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General Certificate of Education
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Mathematics MFP3

Further Pure 3

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	$y(1.1) = y(1) + 0.1[1+3+\sin 1]$	M1A1		
	$= 1 + 0.1\times 4.84147. = 1.4841(47..)$ $= 1.4841$ to 4dp	A1	3	Condone > 4dp
(b)	$y(1.2) = y(1) + 2(0.1)\{f[1.1, y(1.1)]\}$	M1		
	…. = 1 + 2(0.1){1.1+3+sin[1.4841(47..)]} = 2.019 to 3dp	A1F A1	3	Ft on cand's answer to **(a)** CAO Must be 2.019 **Note:** If using degrees max mark is 4/6 ie M1A1A0;M1A1FA0
	Total		**6**	
2(a)	$-4k\sin 2x + k\sin 2x = \sin 2x$	M1 A1		Substituting into the differential equation
	$k = -\frac{1}{3}$	A1	3	Accept correct PI
(b)	(Aux. eqn $m^2 + 1 = 0$) $m = \pm$ i CF: $A\cos x + B\sin x$	B1 M1 A1F		PI M0 if m is real OE Ft on incorrect complex values for m For the A1F do not accept if left in the form $Ae^{ix} + Be^{-ix}$
	(GS: $y =$) $A\cos x + B\sin x - \frac{1}{3}\sin 2x$	B1F	4	c's CF +c's PI but must have 2 constants
	Total		**7**	
3(a)	The interval of integration is infinite	E1	1	OE
(b)	$\int 4xe^{-4x}dx = -xe^{-4x} - \int -e^{-4x}dx$	M1 A1		$kxe^{-4x} - \int ke^{-4x}dx$ for non-zero k
	$= -xe^{-4x} - \frac{1}{4}e^{-4x}$ {+c}	A1F	3	Condone absence of +c
(c)	$I = \int_1^{\infty} 4xe^{-4x}\,dx = \lim_{a\to\infty}\int_1^{a} 4xe^{-4x}\,dx$ $\lim_{a\to\infty}\{-ae^{-4a} - \frac{1}{4}e^{-4a}\} - \left[-\frac{5}{4}e^{-4}\right]$	M1		F(a) – F(1) with an indication of limit '$a \to \infty$'
	$\lim_{a\to\infty} a\,e^{-4a} = 0$	M1		For statement with limit/ limiting process shown
	$I = \frac{5}{4}e^{-4}$	A1	3	CSO
	Total		**7**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
4	IF is exp $\left(\int \frac{3}{x}\,\mathrm{d}x\right)$	M1		and with integration attempted
	$= e^{3\ln x}$	A1		PI
	$= x^3$	A1		
	$\frac{\mathrm{d}}{\mathrm{d}x}\left[\, yx^3 \,\right] = x^3(x^4+3)^{\frac{3}{2}}$	M1 A1		LHS. Use of c’s IF. PI
	$\Rightarrow yx^3 = \frac{1}{10}\left(x^4+3 \right)^{\frac{5}{2}} + A$	m1 A1		$k\left(x^4+3 \right)^{\frac{5}{2}}$ Condone missing ‘A’
	$\Rightarrow \frac{1}{5} = \frac{1}{10}(4)^{\frac{5}{2}} + A$	m1		Use of boundary conditions in attempt to find constant after intgr. Dep on two M marks, not dep on m
	$\Rightarrow A = -3$; (*) $\Rightarrow yx^3 = \frac{1}{10}\left(x^4+3\right)^{\frac{5}{2}} - 3$	A1	9	ACF. The A1 can be awarded at line (*) provided a correct earlier eqn in y, x and ‘A’ is seen immediately before boundary conditions are substituted.
	Total		**9**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\cos 4x \approx 1 - \frac{(4x)^2}{2} + \frac{(4x)^4}{4!} \ldots$	M1		Clear attempt to replace x by $4x$ in expansion of $\cos x$…condone missing brackets for the M mark
	$\approx 1 - 8x^2 + \frac{32}{3}x^4 \ldots$	A1	2	
(b)(i)	$\frac{dy}{dx} = \frac{1}{2 - e^x} \times (-e^x)$	M1 A1		Chain rule
	$\frac{d^2y}{dx^2} = \frac{(2-e^x)(-e^x) - (-e^x)(-e^x)}{(2-e^x)^2}$	M1 A1		Quotient rule OE ACF
	$= \frac{-2e^x}{(2-e^x)^2}$			
	$\frac{d^3y}{dx^3} = \frac{(2-e^x)^2(-2e^x) - (-2e^x)2(2-e^x)(-e^x)}{(2-e^x)^4}$	m1		All necessary rules attempted (dep on previous 2 M marks)
		A1	6	ACF
(ii)	$y(0) = 0$; $y'(0) = -1$; $y''(0) = -2$; $y'''(0) = -6$	M1		At least three attempted
	$\text{Ln}(2-e^x) \approx y(0) + xy'(0) + \frac{x^2}{2}y''(0) + \frac{x^3}{6}y'''(0) \ldots$			
	$\ldots. \approx -x - x^2 - x^3 \ldots$	A1	2	CSO AG (The previous 7 marks must have been awarded and no double errors seen)
(c)	$\left[\frac{x\ln(2-e^x)}{1-\cos 4x}\right] \approx \frac{-x^2 - x^3 - x^4 \ldots}{8x^2 - \frac{32}{3}x^4}$	M1		Using the expansions
	Limit $= \lim_{x \to 0} \frac{-x^2 - o(x^3)}{8x^2 - o(x^4)}$			The notation $o(x^n)$ can be replaced by a term of the form kx^n
	$\ldots.. = \lim_{x \to 0} \frac{-1 - o(x)}{8 - o(x^2)}$	m1		Division by x^2 stage before taking the limit
	$\ldots\ldots = -\frac{1}{8}$	A1	3	CSO
	Total		**13**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$x^2 + y^2 = r^2,\ x = r\cos\theta,\ y = r\sin\theta$	B2,1,0		B1 for one stated or used
	$r^2 = 2r(\cos\theta - \sin\theta)$	M1		
	$x^2 + y^2 = 2(x - y)$	A1	4	ACF
(ii)	$(x-1)^2 + (y+1)^2 = 2$	M1 A1F		
	Centre $(1, -1)$; radius $\sqrt{2}$	A1F	3	
(b)(i)	Area = $\dfrac{1}{2}\int (4+\sin\theta)^2\,\mathrm{d}\theta$	M1		Use of $\dfrac{1}{2}\int r^2\,\mathrm{d}\theta$.
	$= \dfrac{1}{2}\int_0^{2\pi} (16 + 8\sin\theta + \sin^2\theta)\,\mathrm{d}\theta$	B1 B1		Correct expn of $[4+\sin\theta]^2$ Correct limits
	$= \int_0^{2\pi} (8 + 4\sin\theta + 0.25(1-\cos 2\theta))\,\mathrm{d}\theta$	M1		Attempt to write $\sin^2\theta$ in terms of $\cos 2\theta$
	$= \left[8\theta - 4\cos\theta + \dfrac{1}{4}\theta - \dfrac{1}{8}\sin 2\theta\right]_0^{2\pi}$	A1F		Correct integration ft wrong coefficients
	$= 16.5\pi$	A1	6	CSO
(ii)	For the curves to intersect, the eqn $2(\cos\theta - \sin\theta) = 4 + \sin\theta$ must have a solution.	M1		Equating rs and simplifying to a suitable form
	$2\cos\theta - 3\sin\theta = 4$ $R\cos(\theta + \alpha) = 4$,	M1		OE. Forming a relevant eqn from which valid explanation can be stated directly
	where $R = \sqrt{2^2 + 3^2}$ and $\cos\alpha = \dfrac{2}{R}$	A1		OE. Correct relevant equation
	$\cos(\theta+\alpha) = \dfrac{4}{\sqrt{13}} > 1$. Since must have $-1 \le \cos X \le 1$ there are no solutions of the equation $2(\cos\theta - \sin\theta) = 4 + \sin\theta$ so the two curves do not intersect.	E1	4	Accept other valid explanations.
(iii)	Required area = answer (b)(i) $- \pi(\text{radius of } C_1)^2$	M1		
	$= 16.5\pi - 2\pi = 14.5\pi$	A1F	2	Ft on (a)(ii) and (b)(i)
	Total		**19**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$\frac{dx}{dt}\frac{dy}{dx}=\frac{dy}{dt}$	M1		OE Chain rule
	$\frac{1}{2}t^{-\frac{1}{2}}\frac{dy}{dx}=\frac{dy}{dt}$ so $\frac{dy}{dx}=2t^{\frac{1}{2}}\frac{dy}{dt}$	A1	2	CSO A.G.
(a)(ii)	$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(2t^{\frac{1}{2}}\frac{dy}{dt}\right)=\frac{dt}{dx}\frac{d}{dt}\left(2t^{\frac{1}{2}}\frac{dy}{dt}\right)$	M1		$\frac{d}{dx}(\mathrm{f}(t))=\frac{dt}{dx}\frac{d}{dt}(\mathrm{f}(t))$ O.E. eg $\frac{d}{dt}(\mathrm{g}(x))=\frac{dx}{dt}\frac{d}{dx}(\mathrm{g}(x))$
	$\frac{d^2y}{dx^2}=2t^{\frac{1}{2}}\left[2t^{\frac{1}{2}}\frac{d^2y}{dt^2}+t^{-\frac{1}{2}}\frac{dy}{dt}\right]$	m1		Product rule O.E. used dep on previous M1 being awarded at some stage
	$\frac{d^2y}{dx^2}=4t\frac{d^2y}{dt^2}+2\frac{dy}{dt}$	A1	3	CSO A.G.
(b)	$t^{\frac{1}{2}}\left[4t\frac{d^2y}{dt^2}+2\frac{dy}{dt}\right]-(8t+1)2t^{\frac{1}{2}}\frac{dy}{dt}$ $+12t^{\frac{3}{2}}y=12t^{\frac{5}{2}}$	M1		Subst. using (a)(i), (a)(ii) into given DE to eliminate all x
	$4t^{\frac{3}{2}}\frac{d^2y}{dt^2}-16t^{\frac{3}{2}}\frac{dy}{dt}+12t^{\frac{3}{2}}y=12t^{\frac{5}{2}}$			
	Divide by $4t^{\frac{3}{2}}$ gives $\frac{d^2y}{dt^2}-4\frac{dy}{dt}+3y=3t$	A1	2	CSO A.G.
(c)	Solving $\frac{d^2y}{dt^2}-4\frac{dy}{dt}+3y=3t$ (*)			
	Auxl. Eqn. $m^2-4m+3=0$ $(m-1)(m-3)=0$	M1		PI
	$m=1$ and 3	A1		
	CF Ae^t+Be^{3t}	M1		Condone x for t here; ft c's 2 real values for 'm'
	For PI try $y=pt+q$	M1		OE
	$-4p+3pt+3q=3t \Rightarrow p=1,\ q=\frac{4}{3}$	A1		
	GS of (*) is $y=Ae^t+Be^{3t}+t+\frac{4}{3}$	B1F		CF + PI with 2 arb. constants and both CF and PI functions of t only
	GS of $x\frac{d^2y}{dx^2}-(8x^2+1)\frac{dy}{dx}+12x^3y=12x^5$ is $y=Ae^{x^2}+Be^{3x^2}+x^2+\frac{4}{3}$	A1	7	
	Total		**14**	
	TOTAL		**75**	