General Certificate of Education
Advanced Level Examination
January 2010

Mathematics MFP3

Unit Further Pure 3

Tuesday 19 January 2010 9.00 am to 10.30 am

For this paper you must have:
- a 12-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MFP3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The function $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where $$\mathrm{f}(x, y) = x \ln(2x + y)$$

and $$y(3) = 2$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h\,\mathrm{f}(x_r, y_r)$$

with $h = 0.1$, to obtain an approximation to $y(3.1)$, giving your answer to four decimal places. *(3 marks)*

(b) Use the improved Euler formula

$$y_{r+1} = y_r + \tfrac{1}{2}(k_1 + k_2)$$

where $k_1 = h\,\mathrm{f}(x_r, y_r)$ and $k_2 = h\,\mathrm{f}(x_r + h, y_r + k_1)$ and $h = 0.1$, to obtain an approximation to $y(3.1)$, giving your answer to four decimal places. *(5 marks)*

2 (a) Given that $y = \ln(4 + 3x)$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$. *(3 marks)*

(b) Hence, by using Maclaurin's theorem, find the first three terms in the expansion, in ascending powers of x, of $\ln(4 + 3x)$. *(2 marks)*

(c) Write down the first three terms in the expansion, in ascending powers of x, of $\ln(4 - 3x)$. *(1 mark)*

(d) Show that, for small values of x,

$$\ln\left(\frac{4 + 3x}{4 - 3x}\right) \approx \frac{3}{2}x$$ *(2 marks)*

3 (a) A differential equation is given by

$$x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\frac{\mathrm{d}y}{\mathrm{d}x}=3x$$

Show that the substitution

$$u=\frac{\mathrm{d}y}{\mathrm{d}x}$$

transforms this differential equation into

$$\frac{\mathrm{d}u}{\mathrm{d}x}+\frac{2}{x}u=3$$ *(2 marks)*

(b) Find the general solution of

$$\frac{\mathrm{d}u}{\mathrm{d}x}+\frac{2}{x}u=3$$

giving your answer in the form $u=\mathrm{f}(x)$. *(5 marks)*

(c) Hence find the general solution of the differential equation

$$x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\frac{\mathrm{d}y}{\mathrm{d}x}=3x$$

giving your answer in the form $y=\mathrm{g}(x)$. *(2 marks)*

4 (a) Write down the expansion of $\sin 3x$ in ascending powers of x up to and including the term in x^3. *(1 mark)*

(b) Find

$$\lim_{x\to 0}\left[\frac{3x\cos 2x-\sin 3x}{5x^3}\right]$$ *(4 marks)*

5 It is given that y satisfies the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 2e^{-2x}$$

(a) Find the value of the constant p for which $y = pxe^{-2x}$ is a particular integral of the given differential equation. *(4 marks)*

(b) Solve the differential equation, expressing y in terms of x, given that $y = 2$ and $\frac{dy}{dx} = 0$ when $x = 0$. *(8 marks)*

6 (a) Explain why $\int_1^{\infty} \frac{\ln x^2}{x^3}\,dx$ is an improper integral. *(1 mark)*

(b) (i) Show that the substitution $y = \frac{1}{x}$ transforms $\int \frac{\ln x^2}{x^3}\,dx$ into $\int 2y \ln y\,dy$. *(2 marks)*

(ii) Evaluate $\int_0^1 2y \ln y\,dy$, showing the limiting process used. *(5 marks)*

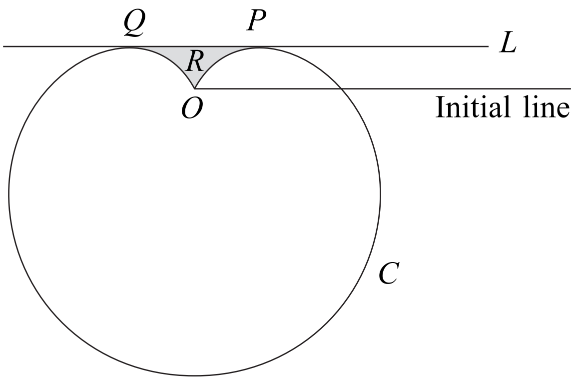
(iii) Hence write down the value of $\int_1^{\infty} \frac{\ln x^2}{x^3}\,dx$. *(1 mark)*

7 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4y = 8x^2 + 9\sin x$$

(8 marks)

8 The diagram shows a sketch of a curve C and a line L, which is parallel to the initial line and touches the curve at the points P and Q.



The polar equation of the curve C is

$$r = 4(1 - \sin\theta), \qquad 0 \leqslant \theta < 2\pi$$

and the polar equation of the line L is

$$r\sin\theta = 1$$

(a) Show that the polar coordinates of P are $\left(2, \frac{\pi}{6}\right)$ and find the polar coordinates of Q. *(5 marks)*

(b) Find the area of the shaded region R bounded by the line L and the curve C. Give your answer in the form $m\sqrt{3} + n\pi$, where m and n are integers. *(11 marks)*

END OF QUESTIONS

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