General Certificate of Education
Advanced Level Examination
January 2011

Mathematics MFP3

Unit Further Pure 3

Monday 24 January 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where $$f(x, y) = x + \sqrt{y}$$

and $$y(3) = 4$$

Use the improved Euler formula

$$y_{r+1} = y_r + \tfrac{1}{2}(k_1 + k_2)$$

where $k_1 = hf(x_r, y_r)$ and $k_2 = hf(x_r + h, y_r + k_1)$ and $h = 0.1$, to obtain an approximation to $y(3.1)$, giving your answer to three decimal places. *(5 marks)*

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2 (a) Find the values of the constants p and q for which $p\sin x + q\cos x$ is a particular integral of the differential equation

$$\frac{dy}{dx} + 5y = 13\cos x$$ *(3 marks)*

(b) Hence find the general solution of this differential equation. *(3 marks)*

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3 A curve C has polar equation $r(1 + \cos\theta) = 2$.

(a) Find the cartesian equation of C, giving your answer in the form $y^2 = f(x)$. *(5 marks)*

(b) The straight line with polar equation $4r = 3\sec\theta$ intersects the curve C at the points P and Q. Find the length of PQ. *(4 marks)*

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4 By using an integrating factor, find the solution of the differential equation

$$\frac{dy}{dx} - \frac{2}{x}y = 2x^3 e^{2x}$$

given that $y = e^4$ when $x = 2$. Give your answer in the form $y = f(x)$. *(9 marks)*

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5 (a) Write $\dfrac{4}{4x+1} - \dfrac{3}{3x+2}$ in the form $\dfrac{C}{(4x+1)(3x+2)}$, where C is a constant. *(1 mark)*

(b) Evaluate the improper integral

$$\int_1^{\infty} \frac{10}{(4x+1)(3x+2)}\,\mathrm{d}x$$

showing the limiting process used and giving your answer in the form $\ln k$, where k is a constant. *(6 marks)*

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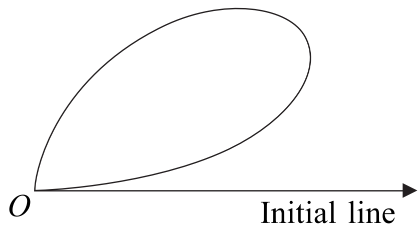
6 The diagram shows a sketch of a curve C.



The polar equation of the curve is

$$r = 2\sin 2\theta\sqrt{\cos\theta}\,, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

Show that the area of the region bounded by C is $\frac{16}{15}$. *(7 marks)*

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7 (a) Write down the expansions in ascending powers of x up to and including the term in x^3 of:

(i) $\cos x + \sin x\,;$ *(1 mark)*

(ii) $\ln(1+3x)\,.$ *(1 mark)*

(b) It is given that $y = \mathrm{e}^{\tan x}$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and show that $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = (1+\tan x)^2\,\dfrac{\mathrm{d}y}{\mathrm{d}x}$. *(5 marks)*

(ii) Find the value of $\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}$ when $x = 0$. *(2 marks)*

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(iii) Hence, by using Maclaurin's theorem, show that the first four terms in the expansion, in ascending powers of x, of $e^{\tan x}$ are

$$1 + x + \frac{1}{2}x^2 + \frac{1}{2}x^3$$ *(2 marks)*

(c) Find

$$\lim_{x \to 0} \left[\frac{e^{\tan x} - (\cos x + \sin x)}{x \ln(1 + 3x)} \right]$$ *(3 marks)*

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8 (a) Given that $x = e^t$ and that y is a function of x, show that

$$x \frac{dy}{dx} = \frac{dy}{dt}$$ *(2 marks)*

(b) Hence show that the substitution $x = e^t$ transforms the differential equation

$$x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 4y = 2 \ln x$$

into

$$\frac{d^2y}{dt^2} - 4 \frac{dy}{dt} + 4y = 2t$$ *(5 marks)*

(c) Find the general solution of the differential equation

$$\frac{d^2y}{dt^2} - 4 \frac{dy}{dt} + 4y = 2t$$ *(6 marks)*

(d) Hence solve the differential equation $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 4y = 2 \ln x$, given that $y = \frac{3}{2}$ and $\frac{dy}{dx} = \frac{1}{2}$ when $x = 1$. *(5 marks)*