Version

General Certificate of Education (A-level)
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Mathematics MFP3

(Specification 6360)

Further Pure 3

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	$y(3.2) = y(3) + 0.2\sqrt{2\times3+5}$	M1		
	$= 5 + 0.2 \times \sqrt{11}$	A1		
	$= 5.66332\ldots = 5.6633$ to 4dp	A1	3	Condone >4dp
(b)	$y(3.4) = y(3) + 2(0.2)\{\mathrm{f}\,[3.2, y(3.2)]\}$	M1		
	$\ldots. = 5 + 2(0.2)\sqrt{2\times3.2+5.6633...}$	A1F		Ft on cand's answer to **(a)**
	$(= 5 + (0.4)\sqrt{12.0633...}\ )$			
	$= 6.389$ to 3dp	A1	3	CAO Must be 6.389
	Total		**6**	
2				Ignore higher powers beyond x^2 throughout this question
(a)	$e^{3x} = 1 + 3x + 4.5x^2$	B1	1	
(b)	$(1+2x)^{-3/2} = 1 - 3x + \frac{15}{2}x^2$	M1		$(1+2x)^{-3/2} = 1\pm3x+kx^2$ or $1+kx\pm7.5x^2$ OE
		A1		$1-3x+7.5x^2$ OE (simplified PI)
	$e^{3x}(1+2x)^{-3/2} = (1 + 3x + 4.5x^2)(1-3x+7.5x^2)$	M1		Product of c's two expansions with an attempt to multiply out to find x^2 term
	x^2 term(s): $7.5x^2 - 9x^2 + 4.5x^2 = 3x^2$.	A1	4	
	Total		**5**	

Q	Solution	Marks	Total	Comments
3	PI: $y_{PI} = kx^2\mathrm{e}^x$	M1		
	$y'_{PI} = 2kx\mathrm{e}^x + kx^2\mathrm{e}^x$ $y''_{PI} = 2k\mathrm{e}^x + 4kx\mathrm{e}^x + kx^2\mathrm{e}^x$	m1		Product rule used in finding both derivatives
	$2k\mathrm{e}^x + 4kx\mathrm{e}^x + kx^2\mathrm{e}^x - 4kx\mathrm{e}^x - 2kx^2\mathrm{e}^x + kx^2\mathrm{e}^x = 6\mathrm{e}^x$	m1		Subst. into DE
	$2k = 6$; $k = 3$; $y_{PI} = 3x^2\mathrm{e}^x$	A1		CSO
	(GS: $y =$) $\mathrm{e}^x(Ax+B) + 3x^2\mathrm{e}^x$	B1F	5	$\mathrm{e}^x(Ax+B) + kx^2\mathrm{e}^x$, ft c's k .
	Total		**5**	
4(a)	Integrand is not defined at $x = 0$	E1	1	OE
(b)	$\int x^4 \ln x \, \mathrm{d}x = \frac{x^5}{5}\ln x - \int \frac{x^5}{5}\left(\frac{1}{x}\right)\mathrm{d}x$	M1 A1		$\ldots = kx^5 \ln x \pm \int \mathrm{f}(x)$, with f($x$) not involving the 'original' ln x
	$\ldots\ldots = \frac{x^5}{5}\ln x - \frac{x^5}{25}$ $(+\,c)$	A1		
	$\int_0^1 x^4 \ln x \, \mathrm{d}x = \{ \lim_{a \to 0} \int_a^1 x^4 \ln x \, \mathrm{d}x \}$ $= -\frac{1}{25} - \lim_{a \to 0} [\frac{a^5}{5}\ln a - \frac{a^5}{25}]$	M1		Limit 0 replaced by a limiting process and F(1)−F(a) OE
	But $\lim_{a \to 0} a^5 \ln a = 0$	E1		Accept $\lim_{x \to 0} x^k \ln x = 0$ for any k>0
	So $\int_0^1 x^4 \ln x \, \mathrm{d}x = -\frac{1}{25}$	A1	6	Dep on M and A marks all scored
	Total		**7**	

Q	Solution	Marks	Total	Comments
5	$\frac{dy}{dx} + \frac{\sec^2 x}{\tan x} y = \tan x$			
(a)	IF is $\exp\left(\int \frac{\sec^2 x}{\tan x} \, dx\right)$	M1		and with integration attempted
	$= e^{\ln(\tan x)} = \tan x$	A1	2	AG Be convinced
(b)	$\tan x \frac{dy}{dx} + (\sec^2 x) y = \tan^2 x$			
	$\frac{d}{dx}[y \tan x] = \tan^2 x$	M1		LHS as differential of $y \times$IF PI
	$y \tan x = \int \tan^2 x \, dx$	A1		
	$\Rightarrow y \tan x = \int (\sec^2 x - 1) \, dx$	m1		Using $\tan^2 x = \pm \sec^2 x \pm 1$ PI or other valid methods to integrate $\tan^2 x$
	$y \tan x = \tan x - x$ $(+c)$	A1		Correct integration of $\tan^2 x$; condone absence of $+c$.
	$3 \tan \frac{\pi}{4} = \tan \frac{\pi}{4} - \frac{\pi}{4} + c$	m1		Boundary condition used in attempt to find value of c
	$c = 2 + \frac{\pi}{4}$ so $y \tan x = \tan x - x + 2 + \frac{\pi}{4}$ $y = 1 + (2 - x + \frac{\pi}{4}) \cot x$	A1	6	ACF
	Total		**8**	

Q	Solution	Marks	Total	Comments
6(a)(i)	$y = \ln\left(e^{3x}\cos x\right) = \ln e^{3x} + \ln\cos x = 3x + \ln\cos x$	B1		
	$\dfrac{dy}{dx} = 3 + \dfrac{1}{\cos x} \times (-\sin x)$	M1		Chain rule for derivative of ln cos$x$
	$\dfrac{dy}{dx} = 3 - \tan x$	A1	3	CSO AG
(ii)	$\dfrac{d^2y}{dx^2} = -\sec^2 x$; $\quad \dfrac{d^3y}{dx^3} = -2\sec x(\sec x\tan x)$	B1; M1		M1 for d/dx\{ $[f(x)]^2$ \}= $2f(x)f'(x)$
	$\dfrac{d^4y}{dx^4} = -4\sec x(\sec x\tan x)\tan x - 2\sec^4 x$	A1	3	ACF
(b)	Maclaurin's Thm: $y(0)+x\,y'(0)+\dfrac{x^2}{2!}\,y''(0)+\dfrac{x^2}{3!}\,y'''(0)+\dfrac{x^4}{4!}\,y^{(iv)}(0)$ $y(0) = \ln 1 = 0$; $\quad y'(0) = 3$; $\quad y''(0) = -1$; $y'''(0) = 0$; $\quad y^{(iv)}(0) = -2$	M1		Mac. Thm with attempt to evaluate at least two derivatives at x=0
	$\ln\left(e^{3x}\cos x\right) = 0 + 3x + \dfrac{-1}{2!}x^2 + \dfrac{0}{3!}x^3 + \dfrac{-2}{4!}x^4 \ldots$	A1F		At least 3 of 5 terms correctly obtained. Ft one miscopy in (a)
	$= 3x - \dfrac{1}{2}x^2 - \dfrac{1}{12}x^4$	A1	3	CSO AG Be convinced
(c)	$\{\ln(1+px)\} = px - \dfrac{1}{2}p^2x^2$	B1	1	accept $(px)^2$ for p^2x^2; ignore higher powers;
(d)(i)	$\left[\dfrac{1}{x^2}\{\ln\left(e^{3x}\cos x\right) - \ln(1+px)\}\right] =$			
	$\left[\dfrac{1}{x^2}\left\{3x - \dfrac{1}{2}x^2 - O(x^4) - \left(px - \dfrac{1}{2}p^2x^2 + O(x^3)\right)\right\}\right]$	M1		Law of logs and expansions used;
	For $\lim\limits_{x\to 0}\left[\dfrac{1}{x^2}\ln\left(\dfrac{e^{3x}\cos x}{1+px}\right)\right]$ to exist, $p = 3$	A1		p=3 convincingly found
(ii)	…… $= \lim\limits_{x\to 0}\left[\left(\dfrac{3-p}{x}\right) - \dfrac{1}{2} + \dfrac{p^2}{2} - O(x)\right]$	m1		Divide throughout by x^2 before taking limit. (m1 can be awarded before or after the A1 above)
	Value of limit $= -\dfrac{1}{2} + \dfrac{p^2}{2} = 4.$	A1	4	Must be convincingly obtained
	Total		**14**	

Q	Solution	Marks	Total	Comments
7(a)	Solving $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 10y = e^{2t}$ (*)			
	Auxl. Eqn. $m^2 - 6m + 10 = 0$ $(m-3)^2 + 1 = 0$	M1		PI Completing sq or using quadratic formula to find m.
	$m = 3 \pm i$	A1		
	CF $(y_{CF} =)\ e^{3t}(A\cos t + B\sin t)$	M1		OE Condone x for t here; ft c's 2 **non-real** values for 'm'.
	For PI try $(y_{PI} =)\ ke^{2t}$	M1		Condone x for t here
	$4k - 12k + 10k = 1 \Rightarrow k = \frac{1}{2}$	A1		
	GS of (*) is $(y_{GS} =)\ e^{3t}(A\cos t + B\sin t) + \frac{1}{2}e^{2t}$	B1F	6	CF +PI with 2 arb. constants and both CF and PI functions of t only
(b)	$\frac{dy}{dx} = \frac{dt}{dx}\frac{dy}{dt}$	M1		OE Chain rule
	$\frac{dy}{dx} = 2x\frac{dy}{dt}$	A1		OE
	$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(2x\frac{dy}{dt}\right) = (2x)\ \frac{dt}{dx}\frac{d}{dt}\left(\frac{dy}{dt}\right) + 2\frac{dy}{dt}$	M1		$\frac{d}{dx}(\,f(t)\,) = \frac{dt}{dx}\frac{d}{dt}(\,f(t)\,)$ OE
	$= (2x)(2x)\ \frac{d^2y}{dt^2} + 2\frac{dy}{dt}$			eg $\frac{d}{dt}(\,g(x)\,) = \frac{dx}{dt}\frac{d}{dx}(\,g(x)\,)$
		m1		Product rule OE used dep on previous M1 being awarded at some stage
	$\frac{d^2y}{dx^2} = 4t\frac{d^2y}{dt^2} + 2\frac{dy}{dt}$	A1	5	CSO A.G.
(c)	$t^{\frac{1}{2}}\left[4t\frac{d^2y}{dt^2} + 2\frac{dy}{dt}\right] - (12t+1)2t^{\frac{1}{2}}\frac{dy}{dt} + 40t^{\frac{3}{2}}y = 4t^{\frac{3}{2}}e^{2t}$	M1		Subst. using (b) into given DE to eliminate all x
	$4t^{\frac{3}{2}}\{\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 10y\} = 4t^{\frac{3}{2}}e^{2t}$			
	$t \neq 0$ so divide by $4t^{\frac{3}{2}}$ gives $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 10y = e^{2t}$ (*)	A1	2	CSO A.G.
(d)	$y =\ e^{3x^2}(A\cos x^2 + B\sin x^2) + \frac{1}{2}e^{2x^2}$	B1	1	OE Must include y=
	Total		**14**	

Q	Solution	Marks	Total	Comments
8(a)(i)	$r=\sin\frac{2\pi}{3}\sqrt{(2+\frac{1}{2}\cos\frac{\pi}{3})} = \frac{\sqrt{3}}{2}\times\sqrt{\frac{9}{4}} = \frac{3\sqrt{3}}{4}$	M1; A1	2	
(ii)	$x = ON = (3\sqrt{3})/8$			
	Polar eqn of PN is $r\cos\theta = ON$	M1		
	$r = \frac{3\sqrt{3}}{8}\sec\theta$	A1	2	AG Be convinced
(iii)	Area $\Delta ONP = 0.5 \times r_N \times r_P \times \sin(\pi/3)$	M1		OE With correct or ft from (a)(i) (ii), values for r_P and r_N .
	$= \frac{1}{2}\times\frac{3\sqrt{3}}{8}\times\frac{3\sqrt{3}}{4}\times\frac{\sqrt{3}}{2} = \frac{27\sqrt{3}}{128}$	A1	2	Be convinced
(b)(i)	$\int \sin^n\theta\cos\theta\,\mathrm{d}\theta = \int u^n\,\mathrm{d}u$	M1		PI
	$= \frac{\sin^{n+1}\theta}{n+1}\quad(+c)$	A1	2	
(ii)	Area of shaded region bounded by line OP and arc OP = $\frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\sin^2 2\theta\left(2+\frac{1}{2}\cos\theta\right)\mathrm{d}\theta$	M1		Use of $\frac{1}{2}\int r^2\,\mathrm{d}\theta$
		B1		Correct limits
	$\frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}(1-\cos 4\theta)\,\mathrm{d}\theta+\frac{1}{4}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}4\sin^2\theta\cos^2\theta\cos\theta\,\mathrm{d}\theta$	M1		$2\sin^2 2\theta = \pm1\pm\cos 4\theta$
		B1		$\sin^2 2\theta\ \cos\theta = 4\sin^2\theta\cos^2\theta\ \cos\theta$
	$=\left[\frac{\theta}{2}-\frac{\sin 4\theta}{8}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}+\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}(\sin^2\theta-\sin^4\theta)\cos\theta\,\mathrm{d}\theta$	A1		Correct integration of $0.5(1-\cos 4\theta)$
		m1		Writing 2nd integrand in a suitable form to be able to use (b)(i) OE PI
	$=\left[\frac{\theta}{2}-\frac{\sin 4\theta}{8}+\frac{\sin^3\theta}{3}-\frac{\sin^5\theta}{5}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$	A1		Last two terms OE
	$=\frac{\pi}{12}-\frac{21\sqrt{3}}{160}+\frac{2}{15}$	A1	8	CSO
	Total		**16**	
	TOTAL		**75**	