General Certificate of Education
June 2009
Advanced Level Examination

**MATHEMATICS** **MFP3**
**Unit Further Pure 3**

Thursday 11 June 2009 9.00 am to 10.30 am

**For this paper you must have:**
- a 12-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

**1** The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where $$f(x, y) = \sqrt{x^2 + y + 1}$$

and $$y(3) = 2$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h\,f(x_r, y_r)$$

with $h = 0.1$, to obtain an approximation to $y(3.1)$, giving your answer to four decimal places. *(3 marks)*

(b) Use the formula

$$y_{r+1} = y_{r-1} + 2h\,f(x_r, y_r)$$

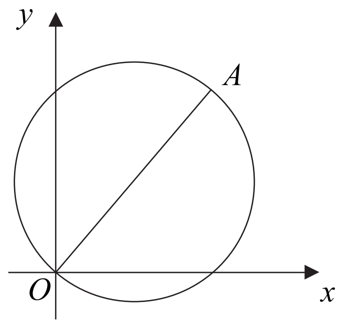
with your answer to part (a), to obtain an approximation to $y(3.2)$, giving your answer to three decimal places. *(3 marks)*

**2** By using an integrating factor, find the solution of the differential equation

$$\frac{dy}{dx} - y\tan x = 2\sin x$$

given that $y = 2$ when $x = 0$. *(9 marks)*

**3** The diagram shows a sketch of a circle which passes through the origin $O$.




The equation of the circle is $(x-3)^2+(y-4)^2=25$ and $OA$ is a diameter.

(a) Find the cartesian coordinates of the point $A$. *(2 marks)*

(b) Using $O$ as the pole and the positive $x$-axis as the initial line, the polar coordinates of $A$ are $(k, \alpha)$.

(i) Find the value of $k$ and the value of $\tan\alpha$. *(2 marks)*

(ii) Find the polar equation of the circle $(x-3)^2+(y-4)^2=25$, giving your answer in the form $r = p\cos\theta + q\sin\theta$. *(4 marks)*

**4** Evaluate the improper integral

$$\int_1^{\infty}\left(\frac{1}{x}-\frac{4}{4x+1}\right)\mathrm{d}x$$

showing the limiting process used and giving your answer in the form $\ln k$, where $k$ is a constant to be found. *(5 marks)*

**5** It is given that $y$ satisfies the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\frac{\mathrm{d}y}{\mathrm{d}x}+5y=8\sin x+4\cos x$$

(a) Find the value of the constant $k$ for which $y = k\sin x$ is a particular integral of the given differential equation. *(3 marks)*

(b) Solve the differential equation, expressing $y$ in terms of $x$, given that $y=1$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x}=4$ when $x=0$. *(8 marks)*

**Turn over ▶**

**6** The function f is defined by

$$f(x) = (9 + \tan x)^{\frac{1}{2}}$$

(a) (i) Find $f''(x)$. *(4 marks)*

(ii) By using Maclaurin's theorem, show that, for small values of $x$,

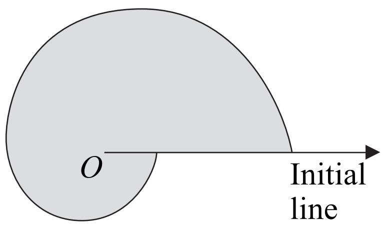
$$(9 + \tan x)^{\frac{1}{2}} \approx 3 + \frac{x}{6} - \frac{x^2}{216}$$ *(3 marks)*

(b) Find

$$\lim_{x \to 0} \left[ \frac{f(x) - 3}{\sin 3x} \right]$$ *(3 marks)*

**7** The diagram shows the curve $C_1$ with polar equation

$$r = 1 + 6e^{-\frac{\theta}{\pi}}, \quad 0 \leqslant \theta \leqslant 2\pi$$




(a) Find, in terms of π and e, the area of the shaded region bounded by $C_1$ and the initial line. *(5 marks)*

(b) The polar equation of a curve $C_2$ is

$$r = e^{\frac{\theta}{\pi}}, \quad 0 \leqslant \theta \leqslant 2\pi$$

Sketch the curve $C_2$ and state the polar coordinates of the end-points of this curve. *(4 marks)*

(c) The curves $C_1$ and $C_2$ intersect at the point $P$. Find the polar coordinates of $P$. *(5 marks)*

**8** (a) Given that $x = t^2$, where $t \geqslant 0$, and that $y$ is a function of $x$, show that:

(i) $2\sqrt{x}\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}t}$; *(3 marks)*

(ii) $4x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}$. *(3 marks)*

(b) Hence show that the substitution $x = t^2$, where $t \geqslant 0$, transforms the differential equation

$$4x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2(1 + 2\sqrt{x})\frac{\mathrm{d}y}{\mathrm{d}x} - 3y = 0$$

into

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + 2\frac{\mathrm{d}y}{\mathrm{d}t} - 3y = 0$$ *(2 marks)*

(c) Hence find the general solution of the differential equation

$$4x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2(1 + 2\sqrt{x})\frac{\mathrm{d}y}{\mathrm{d}x} - 3y = 0$$

giving your answer in the form $y = \mathrm{g}(x)$. *(4 marks)*

**END OF QUESTIONS**

**There are no questions printed on this page**

**There are no questions printed on this page**

**There are no questions printed on this page**