General Certificate of Education
January 2007
Advanced Level Examination

MATHEMATICS **MFP3**
Unit Further Pure 3

Friday 26 January 2007 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where $$f(x, y) = \ln(1 + x^2 + y)$$

and $$y(1) = 0.6$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h\,f(x_r, y_r)$$

with $h = 0.05$, to obtain an approximation to $y(1.05)$, giving your answer to four decimal places. *(3 marks)*

(b) Use the improved Euler formula

$$y_{r+1} = y_r + \tfrac{1}{2}(k_1 + k_2)$$

where $k_1 = h\,f(x_r, y_r)$ and $k_2 = h\,f(x_r + h, y_r + k_1)$ and $h = 0.05$, to obtain an approximation to $y(1.05)$, giving your answer to four decimal places. *(6 marks)*

2 A curve has polar equation $r(1 - \sin\theta) = 4$. Find its cartesian equation in the form $y = f(x)$. *(6 marks)*

3 (a) Show that x^2 is an integrating factor for the first-order differential equation

$$\frac{dy}{dx} + \frac{2}{x}y = 3(x^3 + 1)^{\frac{1}{2}}$$ *(3 marks)*

(b) Solve this differential equation, given that $y = 1$ when $x = 2$. *(6 marks)*

4 (a) Explain why $\int_0^{e} \frac{\ln x}{\sqrt{x}}\, dx$ is an improper integral. *(1 mark)*

(b) Use integration by parts to find $\int x^{-\frac{1}{2}} \ln x \, dx$. *(3 marks)*

(c) Show that $\int_0^{e} \frac{\ln x}{\sqrt{x}}\, dx$ exists and find its value. *(4 marks)*

5 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 6 + 5\sin x$$ *(12 marks)*

6 The function f is defined by $f(x) = (1 + 2x)^{\frac{1}{2}}$.

(a) (i) Find $f'''(x)$. *(4 marks)*

(ii) Using Maclaurin's theorem, show that, for small values of x,

$$f(x) \approx 1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3$$ *(4 marks)*

(b) Use the expansion of e^x together with the result in part (a)(ii) to show that, for small values of x,

$$e^x(1 + 2x)^{\frac{1}{2}} \approx 1 + 2x + x^2 + kx^3$$

where k is a rational number to be found. *(3 marks)*

(c) Write down the first four terms in the expansion, in ascending powers of x, of e^{2x}. *(1 mark)*

(d) Find

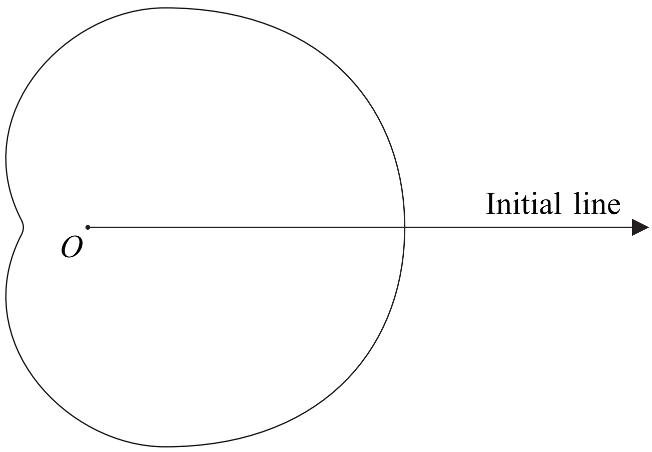
$$\lim_{x \to 0} \frac{e^x(1 + 2x)^{\frac{1}{2}} - e^{2x}}{1 - \cos x}$$ *(4 marks)*

Turn over for the next question

7 A curve C has polar equation

$$r = 6 + 4\cos\theta, \qquad -\pi \leqslant \theta \leqslant \pi$$

The diagram shows a sketch of the curve C, the pole O and the initial line.



(a) Calculate the area of the region bounded by the curve C. *(6 marks)*

(b) The point P is the point on the curve C for which $\theta = \dfrac{2\pi}{3}$.

The point Q is the point on C for which $\theta = \pi$.

Show that QP is parallel to the line $\theta = \dfrac{\pi}{2}$. *(4 marks)*

(c) The line PQ intersects the curve C again at a point R.

The line RO intersects C again at a point S.

(i) Find, in surd form, the length of PS. *(4 marks)*

(ii) Show that the angle OPS is a right angle. *(1 mark)*

END OF QUESTIONS