General Certificate of Education
Advanced Level Examination
January 2012

Mathematics MFP3

Unit Further Pure 3

Monday 23 January 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The function $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where $$\mathrm{f}(x, y) = \frac{y - x}{y^2 + x}$$

and $$y(1) = 2$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h\,\mathrm{f}(x_r, y_r)$$

with $h = 0.1$, to obtain an approximation to $y(1.1)$. *(3 marks)*

(b) Use the formula

$$y_{r+1} = y_{r-1} + 2h\,\mathrm{f}(x_r, y_r)$$

with your answer to part **(a)**, to obtain an approximation to $y(1.2)$, giving your answer to three decimal places. *(3 marks)*

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2 Find

$$\lim_{x \to 0}\left[\frac{\sqrt{4+x} - 2}{x + x^2}\right]$$ *(3 marks)*

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3 Solve the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}y}{\mathrm{d}x} + 10y = 26\mathrm{e}^x$$

given that $y = 5$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 11$ when $x = 0$. Give your answer in the form $y = \mathrm{f}(x)$. *(10 marks)*

4 (a) By using an integrating factor, find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2}{x}y = \ln x$$ *(7 marks)*

(b) Hence, given that $y \to 0$ as $x \to 0$, find the value of y when $x = 1$. *(3 marks)*

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5 (a) Explain why $\int_{\frac{1}{2}}^{\infty} \frac{x(1-2x)}{x^2 + 3\mathrm{e}^{4x}}\,\mathrm{d}x$ is an improper integral. *(1 mark)*

(b) By using the substitution $u = x^2\mathrm{e}^{-4x} + 3$, find

$$\int \frac{x(1-2x)}{x^2 + 3\mathrm{e}^{4x}}\,\mathrm{d}x$$ *(3 marks)*

(c) Hence evaluate $\int_{\frac{1}{2}}^{\infty} \frac{x(1-2x)}{x^2 + 3\mathrm{e}^{4x}}\,\mathrm{d}x$, showing the limiting process used. *(4 marks)*

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6 (a) Given that $y = \ln\cos 2x$, find $\frac{\mathrm{d}^4y}{\mathrm{d}x^4}$. *(6 marks)*

(b) Use Maclaurin's theorem to show that the first two non-zero terms in the expansion, in ascending powers of x, of $\ln\cos 2x$ are $-2x^2 - \frac{4}{3}x^4$. *(3 marks)*

(c) **Hence** find the first two non-zero terms in the expansion, in ascending powers of x, of $\ln\sec^2 2x$. *(2 marks)*

7 It is given that, for $x \neq 0$, y satisfies the differential equation

$$x\frac{d^2y}{dx^2} + 2(3x+1)\frac{dy}{dx} + 3y(3x+2) = 18x$$

(a) Show that the substitution $u = xy$ transforms this differential equation into

$$\frac{d^2u}{dx^2} + 6\frac{du}{dx} + 9u = 18x$$ *(4 marks)*

(b) Hence find the general solution of the differential equation

$$x\frac{d^2y}{dx^2} + 2(3x+1)\frac{dy}{dx} + 3y(3x+2) = 18x$$

giving your answer in the form $y = f(x)$. *(8 marks)*

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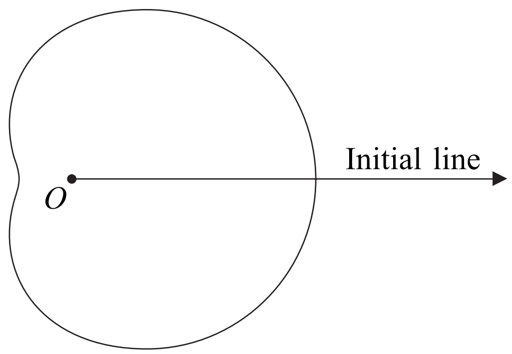
8 The diagram shows a sketch of the curve C with polar equation

$$r = 3 + 2\cos\theta, \quad 0 \leqslant \theta \leqslant 2\pi$$



(a) Find the area of the region bounded by the curve C. *(6 marks)*

(b) A circle, whose cartesian equation is $(x-4)^2 + y^2 = 16$, intersects the curve C at the points A and B.

(i) Find, in surd form, the length of AB. *(6 marks)*

(ii) Find the perimeter of the segment AOB of the circle, where O is the pole. *(3 marks)*