Version 1.0

General Certificate of Education (A-level)
January 2012

Mathematics MFP3

(Specification 6360)

Further Pure 3

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	$y(1.1) = y(1) + 0.1\left[\frac{2-1}{4+1}\right]$	M1A1		
	$= 2 + 0.02 = 2.02$	A1	3	
(b)	$y(1.2) = y(1) + 2(0.1)\{f[1.1, y(1.1)]\}$	M1		
	$= 2 + 2(0.1)\left[\frac{2.02-1.1}{2.02^2+1.1}\right]$	A1F		ft on c's answer to (a)
	$= 2.035518\ldots = 2.036$ to 3dp	A1	3	CAO Must be 2.036
	Total		**6**	
2	$\sqrt{4+x} = 2\left(1+\frac{x}{4}\right)^{\frac{1}{2}} = 2\left[1+\frac{1}{2}\left(\frac{x}{4}\right)+O(x^2)\right]$	M1		Attempt to use binomial theorem OE The notation $O(x^n)$ can be replaced by a term of the form kx^n
	$\left[\frac{\sqrt{4+x}-2}{x+x^2}\right] = \left[\frac{\frac{x}{4}+O(x^2)}{x+x^2}\right] = \left[\frac{\frac{1}{4}+O(x)}{1+x}\right]$	m1		Division by x stage before taking the limit
	$\lim_{x\to 0}\left[\frac{\sqrt{4+x}-2}{x+x^2}\right] = \frac{1}{4}$	A1	3	CSO NMS 0/3
	Total		**3**	
3	$m^2+2m+10=0$	M1		PI
	$m = -1 \pm 3\text{i}$	A1		
	Complementary function is $(y =)\ \text{e}^{-x}(A\cos 3x + B\sin 3x)$	A1F		OE Ft on incorrect **complex value** of m
	Particular integral: try $y = k\text{e}^x$	M1		
	$k+2k+10k=26 \Rightarrow k=2$	A1		
	(GS $y =$) $\text{e}^{-x}(A\cos 3x + B\sin 3x) + 2\text{e}^x$	B1F		c's CF+ c's non-zero PI but must have 2 arb consts
	$x=0, y=5 \Rightarrow 5 = A+2$ so $A=3$	B1F		ft c's k ie $A = 5-k, k \neq 0$
	$\frac{\text{d}y}{\text{d}x} =$ $\text{e}^{-x}(-3A\sin 3x + 3B\cos 3x - A\cos 3x - B\sin 3x) + 2\text{e}^x$	M1		Attempt to differentiate c's **GS** (ie CF + PI)
	$11 = 3B - A + 2 \qquad (B=4)$	A1		
	$y = \text{e}^{-x}(3\cos 3x + 4\sin 3x) + 2\text{e}^x$	A1	10	CSO
	Total		**10**	

Q	Solution	Marks	Total	Comments
4(a)	IF is $\exp\left(\int \frac{2}{x}\mathrm{d}x\right)$	M1		and with integration attempted
	$= \mathrm{e}^{2\ln x}$	A1		PI
	$= x^2$	A1		
	$\frac{\mathrm{d}}{\mathrm{d}x}\left[yx^2\right] = x^2 \ln x$	M1		LHS; PI
	$\Rightarrow yx^2 = \int (\ln x)\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x^3}{3}\right)$	M1		Attempt integration by parts in correct direction to integrate $x^p \ln x$
	$= \frac{x^3}{3}\ln x - \int \frac{x^2}{3}\mathrm{d}x$	A1		RHS
	$yx^2 = \frac{x^3}{3}\ln x - \frac{x^3}{9} + A$			
	$\{\, y = \frac{x}{3}\ln x - \frac{x}{9} + Ax^{-2} \,\}$	A1	7	
(b)	Now, as $x \to 0$, $x^k \ln x \to 0$	E1		Must be stated explicitly for a value of $k > 0$
	As $x \to 0$, $y \to 0 \Rightarrow A = 0$	B1		Const of int = 0 must be convincing
	$yx^2 = \frac{x^3}{3}\ln x - \frac{x^3}{9}$			
	When $x = 1$, $y = -\frac{1}{9}$	B1F	3	ft on one slip but must have made a realistic attempt to find A
	Total		**10**	

Q	Solution	Marks	Total	Comments
5(a)	The interval of integration is infinite	E1	1	OE
(b)	$u = x^2 \text{ e}^{-4x} + 3 \Rightarrow \text{d}u = (2x\text{e}^{-4x} - 4x^2\text{e}^{-4x})\text{ d}x$	M1		d*u*/d*x* or 'better'
	$\int \frac{x(1-2x)}{x^2+3\text{e}^{4x}}\text{ d}x = \int \frac{1}{2} \times \frac{2x(1-2x)\text{e}^{-4x}}{x^2\text{e}^{-4x}+3}\text{ d}x$			
	$= \frac{1}{2} \times \int \frac{1}{u}\text{ d}u$	A1		
	$= \frac{1}{2}\ln u + c = \frac{1}{2}\ln\left(x^2\text{e}^{-4x}+3\right)\ \{+c\}$	A1	3	OE Condone missing *c*. Accept later substitution back if explicit
(c)	$\text{I} = \int_{\frac{1}{2}}^{\infty} \frac{x(1-2x)}{x^2+3\text{e}^{4x}}\text{ d}x$			
	$= \lim_{a\to\infty} \int_{\frac{1}{2}}^{a} \frac{x(1-2x)}{x^2+3\text{e}^{4x}}\text{ d}x$	M1		
	$= \lim_{a\to\infty} \frac{1}{2}\{\ln\left(a^2\text{e}^{-4a}+3\right) - \ln(\frac{\text{e}^{-2}}{4}+3)\}$	M1		Uses part (b) and F(*a*) – F(1/2)
	$= \frac{1}{2}\ln\{\lim_{a\to\infty}\left(a^2\text{e}^{-4a}+3\right)\} - \frac{1}{2}\ln(\frac{\text{e}^{-2}}{4}+3)$			
	Now $\lim_{a\to\infty}\left(a^2\text{e}^{-4a}\right) = 0$	E1		Stated explicitly (could be in general form)
	$\text{I} = \frac{1}{2}\ln 3 - \frac{1}{2}\ln(\frac{\text{e}^{-2}}{4}+3)$	A1	4	CSO ACF
	Total		**8**	

Q	Solution	Marks	Total	Comments
6(a)	$y = \ln\cos 2x \Rightarrow y'(x) = \dfrac{1}{\cos 2x}(-2\sin 2x)$	M1 A1		Chain rule
	$y''(x) = -4\sec^2 2x$	m1		$\lambda\sec^2 2x$ OE
	$y'''(x) = -8\sec 2x\,(2\sec 2x\tan 2x)$	M1		$K\sec^2 2x\tan 2x$ OE
	$\{y'''(x) = -16\tan 2x\,(\sec^2 2x)\}$			
	$y''''(x) = -16[2\sec^2 2x(\sec^2 2x) +$ $\tan 2x(2\sec 2x\,(2\sec 2x\tan 2x))]$	M1 A1	6	Product rule OE ACF
(b)	$y(0) = 0,\ y'(0) = 0,\ y''(0) = -4,\ y'''(0) = 0,$ $y''''(0) = -32$	B1F		ft c's derivatives
	$\ln\cos 2x \approx 0 + 0 + \dfrac{x^2}{2}(-4) + 0 + \dfrac{x^4}{4!}(-32)$	M1		
	$\approx -2x^2 - \dfrac{4}{3}x^4$	A1	3	CSO throughout parts (a) and (b) AG
(c)	$\ln(\sec^2 2x) = -2\ln(\cos 2x)$	M1		PI
	$\approx 4x^2 + \dfrac{8}{3}x^4$	A1	2	
	Total		**11**	

Q	Solution	Marks	Total	Comments
7(a)	$u = xy$			
	$\frac{\mathrm{d}u}{\mathrm{d}x} = y + x\frac{\mathrm{d}y}{\mathrm{d}x}$	M1 A1		Product rule OE OE
	$\frac{\mathrm{d}^2u}{\mathrm{d}x^2} = \frac{\mathrm{d}y}{\mathrm{d}x} + (\frac{\mathrm{d}y}{\mathrm{d}x} + x\frac{\mathrm{d}^2y}{\mathrm{d}x^2})$	A1		OE
	$x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2(3x+1)\frac{\mathrm{d}y}{\mathrm{d}x} + 3y(3x+2) = 18x$			
	$(x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}y}{\mathrm{d}x}) + 6(x\frac{\mathrm{d}y}{\mathrm{d}x} + y) + 9xy = 18x$			
	$\frac{\mathrm{d}^2u}{\mathrm{d}x^2} + 6\frac{\mathrm{d}u}{\mathrm{d}x} + 9u = 18x$	A1	4	CSO AG Be convinced
(b)	$\frac{\mathrm{d}^2u}{\mathrm{d}x^2} + 6\frac{\mathrm{d}u}{\mathrm{d}x} + 9u = 18x$			
	CF: Aux eqn $m^2 + 6m + 9 = 0$	M1		PI
	$(m+3)^2 = 0$ so $m = -3$	A1		PI
	CF: $(u =)\ \mathrm{e}^{-3x}(Ax + B)$	A1F		
	PI: Try $(u =)\ px + q$ $0 + 6p + 9(px + q) = 18x$	M1		PI. Must be more than just stated
	$9p = 18,\quad 6p + 9q = 0$	m1		
	$p = 2;\quad q = -\frac{12}{9}$	A1		Both
	$u = \mathrm{e}^{-3x}(Ax + B) + 2x - \frac{4}{3}$	B1F		c's CF + c's PI but must have 2 constants, also must be in the form $u = \mathrm{f}(x)$
	$xy = \mathrm{e}^{-3x}(Ax + B) + 2x - \frac{4}{3}$			
	$y = \frac{1}{x}\{\mathrm{e}^{-3x}(Ax + B) + 2x - \frac{4}{3}\}$	A1	8	
	Total		**12**	

Q	Solution	Marks	Total	Comments
8(a)	Area = $\frac{1}{2}\int(3+2\cos\theta)^2\,\mathrm{d}\theta$	M1		Use of $\frac{1}{2}\int r^2\,\mathrm{d}\theta$ or $\int_0^{\pi} r^2\,\mathrm{d}\theta$
	$= \frac{1}{2}\int_0^{2\pi}(9+12\cos\theta+4\cos^2\theta)\,\mathrm{d}\theta$	B1 B1		Correct expn of $[3+2\cos\theta]^2$ Correct limits
	$= \int_0^{2\pi}(4.5+6\cos\theta+(1+\cos 2\theta))\,\mathrm{d}\theta$	M1		Attempt to write $\cos^2\theta$ in terms of $\cos 2\theta$
	$= \left[4.5\theta+6\sin\theta+\theta+\frac{1}{2}\sin 2\theta\right]_0^{2\pi}$	A1F		Correct integration ft wrong coefficients
	$= 11\pi$	A1	6	CSO
(b)(i)	$x^2+y^2-8x+16=16$	M1		Use of **any two** of $x = r\cos\theta$, $y = r\sin\theta$, $x^2+y^2=r^2$
	$r^2-8r\cos\theta+16=16 \;\Rightarrow\; r=8\cos\theta$	A1		
	At intersection, $8\cos\theta = 3+2\cos\theta$ $\Rightarrow \cos\theta = \frac{3}{6}$	M1		Equating rs or equating $\cos\theta$s with a further step to solve eqn. (OE eg $4r = 12 + r \Rightarrow 4r - r = 12$)
	Points $\left(4, \frac{\pi}{3}\right)$ and $\left(4, \frac{5\pi}{3}\right)$	A1		OE
	$AB = 2\times\left(4\sin\frac{\pi}{3}\right)$	M1		Valid method to find AB, ft c's r and θ values
	$= 4\sqrt{3}$	A1	6	OE surd
(ii)	Let M=centre of circle then $\angle AMB = \frac{2\pi}{3}$	B1		Accept equiv eg $\angle AMO = \frac{\pi}{3}$
	Length of arc AOB of circle = $4\times\frac{2\pi}{3}$	M1		Use of arc = 4 × ($\angle AMB$ in rads)
	Perimeter of segment AOB = $\frac{8\pi}{3}+4\sqrt{3}$	A1	3	
	Total		**15**	
	Alternative to (b)(i): Writing $r = 3+2\cos\theta$ in cartesian form (M1A1) Finding cartesian coordinates of points A and B ie $(2, \pm 2\sqrt{2})$ (M1A1) Finding length AB (M1A1)			
	TOTAL		**75**	