	$\ln(1-x) = -x - \frac{1}{2}x^2....$	B1		Ignore higher power terms
	$\left[\frac{\ln(1+2\tan x)}{\ln(1-x)}\right] \approx \frac{2x-2x^2...}{-x-\frac{1}{2}x^2...}$	M1		Expansions used
	$= \frac{2-2x...}{-1-\frac{1}{2}x...}$	m1		Dividing num. and den. by x to get constant term in each and non-const term in at least num. or den.
	So $\lim_{x\to 0}\left[\frac{\ln(1+2\tan x)}{\ln(1-x)}\right] = \frac{2}{-1} = -2$	A1F	4	ft c's answer to (b) provided answer (b) is in the form $\pm px \pm qx^2...$ and B1 awarded
	Total		**10**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$u=\frac{\mathrm{d}y}{\mathrm{d}x}-2x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x}=\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-2$	M1 A1		Differentiating subst wrt x, $\geq$ two terms correct
	DE becomes			
	$(x^3+1)(\frac{\mathrm{d}u}{\mathrm{d}x}+2)-3x^2(u+2x)=2-4x^3$	M1		Substitute into LHS of DE as far as no ys
	$(x^3+1)\frac{\mathrm{d}u}{\mathrm{d}x}+2x^3+2-3x^2u-6x^3=2-4x^3$			
	DE becomes $(x^3+1)\frac{\mathrm{d}u}{\mathrm{d}x}=3x^2u$	A1	4	CSO AG
(b)	$\int \frac{1}{u}\,\mathrm{d}u=\int \frac{3x^2}{x^3+1}\,\mathrm{d}x$	M1		Separate variables OE PI
	$\ln u=\ln(x^3+1)+\ln A$	A1;A1		$\ln u$; $\ln(x^3+1)$
		A1F		Applying law of logs to correctly combine two log terms or better
	$u=A(x^3+1)$	A1		OE RHS
	$\frac{\mathrm{d}y}{\mathrm{d}x}=A(x^3+1)+2x$	m1		$u=\mathrm{f}(x)$ to $\frac{\mathrm{d}y}{\mathrm{d}x}=\pm\mathrm{f}(x)\pm 2x$
	$y=A\left(\frac{x^4}{4}+x\right)+x^2+B$	m1 A1	8	Solution with two arbitrary constants and both previous M and m scored OE RHS
	Total		**12**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$r = 2\sin\theta \Rightarrow \quad r^2 = 2r\sin\theta$	M1		
	$x^2 + y^2 = 2y$	A2,1	3	OE (A1) either for $r^2=x^2+y^2$ or for $r\sin\theta=y$
				SC If M0 give B1 for $r^2=x^2+y^2$ or for $r\sin\theta=y$ used
(b)(i)	$2\sin\theta = \tan\theta$	M1		Equating rs
	$2\sin\theta\cos\theta = \sin\theta$			
	$\sin\theta(2\cos\theta - 1) = 0$	m1		Both solutions have to be considered if not in factorised form
	$\sin\theta = 0 \Rightarrow \theta = 0$; $\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$			Alternative: $\sin 2\theta = \sin\theta \Rightarrow \theta = 0, \frac{\pi}{3}$
	$\theta = 0 \Rightarrow r = 0$ ie pole O (0,0)	B1		Indep. Can just verify using both eqns +statement.
	$\theta = \frac{\pi}{3} \Rightarrow r = \sqrt{3} \quad \left(P\left(\sqrt{3}, \frac{\pi}{3}\right)\right)$	A1	4	CSO
(ii)	At A, $\theta = \frac{\pi}{4}$, $r = 2\sin\frac{\pi}{4} = \sqrt{2}$			
	At B, $\theta = \frac{\pi}{4}$, $r = \tan\frac{\pi}{4} = 1$	M1		Substitute $\theta = \frac{\pi}{4}$ into the equations of both curves.
	Since $\sqrt{2} > 1$, A is further away (from the pole than B.)	E1	2	CSO
(iii)				
	Area bounded by line OP and curve C_1			
	$= \frac{1}{2}\int_0^{\frac{\pi}{3}} 4\sin^2\theta \, d\theta$	M1		Use of $\frac{1}{2}\int r^2 \, d\theta$; ignore limits here
	$= \int (1 - \cos 2\theta) \, d\theta$	m1		Attempt to write $\sin^2\theta$ in terms of $\cos 2\theta$ only
	$= \left[\theta - \frac{1}{2}\sin 2\theta\right]$	A1		Ignore limits here
	$= \left(\frac{\pi}{3} - \frac{1}{2} \times \frac{\sqrt{3}}{2}\right) - 0 = \frac{\pi}{3} - \frac{\sqrt{3}}{4}$	A1		PI
	Area bounded by line OP and curve C_2			
	$= \frac{1}{2}\int_0^{\frac{\pi}{3}} \tan^2\theta \, d\theta$	M1		Use of $\frac{1}{2}\int r^2 \, d\theta$; ignore limits here
	$= \frac{1}{2}\int (\sec^2\theta - 1) \, d\theta$	m1		Using $\tan^2\theta = \pm \sec^2\theta \pm 1$ PI
	$= \frac{1}{2}[\tan\theta - \theta]$	A1		Ignore limits here
	$= \frac{1}{2}\left(\sqrt{3} - \frac{\pi}{3}\right) - 0 = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$	A1		PI
	Required area = $\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) - \left(\frac{\sqrt{3}}{2} - \frac{\pi}{6}\right)$	M1		Can award earlier eg if we see $\frac{1}{2}\int_0^{\frac{\pi}{3}} 4\sin^2\theta \, d\theta - \frac{1}{2}\int_0^{\frac{\pi}{3}} \tan^2\theta \, d\theta$
	$= \frac{1}{2}\pi - \frac{3}{4}\sqrt{3} \quad \left(a = \frac{1}{2}, \quad b = -\frac{3}{4}\right)$	A1	10	CSO
	Total		**19**	
	TOTAL		**75**	