

General Certificate of Education  
June 2008  
Advanced Level Examination



**MATHEMATICS**  
**Unit Further Pure 3**

**MFP3**

Monday 16 June 2008 1.30 pm to 3.00 pm

**For this paper you must have:**

- a 12-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

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- 1 The function  $y(x)$  satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where

$$f(x, y) = \ln(x + y)$$

and

$$y(2) = 3$$

Use the improved Euler formula

$$y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$$

where  $k_1 = hf(x_r, y_r)$  and  $k_2 = hf(x_r + h, y_r + k_1)$  and  $h = 0.1$ , to obtain an approximation to  $y(2.1)$ , giving your answer to four decimal places. (6 marks)

- 2 (a) Find the values of the constants  $a$ ,  $b$ ,  $c$  and  $d$  for which  $a + bx + c \sin x + d \cos x$  is a particular integral of the differential equation

$$\frac{dy}{dx} - 3y = 10 \sin x - 3x \quad (4 \text{ marks})$$

- (b) Hence find the general solution of this differential equation. (3 marks)

- 3 (a) Show that  $x^2 = 1 - 2y$  can be written in the form  $x^2 + y^2 = (1 - y)^2$ . (1 mark)

- (b) A curve has cartesian equation  $x^2 = 1 - 2y$ .

Find its polar equation in the form  $r = f(\theta)$ , given that  $r > 0$ . (5 marks)

- 4 (a) A differential equation is given by

$$x \frac{d^2y}{dx^2} - \frac{dy}{dx} = 3x^2$$

Show that the substitution

$$u = \frac{dy}{dx}$$

transforms this differential equation into

$$\frac{du}{dx} - \frac{1}{x}u = 3x \quad (2 \text{ marks})$$

- (b) By using an integrating factor, find the general solution of

$$\frac{du}{dx} - \frac{1}{x}u = 3x$$

giving your answer in the form  $u = f(x)$ . (6 marks)

- (c) Hence find the general solution of the differential equation

$$x \frac{d^2y}{dx^2} - \frac{dy}{dx} = 3x^2$$

giving your answer in the form  $y = g(x)$ . (2 marks)

- 5 (a) Find  $\int x^3 \ln x \, dx$ . (3 marks)

- (b) Explain why  $\int_0^e x^3 \ln x \, dx$  is an improper integral. (1 mark)

- (c) Evaluate  $\int_0^e x^3 \ln x \, dx$ , showing the limiting process used. (3 marks)

- 6 (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 10e^{-2x} - 9 \quad (10 \text{ marks})$$

- (b) Hence express  $y$  in terms of  $x$ , given that  $y = 7$  when  $x = 0$  and that  $\frac{dy}{dx} \rightarrow 0$  as  $x \rightarrow \infty$ . (4 marks)

Turn over for the next question

Turn over ►

7 (a) Write down the expansion of  $\sin 2x$  in ascending powers of  $x$  up to and including the term in  $x^3$ . (1 mark)

(b) (i) Given that  $y = \sqrt{3 + e^x}$ , find the values of  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  when  $x = 0$ . (5 marks)

(ii) Using Maclaurin's theorem, show that, for small values of  $x$ ,

$$\sqrt{3 + e^x} \approx 2 + \frac{1}{4}x + \frac{7}{64}x^2 \quad (2 \text{ marks})$$

(c) Find

$$\lim_{x \rightarrow 0} \left[ \frac{\sqrt{3 + e^x} - 2}{\sin 2x} \right] \quad (3 \text{ marks})$$

8 The polar equation of a curve  $C$  is

$$r = 5 + 2 \cos \theta, \quad -\pi \leq \theta \leq \pi$$

(a) Verify that the points  $A$  and  $B$ , with **polar coordinates**  $(7, 0)$  and  $(3, \pi)$  respectively, lie on the curve  $C$ . (2 marks)

(b) Sketch the curve  $C$ . (2 marks)

(c) Find the area of the region bounded by the curve  $C$ . (6 marks)

(d) The point  $P$  is the point on the curve  $C$  for which  $\theta = \alpha$ , where  $0 < \alpha \leq \frac{\pi}{2}$ . The point  $Q$  lies on the curve such that  $POQ$  is a straight line, where the point  $O$  is the pole. Find, in terms of  $\alpha$ , the area of triangle  $OQB$ . (4 marks)

**END OF QUESTIONS**