AQA Maths Further Pure 3

Mark Scheme Pack

2006-2015

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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MFP3 Further Pure 3

Mark Scheme

2006 examination - January series

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Key To Mark Scheme And Abbreviations Used In Marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–x EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP3

Q	Solution	Marks	Total	Comments
1(a)	$(m+1)^2 = -1$	M1		Completing sq or formula
	$m = -1 \pm i$	A1	2	
(b)(i)	CF is $e^{-x}(A\cos x + B\sin x)$	M1		If m is real give M0
	{or $e^{-x}A\cos(x+B)$ **but not** $Ae^{(-1+i)x} + Be^{(-1-i)x}$}	A1√		On wrong a's and b's but roots must be complex.
	{P.Int.} try $y = px + q$	M1		OE
	$2p + 2(px+q) = 4x$	A1		
	$p = 2,\quad q = -2$	A1√		On one slip
	GS $y = e^{-x}(A\cos x + B\sin x) + 2x - 2$	B1√	6	Their CF + their PI with two arbitrary constants.
(ii)	$x=0,\ y=1 \Rightarrow A = 3$	B1√		Provided an M1gained in (b)(i)
	$y'(x) = -e^{-x}(A\cos x + B\sin x) +$	M1		Product rule used
	$+ e^{-x}(-A\sin x + B\cos x) + 2$	A1√		
	$y'(0) = 2 \Rightarrow 2 = -A+B+2 \Rightarrow B = 3$	A1√		Slips
	$y = 3e^{-x}(\cos x + \sin x) + 2x - 2$		4	
	Total		**12**	
2(a)	$\int xe^{-2x}dx = -\frac{1}{2}xe^{-2x} - \int -\frac{1}{2}e^{-2x}dx$	M1 A1		Reasonable attempt at parts
	$= -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}\ \{+c\}$	A1√		Condone absence of $+c$
	$\int_0^a xe^{-2x}dx = -\frac{1}{2}ae^{-2a} - \frac{1}{4}e^{-2a} - (0 - \frac{1}{4})$	M1		F(a) – F(0)
	$= \frac{1}{4} - \frac{1}{2}ae^{-2a} - \frac{1}{4}e^{-2a}$	A1	5	
(b)	$\lim_{a\to\infty} a^k e^{-2a} = 0$	B1	1	
(c)	$\int_0^\infty xe^{-2x}dx =$			
	$= \lim_{a\to\infty}\{\frac{1}{4} - \frac{1}{2}ae^{-2a} - \frac{1}{4}e^{-2a}\}$	M1		If this line oe is missing then 0/2
	$= \frac{1}{4} - 0 - 0 = \frac{1}{4}$	A1√	2	On candidate's "1/4" in part (a). B1 must have been earned
	Total		**8**	

MFP3

Q	Solution	Marks	Total	Comments
3(a)	$y = x^3 - x \Rightarrow y'(x) = 3x^2 - 1$	B1		Accept general cubic.
	$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2xy}{x^2-1} = 3x^2 - 1 + \frac{2x(x^3-x)}{x^2-1}$	M1		Substitution into LHS of DE
	$= 3x^2 - 1 + \frac{2x^2(x^2-1)}{x^2-1} = 5x^2 - 1$	A1	3	Completion. If using general cubic all unknown constants must be found
(b)	$\frac{\mathrm{d}}{\mathrm{d}x}\left[(x^2-1)y\right] = 2xy + (x^2-1)\frac{\mathrm{d}y}{\mathrm{d}x}$	M1A1		
	Differentiating $(x^2-1)y = c$ wrt x leads to $2xy + (x^2-1)\frac{\mathrm{d}y}{\mathrm{d}x} = 0$			**SC** Differentiated but not implicitly give max of 1/3 for complete solution
	$\Rightarrow y = \frac{c}{x^2-1}$ is a soln. of $\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2xy}{x^2-1} = 0$	A1	3	Be generous
(c)	$\Rightarrow y = \frac{c}{x^2-1}$ is a soln with one arb. constant of $\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2xy}{x^2-1} = 0$			
	$\Rightarrow y = \frac{c}{x^2-1}$ is a CF of the DE			
	GS is CF + PI	M1		Must be using 'hence'; CF and PI functions of x only
	$y = \frac{c}{x^2-1} + x^3 - x$	A1	2	CSO Must have explicitly considered the link between one arbitrary constant and the GS of a first order differential equation.
	Total		**8**	

MFP3

Q	Solution	Marks	Total	Comments
4(a)	$\ln(1-x) = -x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4 \ldots$	B1	1	
(b)(i)	$f(x) = e^{\sin x} \Rightarrow f(0) = 1$	B1		
	$f'(x) = \cos x\, e^{\sin x}$ $\Rightarrow f'(0) = 1$	M1A1		
	$f''(x) = -\sin x\, e^{\sin x} + \cos^2 x\, e^{\sin x}$ $f''(0) = 1$	M1A1		Product rule used
	Maclaurin $f(x) = f(0) + xf'(0) + \frac{x^2}{2}f''(0)$			
	so 1st three terms are $1 + x + \frac{1}{2}x^2$	A1	6	**CSO AG**
(ii)	$f'''(x) = \cos x(\cos^2 x - \sin x)\, e^{\sin x} +$ $+\{2\cos x(-\sin x) - \cos x\}\, e^{\sin x}$	M1A1		
	$f'''(0) = 0$ so the coefficient of x^3 in the series is zero	A1	3	**CSO AG** **SC** for (b): Use of series expansions….max of 4/9
(c)	$\sin x \approx x$.	B1		Ignore higher power terms in sin$x$ expansion
	$\frac{e^{\sin x} - 1 + \ln(1-x)}{x^2 \sin x} = \frac{-\frac{1}{3}x^3 + o(x^4)}{x^3}$	M1 A1		Series from (a) & (b) used Numerator kx^3 (+…)
	$= \frac{-\frac{1}{3} + o(x)}{1 + o(x^2)}$			Condone if this step is missing
	$\lim_{x \to 0} \frac{e^{\sin x} - 1 + \ln(1-x)}{x^2 \sin x} = -\frac{1}{3}$	A1√	4	On candidate's x^3 coefficient in (a) provided lower powers cancel
	Total		**14**	

MFP3

Q	Solution	Marks	Total	Comments
5(a)(i)	$y(1.1) = y(1) + 0.1[1\ln 1+1/1]$	M1A1		
	$= 1+0.1 = 1.1$	A1	3	
(ii)	$y(1.2) = y(1) + 2(0.1)[f(1.1, y(1.1)]$	M1A1		
	$\ldots. = 1+2(0.1)[1.1\ln 1.1+(1.1)/1.1]$	A1√		On answer to (a)(i)
	$\ldots. = 1+0.2\times1.104841198\ldots.$			
	$\ldots. = 1.22096824 = 1.221$ to 3dp	A1	4	CAO
(b)(i)	IF is $e^{\int -\frac{1}{x}\,dx}$	M1		Condone $e^{\int \frac{1}{x}dx}$ for M mark
	$= e^{-\ln x}$	A1		
	$= e^{\ln x^{-1}} = x^{-1} = \frac{1}{x}$	A1	3	**AG** (be convinced) (b)(i) Solutions using the printed answer must be convincing before any marks are awarded
(ii)	$\frac{d}{dx}\left(\frac{y}{x}\right) = \ln x$	M1A1		
	$\frac{y}{x} = \int \ln x\,dx = x\ln x - \int x\left(\frac{1}{x}\right)dx$	M1		Integration by parts for $x^k \ln x$
	$\frac{y}{x} = x\ln x - x + c$	A1		Condone missing c.
	$y(1) = 1 \Rightarrow 1 = \ln 1 - 1 + c$	m1		Dependent on at least one of the two previous M marks
	$\Rightarrow c = 2 \Rightarrow y = x^2\ln x - x^2 + 2x$	A1	6	OE eg $\frac{y}{x} = x\ln x - x + 2$
(iii)	$y(1.2) = 1.222543\ldots = 1.223$ to 3dp	B1	1	
	Total		**17**	

MFP3

Q	Solution	Marks	Total	Comments
6(a)	$x^2+y^2-12y+36=36$	M1 M1 m1		Use of $y=r\sin\theta$ ($x=r\cos\theta$ PI) Use of $x^2+y^2=r^2$
	$r^2-12r\sin\theta+36=36$			
	$\Rightarrow r=12\sin\theta$	A1	4	**CSO AG**
(b)	Area = $\frac{1}{2}\int(2\sin\theta+5)^2\,\mathrm{d}\theta$.	M1		Use of $\frac{1}{2}\int r^2\,\mathrm{d}\theta$.
	.. = $\frac{1}{2}\int_0^{2\pi}(4\sin^2\theta+20\sin\theta+25)\mathrm{d}\theta$	B1 B1		Correct expn. of $(2\sin\theta+5)^2$ Correct limits
	$=\frac{1}{2}\int_0^{2\pi}(2(1-\cos2\theta)+20\sin\theta+25)\mathrm{d}\theta$	M1		Attempt to write $\sin^2\theta$ in terms of $\cos 2\theta$.
	$=\frac{1}{2}\left[27\theta-\sin2\theta-20\cos\theta\right]_0^{2\pi}$	A1√		Correct integration ft wrong coeffs
	$=27\pi$.	A1	6	CSO
(c)	At intersection $12\sin\theta=2\sin\theta+5$	M1		OE eg $r=6(r-5)$
	$\Rightarrow \sin\theta=\frac{5}{10}$	A1		OE eg $r=6$
	Points $\left(6,\frac{\pi}{6}\right)$ and $\left(6,\frac{5\pi}{6}\right)$	A1		OE
	OPMQ is a rhombus of side 6			Or two equilateral triangles of side 6
	Area = $6\times6\times\sin\frac{2\pi}{3}$ oe	M1 A1		Any valid complete method to find the area (or half area) of quadrilateral.
	$=18\sqrt{3}$	A1	6	Accept unsimplified surd
	Total		**16**	
	Total		**75**	

Extra notes:

The SC for Q4

$$e^{\sin x}=1+\left(x-\frac{x^3}{3!}\dots\right)+\frac{1}{2!}\left(x-\frac{x^3}{3!}\dots\right)^2+\frac{1}{3!}\left(x-\frac{x^3}{3!}\dots\right)^3\dots$$

M1 for 1st 3 terms ignoring any higher powers than those shown.

A1 for all 4 terms (could be treated separately ie last term often only comes into (b)(ii)

$= 1 + x - \frac{x^3}{6} + \frac{1}{2}\left(x^2 - ....\right) + \frac{1}{6}\left(x^3 - ....\right)$

$= 1 + x + \frac{1}{2}x^2$ **A1 (be convinced…..ignore any powers of *x* above power 2)**

Coefficient of x^3: $-\frac{x^3}{6} + \frac{1}{6}x^3 \quad = 0$ **A1 (be convinced…..ignore any powers of *x* above power 3)**

Quite often the 2nd A mark is awarded before the 1st A1

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1(a)	$y = 2x + \sin 2x \Rightarrow y' = 2 + 2\cos 2x$ $\Rightarrow y'' = -4\sin 2x$	M1 A1		Need to attempt both y' and y''
	$-4\sin 2x - 5(2 + 2\cos 2x) + 4(2x + \sin 2x) =$ $8x - 10 - 10\cos 2x$	A1	3	CSO AG Substitute. and confirm correct
(b)	Auxiliary equation $m^2 - 5m + 4 = 0$	M1		
	$m = 4$ and 1	A1		
	CF: $A\,\mathrm{e}^{4x} + B\,\mathrm{e}^{x}$	M1		
	GS: $y = A\,\mathrm{e}^{4x} + B\,\mathrm{e}^{x} + 2x + \sin 2x$	B1√	4	Their CF + $2x$ + $\sin 2x$
(c)	$x = 0, y = 2 \Rightarrow \quad 2 = A + B$	B1√		Only ft if exponentials in GS
	$x = 0, y' = 0 \Rightarrow \quad 0 = 4A + B + 4$	B1√		Only ft if exponentials in GS and differentiated four terms at least
	Solving the simultaneous equations	M1		
	gives $A = -2$ and $B = 4$ $y = -2\mathrm{e}^{4x} + 4\,\mathrm{e}^{x} + 2x + \sin 2x$	A1	4	
	Total		**11**	
2(a)	$y_1 = 2 + 0.1 \times \left[\dfrac{1^2 + 2^2}{1 \times 2}\right]$	M1 A1		
	$= 2 + 0.1 \times 2.5 = 2.25$	A1	3	
(b)	$k_1 = 0.1 \times 2.5 = 0.25$	M1 A1√		PI ft from (a)
	$k_2 = 0.1 \times \mathrm{f}(1.1,\ 2.25)$	M1		
	$\ldots = 0.1 \times 2.53434\ldots = 0.2534(34\ldots)$	A1√		PI
	$y(1.1) = y(1) + \frac{1}{2}[0.25 + 0.253434\ldots]$	m1		
	$= 2.2517$ to 4dp	A1√	6	If answer not to 4dp withhold this mark
	Total		**9**	
3(a)	IF is $\mathrm{e}^{\int \cot x \mathrm{d}x}$	M1		
	$= \mathrm{e}^{\ln \sin x}$	A1		
	$= \sin x$	A1	3	AG
(b)	$\dfrac{\mathrm{d}}{\mathrm{d}x}(y \sin x) = 2\sin x \cos x$	M1 A1		
	$y \sin x = \int \sin 2x \,\mathrm{d}x$	M1		Method to integrate $2\sin x\cos x$
	$y \sin x = -\frac{1}{2}\cos 2x + c$	A1		OE
	$y = 2$ when $x = \dfrac{\pi}{2} \Rightarrow$			
	$2\sin\dfrac{\pi}{2} = -\dfrac{1}{2}\cos \pi + c$	m1		Depending on at least one M
	$c = \dfrac{3}{2} \Rightarrow y \sin x = \dfrac{1}{2}(3 - \cos 2x)$	A1	6	OE eg $y \sin x = \sin^2 x + 1$
	Total		**9**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
4(a)	Area = $\frac{1}{2}\int 36(1-\cos\theta)^2\,\mathrm{d}\theta$	M1		use of $\frac{1}{2}\int r^2\,\mathrm{d}\theta$
	$\ldots = \frac{1}{2}\int_0^{2\pi} 36(1-2\cos\theta+\cos^2\theta)\,\mathrm{d}\theta$	B1 B1		for correct explanation of $[6(1-\cos\theta)]^2$ for correct limits
	$= 9\int_0^{2\pi} 2-4\cos\theta+(\cos 2\theta+1)\,\mathrm{d}\theta$	M1		Attempt to write $\cos^2\theta$ in terms of $\cos 2\theta$.
	$= \left[27\theta - 36\sin\theta + \frac{9}{2}\sin 2\theta\right]_0^{2\pi}$	A1√		Correct integration; only ft if integrating $a + b\cos\theta + c\cos 2\theta$ with non-zero a, b, c.
	$= 54\pi$	A1	6	CSO
(b)(i)	$x^2+y^2=9 \Rightarrow r^2=9$	B1		PI
	A & B: $3 = 6-6\cos\theta \Rightarrow \cos\theta = \frac{1}{2}$	M1		
	Pts of intersection $\left(3, \frac{\pi}{3}\right)$; $\left(3, \frac{5\pi}{3}\right)$	A1 A1√	4	OE (accept 'different' values of θ not in the given interval)
(ii)	Length $AB = 2\times r\sin\theta$	M1		
	……….. $= 2\times 3\times\frac{\sqrt{3}}{2} = 3\sqrt{3}$	A1	2	OE exact surd form
	Total		**12**	
5(a)	$\Rightarrow \lim_{a\to\infty}\left(\dfrac{3+\frac{2}{a}}{2+\frac{3}{a}}\right) = \dfrac{3+0}{2+0} = \dfrac{3}{2}$	M1 A1	2	
(b)	$\int_1^{\infty} \frac{3}{(3x+2)} - \frac{2}{2x+3}\,\mathrm{d}x$			
	$= \left[\ln(3x+2) - \ln(2x+3)\right]_1^{\infty}$	M1 A1		$a\ln(3x+2) + b\ln(2x+3)$
	$= \left[\ln\left(\frac{3x+2}{2x+3}\right)\right]_1^{\infty}$	m1		
	$= \ln\left\{\lim_{a\to\infty}\left(\frac{3a+2}{2a+3}\right)\right\} - \ln 1$	M1		
	$= \ln\frac{3}{2} - \ln 1 = \ln\frac{3}{2}$	A1	5	CSO
	Total		**7**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$u=\frac{\mathrm{d}y}{\mathrm{d}x}+2y \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x}=\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\frac{\mathrm{d}y}{\mathrm{d}x}$	M1 A1		2 terms correct
	LHS of DE$\Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x}-2\frac{\mathrm{d}y}{\mathrm{d}x}+4\frac{\mathrm{d}y}{\mathrm{d}x}+4y$			
	LHS: $\frac{\mathrm{d}u}{\mathrm{d}x}+2(u-2y)+4y$	M1		Substitution into LHS of DE as far as no derivatives of y
	$\Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x}+2u=\mathrm{e}^{-2x}$	A1	4	CSO AG
(b)	IF is $\mathrm{e}^{\int 2\mathrm{d}x}=\mathrm{e}^{2x}$	B1		
	$\frac{\mathrm{d}}{\mathrm{d}x}\left[u\mathrm{e}^{2x}\right]=1$	M1 A1		
	$\Rightarrow u\mathrm{e}^{2x}=x+A$	A1		
	$\Rightarrow u=x\mathrm{e}^{-2x}+A\mathrm{e}^{-2x}$	A1	5	
	Alternative : Those using CF+PI Auxiliary equation,			
	$m+2=0 \Rightarrow u_{CF}=A\mathrm{e}^{-2x}$	B1		
	For u_{PI} try $u_{PI}=kx\mathrm{e}^{-2x} \Rightarrow$	M1		
	$k\mathrm{e}^{-2x}-2kx\mathrm{e}^{-2x}+2kx\mathrm{e}^{-2x}\{=\mathrm{e}^{-2x}\}$	A1		LHS
	$\Rightarrow k=1 \Rightarrow u_{PI}=x\mathrm{e}^{-2x}$	A1		
	$\Rightarrow u_{GS}=A\mathrm{e}^{-2x}+x\mathrm{e}^{-2x}$	A1		
(c)	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}+2y=x\mathrm{e}^{-2x}+A\mathrm{e}^{-2x}$	M1		Use (b) to reach a 1st order DE in y and x
	IF is $\mathrm{e}^{\int 2\mathrm{d}x}=\mathrm{e}^{2x}$	B1		
	$\Rightarrow \frac{\mathrm{d}}{\mathrm{d}x}\left[y\mathrm{e}^{2x}\right]=x+A$	A1√		
	$\Rightarrow y\mathrm{e}^{2x}=\frac{x^2}{2}+Ax+B$	A1√		
	$\Rightarrow y=\mathrm{e}^{-2x}\left(\frac{x^2}{2}+Ax+B\right)$	A1	5	
	Total		**14**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$(1+y)^{-1}=1-y+y^2......$	B1	1	
(ii)	$\sec x \approx \dfrac{1}{1-\dfrac{x^2}{2}+\dfrac{x^4}{24}....}$	B1		
	$=\left[1-\dfrac{x^2}{2}+\dfrac{x^4}{24}....\right]^{-1}=$	M1		
	$\left\{1-\left(-\dfrac{x^2}{2}+\dfrac{x^4}{24}\right)+\left(-\dfrac{x^2}{2}+\dfrac{x^4}{24}\right)^2\right\}$	M1		
	$=\left\{1+\dfrac{x^2}{2}-\dfrac{x^4}{24}+\dfrac{x^4}{4}+...\right\}$			
	$=1+\dfrac{x^2}{2};+\dfrac{5x^4}{24}$	A1;A1	5	AG be convinced
	Alternative: Those using Maclaurin			
	f(x) = sec x			
	f(0) = 1; f′(x) = sec$x$ tan$x$; {f′(0) = 0}	(B1)		
	f″(x) = sec$x$ tan^2x + sec^3x; f ″(0) = 1	(M1)		Product rule oe
	f‴(x) = sec$x$ tan^3x + 5tan$x$ sec^3x;	(m1)		Chain rule with product rule OE
	f$^{(iv)}$(x) = sec$x$ tan^4x +18tan^2x sec^3x … +5sec^5x $\Rightarrow$ f$^{(iv)}$(0) = 5			
	sec $x \approx$ printed result	(A2)		CSO AG
(b)	f(x) = tan x;			
	f(0) = 0; f′(x) = sec^2x; {f′(0) = 1}	B1		
	f″(x) = 2sec$x$(sec$x$ tan$x$); f ″(0) = 0			
	f‴(x) = 4sec$x$ tan$x$(sec$x$ tan$x$) + 2sec^4x	M1		Chain rule with product rule oe
	f‴(0) = 2			
	$\tan x = 0+1x+0x^2+\dfrac{2}{3!}x^3\ldots= x+\dfrac{1}{3}x^3$	A1	3	CSO AG
	Alternative: Those using otherwise			
	$..=\dfrac{\sin x}{\cos x}\approx\left(x-\dfrac{x^3}{6}...\right)\left(1+\dfrac{x^2}{2}...\right)$	(M1) (A1)		
	$=x+\dfrac{x^3}{2}-\dfrac{x^3}{6}...=x+\dfrac{1}{3}x^3....$	(A1)		
(c)	$\left(\dfrac{x\tan 2x}{\sec x-1}\right)=\dfrac{x\left(2x+o(x^3)\right)}{\dfrac{x^2}{2}+o(x^4)}$	B1 M1		$\tan 2x = 2x+\dfrac{1}{3}(2x)^3$ Condone $o(x^k)$ missing
	$=\dfrac{2+o(x^2)}{\dfrac{1}{2}+o(x^2)}$	M1		
	$\lim\limits_{x\to 0}\left(\dfrac{x\tan 2x}{\sec x-1}\right)=4$	A1√	4	ft on $2k$ after B0 for tan2x = kx+…
	Total		**13**	
	TOTAL		**75**	

Version 1.0: 0107

General Certificate of Education

Mathematics 6360

MFP3 Further Pure 3

Mark Scheme

2007 examination - January series

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Set and published by the Assessment and Qualifications Alliance.

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m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP3

Q	Solution	Marks	Total	Comments
1(a)	$y\ (1.05) = 0.6 + 0.05 \times [\ln(1+1+0.6)]$ $= 0.6477(7557..) = 0.6478$ to 4dp	M1A1 A1	3	Condone >4 dp
(b)	$k_1 = 0.05 \times \ln(1+1+0.6) = 0.0477(75\ldots)$	M1 A1F		PI ft candidate's evaluation in (a)
	$k_2 = 0.05 \times \mathrm{f}\ (1.05,\ 0.6477\ldots)$ $\ldots = 0.05 \times \ln(1 + 1.05^2 + 0.6477\ldots)]$	M1		
	$\ldots = 0.0505(85\ldots)$	A1F		PI
	$y(1.05) = y(1) + \frac{1}{2}[k_1 + k_2]$ $= 0.6 + 0.5 \times 0.09836\ldots$	m1		Dep on previous two Ms and numerical values for k's
	$= 0.6492$ to 4dp	A1F	6	Must be 4 dp… ft one slip
	Total		**9**	
2	$r - r\sin\theta = 4$	M1		
	$r - y = 4$	B1		$r\sin\theta = y$ stated or used
	$r = y + 4$	A1		
	$x^2 + y^2 = (y+4)^2$	M1		$r^2 = x^2 + y^2$ used
	$x^2 + y^2 = y^2 + 8y + 16$	A1F		ft one slip
	$y = \dfrac{x^2 - 16}{8}$	A1	6	
	Total		**6**	
3(a)	IF is $\exp\left(\int \frac{2}{x}\mathrm{d}x\right)$	M1		And with integration attempted
	$= \mathrm{e}^{2\ln x}$	A1		
	$= x^2$	A1	3	CSO **AG** be convinced
(b)	$\frac{\mathrm{d}}{\mathrm{d}x}\left[yx^2\right] = 3x^2(x^3+1)^{\frac{1}{2}}$	M1A1		PI
	$\Rightarrow yx^2 = \frac{2}{3}\left(x^3+1\right)^{\frac{3}{2}} + A$	m1		$k\left(x^3+1\right)^{\frac{3}{2}}$
		A1		Condone missing 'A'
	$\Rightarrow 4 = \frac{2}{3}(9)^{\frac{3}{2}} + A$	m1		Use of boundary conditions to find constant
	$\Rightarrow A = -14$			
	$\Rightarrow y = x^{-2}\left\{\frac{2}{3}\left(x^3+1\right)^{\frac{3}{2}} - 14\right\}$	A1	6	Any correct form
	Total		**9**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
4(a)	Integrand is not defined at $x = 0$	E1	1	OE
(b)	$\int x^{-\frac{1}{2}} \ln x \, \mathrm{d}x = 2x^{\frac{1}{2}} \ln x - \int 2x^{\frac{1}{2}} \left(\frac{1}{x}\right) \mathrm{d}x$	M1		$... = kx^{\frac{1}{2}} \ln x \pm \int \mathrm{f}(x)$, with $\mathrm{f}(x)$ not involving the 'original' $\ln x$
		A1		
	$...... = 2x^{\frac{1}{2}} \ln x - 4x^{\frac{1}{2}} (+c)$	A1	3	Condone absence of '$+ c$'
(c)	$\int_0^{\mathrm{e}} \frac{\ln x}{\sqrt{x}} \mathrm{d}x = \lim_{a \to 0} \int_a^{\mathrm{e}} \frac{\ln x}{\sqrt{x}} \mathrm{d}x$	M1		
	$= -2\mathrm{e}^{\frac{1}{2}} - \lim_{a \to 0} \left[2a^{\frac{1}{2}} \ln a - 4a^{\frac{1}{2}} \right]$	M1		$\mathrm{F}(b) - \mathrm{F}(a)$
	But $\lim_{a \to 0} a^{\frac{1}{2}} \ln a = 0$	B1		Accept a general form e.g. $\lim_{x \to 0} x^k \ln x = 0$
	So $\int_0^{\mathrm{e}} \frac{\ln x}{\sqrt{x}} \mathrm{d}x$ exists and $= -2\mathrm{e}^{\frac{1}{2}}$	A1	4	
	Total		**8**	
5	Auxl. eqn $m^2 - 4m + 3 = 0$	M1		PI
	$m = 3$ and 1	A1		PI
	CF is $A\mathrm{e}^{3x} + B\mathrm{e}^{x}$	A1F		
	PI Try $y = a + b\sin x + c\cos x$	M1		Condone 'a' missing here
	$y'(x) = b\cos x - c\sin x$	A1		
	$y''(x) = -b\sin x - c\cos x$	A1F		ft can be consistent sign error(s)
	Substitute into DE gives	M1		
	$a = 2$	B1		
	$4c + 2b = 5$ and $2c - 4b = 0$	A1		
	$b = 0.5$,	A1F		ft a slip
	$c = 1$	A1F		ft a slip
	GS: $y = A\mathrm{e}^{3x} + B\mathrm{e}^{x} + 2 + 0.5\sin x + \cos x$	B1F	12	$y =$ candidate's CF and candidate's PI (must have exactly two arbitrary constants)
	Total		**12**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$f'(x)=\frac{1}{2}(1+2x)^{-\frac{1}{2}}(2)=(1+2x)^{-\frac{1}{2}}$	M1A1		
	$f''(x)=-(1+2x)^{-\frac{3}{2}}$	A1F		ft a slip
	$f'''(x)=3(1+2x)^{-\frac{5}{2}}$	A1	4	
(ii)	$f(x)=(1+2x)^{\frac{1}{2}}\Rightarrow f(0)=1;$	B1		
	$f'(0)=1;\ f''(0)=-1;\ f'''(0)=3$	M1		All three attempted
		A1F		ft on $k(1+2x)'''$
	$f(x)=f(0)+xf'(0)+\frac{x^2}{2}f''(0)+\frac{x^3}{6}f'''(0)$			
	$\ldots\approx 1+x-\frac{x^2}{2}+\frac{x^3}{2}$	A1	4	CSO **AG**
(b)	$e^x(1+2x)^{\frac{1}{2}}\approx$			
	$\left(1+x+\frac{x^2}{2}+\frac{x^3}{6}\right)\left(1+x-\frac{x^2}{2}+\frac{x^3}{2}\right)$	M1		Attempt to expand needed
	$\approx 1+x(1+1)+x^2(-0.5+1+0.5)$	A1		
	$+\ x^3\left(\frac{1}{2}-\frac{1}{2}+\frac{1}{2}+\frac{1}{6}\right)$			
	$\approx 1+2x+x^2+\frac{2}{3}x^3$	A1	3	CSO
(c)	$e^{2x}=1+2x+\frac{(2x)^2}{2}+\frac{(2x)^3}{6}+\ldots$	B1	1	
	$=1+2x+2x^2+\frac{4}{3}x^3+\ldots$			
(d)	$1-\cos x=\frac{1}{2}x^2+\{o(x^4)\}$	B1		
	$\frac{e^x(1+2x)^{\frac{1}{2}}-e^{2x}}{1-\cos x}=$			
	$\frac{1+2x+x^2+\frac{2}{3}x^3-\left[1+2x+2x^2+\frac{4}{3}x^3\right]}{\frac{1}{2}x^2+\{o(x^4)\}}$	M1		Series used
	$\lim_{x\to 0}\ldots = \lim_{x\to 0}\frac{-x^2+\{o(x^3)\}}{\frac{1}{2}x^2+\{o(x^4)\}}=$	A1F		
	$\lim_{x\to 0}\frac{-1+o(x)}{\frac{1}{2}+o(x^2)}=-2$	A1F	4	ft a slip but must see the intermediate stage
	Total		**16**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
7(a)	Area = $\frac{1}{2}\int(6+4\cos\theta)^2\,\mathrm{d}\theta$	M1		use of $\frac{1}{2}\int r^2\,\mathrm{d}\theta$
	$=\frac{1}{2}\left(\int_{-\pi}^{\pi}36+48\cos\theta+16\cos^2\theta\right)\mathrm{d}\theta$	B1 B1		for correct expansion of $[6+4\cos\theta)]^2$ for limits
	$=\left(\int_{-\pi}^{\pi}18+24\cos\theta+4(\cos 2\theta+1)\right)\mathrm{d}\theta$	M1		Attempt to write $\cos^2\theta$ in terms of $\cos 2\theta$
	$=\left[22\theta+24\sin\theta+2\sin 2\theta\right]_{-\pi}^{\pi}$	A1F		correct integration ft wrong coefficients
	$=44\pi$	A1	6	CSO
(b)	At P, $r=4$; At Q, $r=2$;	B1		PI
	$P\ \{x=\}\ r\cos\theta=4\cos\frac{2\pi}{3}=-2$	M1		Attempt to use $r\cos\theta$
	$Q\ \{x=\}\ r\cos\theta=2\cos\pi=-2$	A1		Both
	Since P and Q have same 'x', PQ is vertical so QP is parallel to the vertical line $\theta=\frac{\pi}{2}$	E1	4	
(c)(i)	$OP=4$; $OS=8$;	B1		
	Angle $POS=\frac{\pi}{3}$	B1		or $S\,(4, 4\sqrt{3})$ and $P\,(-2, 2\sqrt{3})$
	$PS^2=4^2+8^2-2\times4\times8\times\cos\frac{\pi}{3}$ oe	M1		Cosine rule used in triangle POS OE $PS^2=(4+2)^2+(4\sqrt{3}-2\sqrt{3})^2$
	$PS=\sqrt{48}\quad\{=4\sqrt{3}\}$	A1	4	
(ii)	Since $8^2=4^2+\left(\sqrt{48}\right)^2$, $OS^2=OP^2+PS^2\Rightarrow OPS$ is a right angle. (Converse of Pythagoras Theorem)	E1	1	Accept valid equivalents e.g. $PR=2PQ=2(2\sqrt{3})=PS$. $\angle SRP=\angle RSP=\angle RPO=\frac{\pi}{6}$ $\Rightarrow$ OPS is a right angle
	Total		**15**	
	TOTAL		**75**	

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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MFP3 Further Pure 3

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2007 examination - June series

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Registered address: AQA, Devas Street, Manchester M15 6EX *Dr Michael Cresswell Director General*

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MFP3

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1(a)	$y_{\text{PI}} = kx^2e^{5x} \Rightarrow y' = 2kxe^{5x} + 5kx^2e^{5x}$	M1 A1		Product rule to differentiate x^2e^{5x}
	$\Rightarrow y'' = 2ke^{5x} + 10kxe^{5x} + 10kxe^{5x} + 25kx^2e^{5x}$	A1ft		
	$\Rightarrow 2ke^{5x} + 20kxe^{5x} + 25kx^2e^{5x}$ $-10\left(2kxe^{5x} + 5kx^2e^{5x}\right) + 25kx^2e^{5x} = 6e^{5x}$	M1 A1		Substitution into differential equation
	$2k = 6 \Rightarrow k = 3$	A1ft	6	Only ft if xe^{5x} and x^2e^{5x} terms all cancel out
(b)	Aux. eqn. $m^2 - 10m + 25 = 0 \Rightarrow m = 5$	B1		PI
	CF is $(A + Bx)e^{5x}$	M1		
	GS $y = (A + Bx)e^{5x} + 3x^2e^{5x}$	M1 A1ft	4	Their CF + their/our PI ft only on wrong value of k
	Total		**10**	
2(a)	$y_1 = 2 + 0.1 \times \sqrt{1^2 + 2^2 + 3}$	M1		
	$y(1.1) = 2 + 0.1 \times \sqrt{8}$	A1		
	$y(1.1) = 2.28284\ldots = 2.2828$ to 4dp	A1	3	
(b)	$k_1 = 0.1 \times \sqrt{8} = 0.2828$	M1 A1ft		PI
	$k_2 = 0.1 \times \text{f}(1.1,\ 2.2828\ldots)$	M1		
	$= 0.1 \times \sqrt{9.42137\ldots} = 0.3069(425\ldots)$	A1		PI
	$y(1.1) = y(1) + \frac{1}{2}[0.28284\ldots + 0.30694\ldots]$	m1		
	$2.29489\ldots = 2.2949$ to 4dp	A1	6	
	Total		**9**	
3	IF is $e^{\int \tan x\,dx}$	M1		
	$= e^{-\ln\cos x} = e^{\ln\sec x}$	A1		Accept either
	$= \sec x$	A1ft		ft on earlier sign error
	$\frac{d}{dx}(y\sec x) = \sec^2 x$	M1A1		
	$y\sec x = \int \sec^2 x\,dx$			
	$y\sec x = \tan x + c$	A1		Condone missing c
	$y = 3$ when $x = 0 \Rightarrow 3\sec 0 = 0 + c$	m1		
	$c = 3 \Rightarrow y\sec x = \tan x + 3$	A1	8	OE; condone solution finishing at $c = 3$ provided no errors
	Total		**8**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$(\cos\theta+\sin\theta)^2 = \cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta$ $=1+\sin 2\theta$	B1	1	AG (be convinced)
(b)	$(x^2+y^2)^3=(x+y)^4$			
	$(r^2)^3=(r\cos\theta+r\sin\theta)^4$	M2,1,0		[M1 for one of $x^2+y^2=r^2$ OE, $x=r\cos\theta$, $y=r\sin\theta$ used]
	$r^6=r^4(\cos\theta+\sin\theta)^4$			
	$r^6=r^4(1+\sin 2\theta)^2$	M1		Uses (a) OE at any stage
	$r^2=(1+\sin 2\theta)^2$			
	$\Rightarrow r=(1+\sin 2\theta)\ \{r\geq 0\}$	A1	4	CSO; AG
(c)(i)	$r=0\Rightarrow \sin 2\theta=-1$			
	$2\theta=\sin^{-1}(-1);\ =-\frac{\pi}{2},\frac{3\pi}{2}$	M1		
	$\theta=-\frac{\pi}{4};\ \frac{3\pi}{4}$	A1A1ft	3	A1 for either
(ii)	Area $=\frac{1}{2}\int(1+\sin 2\theta)^2\,\mathrm{d}\theta$	M1		Use of $\frac{1}{2}\int r^2\,\mathrm{d}\theta$
	$=\frac{1}{2}\int(1+2\sin 2\theta+\sin^2 2\theta)\,\mathrm{d}\theta$	B1		Correct expansion of $(1+\sin 2\theta)^2$
	$=\frac{1}{2}\int\left(1+2\sin 2\theta+\frac{1}{2}(1-\cos 4\theta)\right)\mathrm{d}\theta$	M1		Attempt to write $\sin^2 2\theta$ in terms of $\cos 4\theta$
	$=\left[\frac{3}{4}\theta-\frac{1}{2}\cos 2\theta-\frac{1}{16}\sin 4\theta\right]$	A1ft		Correct integration ft wrong coefficients only
	$=\left[\frac{3}{4}\theta-\frac{1}{2}\cos 2\theta-\frac{1}{16}\sin 4\theta\right]_{-\frac{\pi}{4}}^{\frac{3\pi}{4}}$			
	$=\left(\frac{9\pi}{16}\right)-\left(-\frac{3\pi}{16}\right)$	m1		Using c's values from (c)(i) as limits or the correct limits
	$=\frac{3\pi}{4}$	A1	6	CSO
	Total		**14**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$u=\dfrac{dy}{dx}+x \Rightarrow \dfrac{du}{dx}=\dfrac{d^2y}{dx^2}+1$	M1A1		
	$\left(x^2-1\right)\left(\dfrac{du}{dx}-1\right)-2x(u-x)=x^2+1$	M1		Substitution into LHS of DE as far as no ys
	DE $\Rightarrow$ $(x^2-1)\dfrac{du}{dx}-2xu=0$			
	$\Rightarrow \dfrac{du}{dx}=\dfrac{2xu}{x^2-1}$	A1	4	CSO; AG
(b)	$\displaystyle\int \frac{1}{u}\,du = \int \frac{2x}{x^2-1}\,dx$	M1 A1		Separate variables
	$\ln u=\ln\left\|x^2-1\right\|+\ln A$	A1A1		
	$u=A(x^2-1)$	A1	5	
(c)	$\dfrac{dy}{dx}+x = A(x^2-1)$	M1		Use (b) $(\neq 0)$ to form DE in y and x
	$\dfrac{dy}{dx} = A(x^2-1)-x$			
	$y= A\left(\dfrac{x^3}{3}-x\right)-\dfrac{x^2}{2}+B$	M1 A1ft	3	Solution must have two different constants and correct method used to solve the DE
	Total		**12**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$\mathrm{f}(x)=\ln\left(1+\mathrm{e}^x\right)$:			
	$\mathrm{f}(0)=\ln 2$	B1		
	$\mathrm{f}'(x)=\dfrac{\mathrm{e}^x}{1+\mathrm{e}^x}$ $\quad \mathrm{f}'(0)=\dfrac{1}{2}$	M1 A1		Chain rule
	$\mathrm{f}''(x)=\dfrac{(1+\mathrm{e}^x)\mathrm{e}^x-\mathrm{e}^x\mathrm{e}^x}{(1+\mathrm{e}^x)^2}=\dfrac{\mathrm{e}^x}{(1+\mathrm{e}^x)^2}$	M1 A1		Quotient rule OE
	$\mathrm{f}''(0)=\dfrac{1}{4}$			
	so first three terms are:			
	$\mathrm{f}(x)=\ln 2+\dfrac{1}{2}x+\dfrac{1}{4}\dfrac{x^2}{2!}=\ln 2+\dfrac{1}{2}x+\dfrac{1}{8}x^2$	A1	6	CSO; AG
(ii)	$\mathrm{f}'''(x)=\dfrac{(1+\mathrm{e}^x)^2\mathrm{e}^x-\mathrm{e}^x\left[2(1+\mathrm{e}^x)\mathrm{e}^x\right]}{(1+\mathrm{e}^x)^4}$	M1 A1ft		Chain rule with quotient/product rule ft on $\mathrm{f}''(x)=k\mathrm{e}^x\left(1+\mathrm{e}^x\right)^n$ (integer $n<0$)
	$\mathrm{f}'''(0)=\dfrac{4-4}{2^4}=0$ {so coefficient of x^3 is zero}	A1	3	CSO; AG; All previous differentiation correct
	SC for those not using Maclaurin's theorem: **maximum** of 4/9			
(b)	$\dfrac{1}{2}x+\dfrac{1}{8}x^2$	B1	1	
(c)	$\ln\left(1-\dfrac{x}{2}\right)=$ $\left(-\dfrac{x}{2}\right)-\dfrac{1}{2}\left(-\dfrac{x}{2}\right)^2+\dfrac{1}{3}\left(-\dfrac{x}{2}\right)^3-\ldots\ldots$	B1	1	
(d)	$\ln\left(\dfrac{1+\mathrm{e}^x}{2}\right)+\ln\left(1-\dfrac{x}{2}\right)=-\dfrac{x^3}{24}+\ldots$	M1		Uses previous expansions to obtain first non-zero term of the form kx^3
	$x-\sin x\approx x-\left[x-\dfrac{x^3}{3!}+\ldots\right]\approx\dfrac{x^3}{3!}+\ldots$	B1		
	$\left[\dfrac{\ln\left(\dfrac{1+\mathrm{e}^x}{2}\right)+\ln\left(1-\dfrac{x}{2}\right)}{x-\sin x}\right]=\dfrac{-\dfrac{1}{24}x^3+\ldots}{\dfrac{1}{6}x^3+o\left(x^5\right)}$	M1		
	$=\dfrac{-\dfrac{1}{24}x^3+\ldots}{x^3\left[\dfrac{1}{6}+o\left(x^2\right)\right]}=\dfrac{-\dfrac{1}{24}+\ldots}{\dfrac{1}{6}+o\left(x^2\right)}$			
	$\lim_{x\to 0}\ldots\ldots=-\dfrac{1}{4}$	A1	4	CSO
	Total		**15**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
7(a)	0	B1	1	
(b)	$u = xe^{-x} + 1 \Rightarrow du = \left(e^{-x} - xe^{-x}\right)dx$	M1		Attempts to find du
	$\int \frac{e^{-x}(1-x)}{xe^{-x}+1}\,dx = \int \frac{1}{u}\,du = \ln u + c$			
	$= \ln\left(xe^{-x}+1\right)\{+c\}$	A1	2	Condone missing c
(c)	$\int \frac{1-x}{x+e^{x}}\,dx = \int \frac{e^{-x}(1-x)}{xe^{-x}+1}dx$	B1		
	$\int_1^{\infty} \frac{1-x}{x+e^{x}}\,dx = \lim_{a\to\infty}\left[\ln(xe^{-x}+1)\right]_1^a$			
	$= \lim_{a\to\infty}\left\{\ln\left(ae^{-a}+1\right)\right\} - \ln\left(e^{-1}+1\right)$	M1		For using part (b) and $F(B)-F(A)$
	$= \ln\left\{\lim_{a\to\infty}\left(ae^{-a}+1\right)\right\} - \ln\left(e^{-1}+1\right)$			
	$= \ln 1 - \ln\left(e^{-1}+1\right) = -\ln\left(e^{-1}+1\right)$	M1 A1	4	For using limiting process
	Total		**7**	
	TOTAL		**75**	

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General Certificate of Education

Mathematics 6360

MFP3 Further Pure 3

Mark Scheme

2008 examination - January series

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Registered address: AQA, Devas Street, Manchester M15 6EX *Dr Michael Cresswell Director General*

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

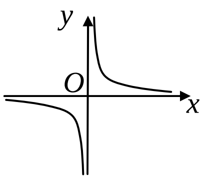
MFP3

Q	Solution	Marks	Total	Comments
1(a)	$y(2.1) = y(2) + 0.1[2^2 - 1^2]$	M1A1		
	$= 1+0.1\times3 = 1.3$	A1	3	
(b)	$y(2.2) = y(2) + 2(0.1)[f(2.1, y(2.1))]$	M1		
	$\ldots = 1+2(0.1)[2.1^2 - 1.3^2]$	A1√		Ft on cand's answer to (a)
	$\ldots = 1+0.2\times2.72 = 1.544$	A1	3	CAO
	Total		**6**	
2(a)	Area $= \frac{1}{2}\int(1+\tan\theta)^2\,\mathrm{d}\theta$	M1		Use of $\frac{1}{2}\int r^2\,\mathrm{d}\theta$
	$\ldots = \frac{1}{2}\int(1+2\tan\theta+\tan^2\theta)\,\mathrm{d}\theta$	B1		Correct expansion of $(1+\tan\theta)^2$
	$=\frac{1}{2}\int(\sec^2\theta+2\tan\theta)\,\mathrm{d}\theta$	M1		$1+\tan^2\theta=\sec^2\theta$ used
	$= \frac{1}{2}\left[\tan\theta+2\ln(\sec\theta)\right]_0^{\frac{\pi}{3}}$	A1√ B1√		Integrating $p\sec^2\theta$ correctly Integrating $q\tan\theta$ correctly
	$=\frac{1}{2}[(\sqrt{3}+2\ln 2)-0]=\frac{\sqrt{3}}{2}+\ln 2$	A1	6	Completion. AG CSO be convinced
(b)	$OP = 1$; $OQ = 1+\tan\frac{\pi}{3}$	B1		Both needed. Accept 2.73 for OQ
	Shaded area =			
	'answer (a)' $-\ \frac{1}{2}OP\times OQ\times\sin\left(\frac{\pi}{3}\right)$	M1		
	$= \frac{\sqrt{3}}{2}+\ln 2-\frac{\sqrt{3}}{4}(1+\sqrt{3})$	A1	3	ACF. Condone 0.376… if exact 'value' for area of triangle seen
	$= \frac{\sqrt{3}}{4}+\ln 2-\frac{3}{4}$			
	Total		**9**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$(m+2)^2 = -1$	M1		Completing sq or formula
	$m = -2 \pm \mathrm{i}$	A1		
	CF is $\mathrm{e}^{-2x}(A\cos x + B\sin x)$	M1		If m is real give M0
	{or $\mathrm{e}^{-x}A\cos(x+B)$ but not $A\mathrm{e}^{(-1+\mathrm{i})x} + B\mathrm{e}^{(-1-\mathrm{i})x}$ }	A1√		Ft on wrong a's and b's but roots must be complex
	PI try $y = p \Rightarrow 5p = 5$ PI is $y = 1$	B1		
	GS $y = \mathrm{e}^{-2x}(A\cos x + B\sin x) + 1$	B1√	6	Their CF + their PI with two arbitrary constants.
(b)	$x=0,\ y = 2 \Rightarrow A = 1$	B1√		Provided previous B1√ awarded
	$y'(x) = -2\mathrm{e}^{-2x}(A\cos x + B\sin x) +$	M1		Product rule used
	$+ \mathrm{e}^{-2x}(-A\sin x + B\cos x)$	A1√		
	$y'(0) = 3 \Rightarrow 3 = -2A + B \Rightarrow B = 5$	A1√	4	Ft on one slip
	$y = \mathrm{e}^{-2x}(\cos x + 5\sin x) + 1$			
	Total		**10**	
4(a)	The interval of integration is infinite	E1	1	OE
(b)	$\int x\mathrm{e}^{-3x}\mathrm{d}x = -\frac{1}{3}x\mathrm{e}^{-3x} - \int -\frac{1}{3}\mathrm{e}^{-3x}\mathrm{d}x$	M1 A1		Reasonable attempt at parts
	$= -\frac{1}{3}x\mathrm{e}^{-3x} - \frac{1}{9}\mathrm{e}^{-3x}$ {+c}	A1√	3	Condone absence of $+c$
(c)	$I = \int_1^{\infty} x\mathrm{e}^{-3x}\,\mathrm{d}x = \lim_{a\to\infty}\int_1^{a} x\mathrm{e}^{-3x}\,\mathrm{d}x$			
	$\lim_{a\to\infty}\{-\frac{1}{3}a\mathrm{e}^{-3a} - \frac{1}{9}\mathrm{e}^{-3a}\} - \left[-\frac{4}{9}\mathrm{e}^{-3}\right]$	M1		F(a) – F(1) with an indication of limit '$a \to \infty$'
	$\lim_{a\to\infty} a\mathrm{e}^{-3a} = 0$	M1		For statement with limit/limiting process shown
	$I = \frac{4}{9}\mathrm{e}^{-3}$	A1	3	
	Total		**7**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
5	IF is $e^{\int \frac{4x}{x^2+1}\,dx}$	M1		
	$= e^{2\ln(x^2+1)}$	A1		
	$= e^{\ln(x^2+1)^2} = (x^2+1)^2$	A1√		Ft on $e^{p\ln(x^2+1)}$
	$\frac{d}{dx}\left(y(x^2+1)^2\right) = x(x^2+1)^2$	M1		LHS as d/dx(y×cand's IF) PI and also RHS of form $kx(x^2+1)^p$
		A1√		
	$y(x^2+1)^2 = \int x(x^2+1)^2\,dx$			
	$y(x^2+1)^2 = \frac{1}{6}(x^2+1)^3 + c$	M1 A1		Use of suitable substitution to find RHS or reaching $k(x^2+1)^3$ OE Condone missing c
	$y(0) = 1 \Rightarrow c = \frac{5}{6}$	m1		
	$y = \frac{1}{6}(x^2+1) + \frac{5}{6(x^2+1)^2}$	A1	9	Accept other forms of f(x) eg $y = \dfrac{\left(\frac{x^6}{6} + \frac{2x^4}{4} + \frac{x^2}{2} + 1\right)}{(x^2+1)^2}$
	Total		**9**	
6(a)	$r^2 2\sin\theta\cos\theta = 8$	M1		$\sin 2\theta = 2\sin\theta\cos\theta$ used
	$x = r\cos\theta \quad y = r\sin\theta$	M1		Either **one** stated or used
	$xy = 4$, $y = \frac{4}{x}$	A1	3	Either OE eg $y = \frac{8}{2x}$
(b)		B1	1	
(c)	$r = 2\sec\theta$ is $x = 2$	B1		
	Sub $x = 2$ in $xy = 4 \Rightarrow 2y = 4$ In cartesian, $A(2, 2)$	M1		
	$\Rightarrow \tan\theta = \frac{y}{x} = 1 \Rightarrow \theta = \frac{\pi}{4}$ $\Rightarrow r = \sqrt{x^2+y^2} = \sqrt{8}$	M1		Used either $\tan\theta = \frac{y}{x}$ or $r = \sqrt{x^2+y^2}$
	$\theta = \frac{\pi}{4}$; $r = \sqrt{8}$	A1	4	r must be given in surd form
	Altn2: Eliminating r to reach eqn. in $\cos\theta$ and $\sin\theta$ only (M1) $\theta = \frac{\pi}{4}$ (A1) Substitution $r = 2\sec\left(\frac{\pi}{4}\right)$ (m1) $r = \sqrt{8}$ (A1) OE surd			Altn3: $r\sin\theta = 2$ (B1) Solving $r\cos\theta = 2$ and $r\sin\theta = 2$ simultaneously (M1) $\tan\theta = 1$ or $r^2 = 2^2 + 2^2$ (M1) $\theta = \frac{\pi}{4}$; $r = \sqrt{8}$ (A1) need both
	Total		**8**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$\ln(1+2x) = 2x - 2x^2 + \frac{8}{3}x^3 ...$	M1 A1	2	Use of expansion of ln(1+*x*) Simplified 'numerators'.
(ii)	$-\frac{1}{2} < x \le \frac{1}{2}$	B1	1	
(b)(i)	$y = \ln\cos x \Rightarrow y'(x) = \frac{1}{\cos x}(-\sin x)$	M1		
	$y''(x) = -\sec^2 x$	A1		ACF
	$y'''(x) = -2\sec x\,(\sec x \tan x)$	M1		Chain rule OE
	$\{y'''(x) = -2\tan x(\sec^2 x)\}$	A1√	4	Ft a slip…accept unsimplified
(ii)	$y''''(x) = -2[\sec^2 x(\sec^2 x) + \tan x(2\sec x\,(\sec x \tan x))]$	M1 A1		Product rule OE ACF
	$y''''(0) = -2[(1)^2 + 0] = -2$	A1√	3	Ft a slip
(iii)	$\ln\cos x \approx 0 + 0 + \frac{x^2}{2}(-1) + 0 + \frac{x^4}{4!}(-2)$	M1		
	$\approx -\frac{x^2}{2} - \frac{x^4}{12}$	A1	2	CSO throughout part (b). AG
(c)	Limit = $\lim_{x \to 0}\left[\frac{x\ln(1+2x)}{x^2 - \ln\cos x}\right]$			
	$= \lim_{x \to 0}\left[\frac{x(2x - 2x^2 + ..)}{x^2 - \left(-\frac{x^2}{2} - \frac{x^4}{12}..\right)}\right]$	M1		Using earlier expansions
	Limit = $\lim_{x \to 0} \frac{2x^2 - o(x^3)}{1.5x^2 + o(x^4)}$			The notation $o(x^n)$ can be replaced by a term of the form kx^n
	$= \lim_{x \to 0} \frac{2 - o(x)}{1.5 + o(x^2)} = \frac{4}{3}$	M1 A1	3	Need to see stage, division by x^2
	Total		**15**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
8(a)(i)	$\frac{dx}{dt} = e^t \ \{= x\}$	B1		
	$x\frac{dy}{dx} = x\frac{dy}{dt}\frac{dt}{dx}$	M1		Chain rule
	$= x\frac{dy}{dt}\frac{1}{x} = \frac{dy}{dt}$	A1	3	Completion. AG
(ii)	$\frac{d^2y}{dt^2} = \frac{d}{dt}\left(x\frac{dy}{dx}\right) =$			
	$= \frac{dx}{dt}\frac{dy}{dx} + x\frac{d}{dt}\left(\frac{dy}{dx}\right)$	M1		Product rule
	….. $= \frac{dy}{dt} + x\frac{dx}{dt}\frac{d}{dx}\left(\frac{dy}{dx}\right)$	M1		
	…. $= \frac{dy}{dt} + x^2\left(\frac{d^2y}{dx^2}\right)$	A1	3	Condone leaving in this form
	$\Rightarrow x^2\frac{d^2y}{dx^2} = \frac{d^2y}{dt^2} - \frac{dy}{dt}$			AG
(b)	$x^2\frac{d^2y}{dx^2} - 6x\frac{dy}{dx} + 6y = 0$			
	$\Rightarrow \frac{d^2y}{dt^2} - 7\frac{dy}{dt} + 6y = 0$	M1		Using results in (a) to reach DE of this form
	Auxl eqn $m^2 - 7m + 6 = 0$			
	$(m-6)(m-1) = 0$	m1		PI
	$m = 1$ and 6	A1		PI
	$y = Ae^{6t} + Be^t$	M1		Must be solving the ‘correct’ DE. (Give M1A0 for $y = Ae^{6x} + Be^x$)
	$y = Ax^6 + Bx$	A1√	5	Ft a minor slip only if previous A0 and all three method marks gained
	Total		**11**	
	TOTAL		**75**	

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General Certificate of Education

Mathematics 6360

MFP3 Further Pure 3

Mark Scheme

2008 examination – June series

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Otherwise we require evidence of a correct method for any marks to be awarded.

MFP3

Q	Solution	Marks	Total	Comments
1	$k_1 = 0.1 \times \ln(2+3)$	M1		
	$= 0.1609(4379\ldots)$ (= *)	A1		PI
	$k_2 = 0.1 \times \mathrm{f}(2.1,\ 3 + *\ldots)$ … = 0.1 × ln(2.1 + 3.16094…)]	M1		
	…. = 0.1660(31…)	A1		PI
	$y(2.1) = y(2) + \frac{1}{2}[k_1 + k_2]$ = 3 + 0.5 × 0.3269748….	m1		Dep on previous two Ms and numerical values for k's
	= 3.163487… = 3.1635 to 4dp	A1	6	Must be 3.1635
	Total		**6**	
2(a)	PI: $y_{PI} = a + bx + c\sin x + d\cos x$ $y'_{PI} = b + c\cos x - d\sin x$			
	$b + c\cos x - d\sin x - 3a - 3bx - 3c\sin x$ $- 3d\cos x = 10\sin x - 3x$	M1		Substituting into DE
	$b - 3a = 0$; $-3b = -3$; $c - 3d = 0$; $-d - 3c = 10$	M1		Equating coefficients (at least 2 eqns)
	$a = \frac{1}{3}$; $b = 1$; $c = -3$; $d = -1$	A2,1	4	A1 for any two correct
	$y_{PI} = \frac{1}{3} + x - 3\sin x - \cos x$			
(b)	Aux. eqn. $m - 3 = 0$	M1		Altn. $\int y^{-1}\,\mathrm{d}y = \int 3\,\mathrm{d}x$ OE (M1)
	$(y_{CF} =)Ae^{3x}$	A1		Ae^{3x} OE
	$(y_{GS} =)Ae^{3x} + \frac{1}{3} + x - 3\sin x - \cos x$	B1F	3	(c's CF + c's PI) with 1 arbitrary constant
	Total		**7**	
3(a)	$x^2 + y^2 = 1 - 2y + y^2 \Rightarrow x^2 + y^2 = (1 - y)^2$	B1	1	AG
(b)	$x^2 + y^2 = r^2$	M1		Or $x = r\cos\theta$
	$y = r\sin\theta$	M1		
	$x^2 = 1 - 2y$ so $x^2 + y^2 = (1 - y)^2$ $\Rightarrow r^2 = (1 - r\sin\theta)^2$	A1		OE eg $r^2\cos^2\theta = 1 - 2r\sin\theta$ PI by the next line
	$r = 1 - r\sin\theta$ or $r = -(1 - r\sin\theta)$ $r(1 + \sin\theta) = 1$ or $r(1 - \sin\theta) = -1$	m1		Either
	$r > 0$ so $r = \frac{1}{1 + \sin\theta}$	A1	5	CSO
	Total		**6**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$u=\frac{\mathrm{d}y}{\mathrm{d}x} \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x}=\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$	M1		
	$x\frac{\mathrm{d}u}{\mathrm{d}x}-u=3x^2 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x}-\frac{1}{x}u=3x$	A1	2	AG Substitution into LHS of DE and completion
(b)	IF is exp $(\int -\frac{1}{x}\,\mathrm{d}x)$	M1		and with integration attempted
	$=\mathrm{e}^{-\ln x}$	A1		
	$=x^{-1}$ or $\frac{1}{x}$	A1		or multiple of x^{-1}
	$\frac{\mathrm{d}}{\mathrm{d}x}\left[ux^{-1}\right]=3$	M1		LHS as differential of $u\times$ IF . PI
	$\Rightarrow ux^{-1}=3x+A$	m1		Must have an arbitrary constant (Dep. on previous M1 only)
	$u=3x^2+Ax$	A1	6	
(c)	$\frac{\mathrm{d}y}{\mathrm{d}x}=3x^2+Ax$	M1		Replaces u by $\frac{\mathrm{d}y}{\mathrm{d}x}$ and attempts to integrate
	$y=x^3+\frac{Ax^2}{2}+B$	A1F	2	ft on cand's u but solution must have two arbitrary constants
	Total		**10**	
5(a)	$\int x^3\ln x\,\mathrm{d}x=\frac{x^4}{4}\ln x-\int\frac{x^4}{4}\left(\frac{1}{x}\right)\mathrm{d}x$	M1		...$=kx^4\ln x\pm\int \mathrm{f}(x)$, with f($x$) not involving the 'original' ln x
		A1		
	…… $=\frac{x^4}{4}\ln x-\frac{x^4}{16}+c$	A1	3	Condone absence of '+ c'
(b)	Integrand is not defined at $x=0$	E1	1	OE
(c)	$\int_0^{\mathrm{e}} x^3\ln x\,\mathrm{d}x=\left\{\lim_{a\to 0}\int_a^{\mathrm{e}} x^3\ln x\,\mathrm{d}x\right\}$			
	$=\frac{3\mathrm{e}^4}{16}-\lim_{a\to 0}\left[\frac{a^4}{4}\ln a-\frac{a^4}{16}\right]$	M1		F(e) – F(a)
	But $\lim_{a\to 0} a^4\ln a=0$	B1		Accept a general form eg $\lim_{x\to 0} x^k\ln x=0$
	So $\int_0^{\mathrm{e}} x^3\ln x\,\mathrm{d}x$ exists and $=\frac{3\mathrm{e}^4}{16}$	A1	3	CSO
	Total		**7**	

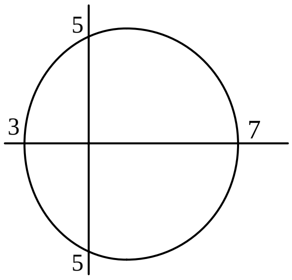
MFP3 (cont)

Q	Solution	Marks	Total	Comments
6(a)	Aux eqn: $m^2 - 2m - 3 = 0$	M1		
	$m = -1,\ 3$	A1		PI
	CF $(y_C =) Ae^{3x} + Be^{-x}$	M1		
	Try $(y_{PI} =) a\,e^{-2x}\ (+b)$	M1		
	$\frac{dy}{dx} = -2ae^{-2x}$	A1		
	$\frac{d^2y}{dx^2} = 4ae^{-2x}$	A1		
	Substitute into DE gives $4ae^{-2x} + 4ae^{-2x} - 3ae^{-2x} - 3b = 10e^{-2x} - 9$	M1		
	$\Rightarrow a = 2$	A1		
	$b = 3$	B1		
	$(y_{GS} =) Ae^{3x} + Be^{-x} + 2e^{-2x} + 3$	B1F	10	(c's CF+c's PI) with 2 arbitrary constants
(b)	$x = 0,\ y = 7 \Rightarrow 7 = A + B + 2 + 3$	B1F		Only ft if exponentials in GS and two arbitrary constants remain
	$\frac{dy}{dx} = 3Ae^{3x} - Be^{-x} - 4e^{-2x}$			
	As $x \to \infty$, $e^{-kx} \to 0$, $\frac{dy}{dx} \to 0$ so $A = 0$	B1		
	When $A = 0$, $5 = 0 + B + 3 \Rightarrow B = 2$	B1F		Must be using 'A' = 0
	$y = 2e^{-x} + 2e^{-2x} + 3$	A1	4	CSO
	Total		**14**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\sin 2x \approx 2x - \frac{(2x)^3}{3!} + .. = 2x - \frac{4}{3}x^3 + ..$	B1	1	
(b)(i)	$\frac{dy}{dx} = \frac{1}{2}\left(3+e^x\right)^{-\frac{1}{2}}(e^x)$	M1 A1		Chain rule
	$\frac{d^2y}{dx^2} = \frac{1}{2}e^x\left(3+e^x\right)^{-\frac{1}{2}} - \frac{1}{4}\left(3+e^x\right)^{-\frac{3}{2}}(e^{2x})$	M1 A1		Product rule OE OE
	$y'(0) = \frac{1}{4}$; $y''(0) = \frac{1}{4} - \frac{1}{32} = \frac{7}{32}$	A1	5	CSO
(ii)	$y(0) = 2$; $y'(0) = \frac{1}{4}$; $y''(0) = \frac{1}{4} - \frac{1}{32} = \frac{7}{32}$ McC. Thm: $y(0) + x\,y'(0) + \frac{x^2}{2}\,y''(0)$ $\sqrt{3+e^x} \approx 2 + \frac{1}{4}x + \frac{7}{64}x^2$	M1 A1	2	CSO; AG
(c)	$\left[\frac{\sqrt{3+e^x}-2}{\sin 2x}\right] = \left[\frac{2+\frac{1}{4}x+\frac{7}{64}x^2-2}{2x-\frac{4}{3}x^3}\right]$	M1		
	$= \left[\frac{\frac{1}{4}+\frac{7}{64}x+...}{2-\frac{4}{3}x^2+..}\right]$	m1		Dividing numerator and denominator by x to get constant term in each
	$\lim_{x\to 0}\left[\frac{\sqrt{3+e^x}-2}{\sin 2x}\right] = \frac{\frac{1}{4}}{2} = \frac{1}{8}$	A1F	3	Ft on cand's answer to (a) provided of the form $ax+bx^3$
	Total		**11**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$\theta = 0$, $r = 5 + 2\cos 0 = 7$ {A lies on C}	B1		
	$\theta = \pi$, $r = 5 + 2\cos\pi = 3$ {B lies on C}	B1	2	
(b)		B1		Closed single loop curve, with (indication of) symmetry
		B1	2	Critical values, 3,5,7 indicated
(c)	Area = $\frac{1}{2}\int(5+2\cos\theta)^2 \mathrm{d}\theta$	M1		Use of $\frac{1}{2}\int r^2 \mathrm{d}\theta$
	= $\frac{1}{2}\int_{-\pi}^{\pi}\left(25+20\cos\theta+4\cos^2\theta\right)\mathrm{d}\theta$	B1 B1		OE for correct expansion of $(5+2\cos\theta)^2$ For correct limits
	$=\frac{1}{2}\int_{-\pi}^{\pi}\left(25+20\cos\theta+2(\cos 2\theta+1)\right)\mathrm{d}\theta$	M1		Attempt to write $\cos^2\theta$ in terms of $\cos 2\theta$
	= $\frac{1}{2}\left[27\theta+20\sin\theta+\sin 2\theta\right]_{-\pi}^{\pi}$	A1F		Correct integration ft wrong non-zero coefficients in $a + b\cos\theta + c\cos 2\theta$
	= 27π	A1	6	CSO
(d)	Triangle OBQ with $OB = 3$ and angle $BOQ = \alpha$	B1		PI
	$OQ = 5 + 2\cos(-\pi + \alpha)$	M1		OE
	Area of triangle OQB = $\frac{1}{2}OB \times OQ \sin\alpha$	m1		Dep. on correct method to find OQ
	$=\frac{3}{2}\left(5-2\cos\alpha\right)\sin\alpha$	A1	4	CSO
	Total		**14**	
	TOTAL		**75**	

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General Certificate of Education

Mathematics 6360

MFP3 Further Pure 3

Mark Scheme

2009 examination - January series

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m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

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Otherwise we require evidence of a correct method for any marks to be awarded.

MFP3

Q	Solution	Marks	Total	Comments
1(a)	$y_1 = 3 + 0.2\times\left[\dfrac{1^2+3^2}{1+3}\right]$	M1A1		
	$= 3.5$	A1	3	
(b)	$k_1 = 0.2\times 2.5 = 0.5$	B1ft		PI ft from (a)
	$k_2 = 0.2\times \mathrm{f}(1.2,\ 3.5)$	M1		ft on (a)
	$\ldots = 0.2\times\dfrac{1.2^2+3.5^2}{1.2+3.5} = 0.5825(53\ldots)$	A1ft		PI condone 3dp
	$y(1.2) = y(1) + \dfrac{1}{2}[0.5 + 0.5825(53...)]$	m1		
	$= 3.54127\ldots = 3.5413$ to 4dp	A1ft	5	ft one slip If answer not to 4dp withhold this mark
	Total		**8**	
2(a)	IF is $\mathrm{e}^{\int -\frac{2}{x}\,\mathrm{d}x}$	M1		$\mathrm{e}^{\int \pm\frac{2}{x}\,\mathrm{d}x}$
	$= \mathrm{e}^{-2\ln x}$	A1		P1
	$= \mathrm{e}^{\ln x^{-2}} = x^{-2} = \dfrac{1}{x^2}$	A1	3	AG Be convinced
(b)	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{y}{x^2}\right) = \dfrac{1}{x^2}x$	M1		LHS as d/dx(y×IF)
		A1		PI
	$\dfrac{y}{x^2} = \int \dfrac{1}{x}\mathrm{d}x = \ln x + c$	A1		RHS Condone missing '+ c' here
	$y = x^2\ln x + cx^2$	A1	4	
	Total		**7**	
3	Area $= \dfrac{1}{2}\displaystyle\int_0^{\pi}(2+\cos\theta)^2\sin\theta\,\mathrm{d}\theta$	M1		use of $\dfrac{1}{2}\int r^2\,\mathrm{d}\theta$
		B1		Correct limits
	$= \dfrac{1}{2}\left[-\dfrac{1}{3}(2+\cos\theta)^3\right]_0^{\pi}$	M2		Valid method to reach $k(2+\cos\theta)^3$ or $a\cos\theta + b\cos 2\theta + c\cos^3\theta$ OE {SC: M1 if expands then integrates to get either $a\cos\theta$ + $b\cos 2\theta$ OE or $c\cos^3\theta$ OE in a valid way}
		A1		OE eg $-4\cos\theta - \cos 2\theta - \dfrac{1}{3}\cos^3\theta$
	$= \dfrac{1}{2}\left\{-\dfrac{1}{3} + \dfrac{1}{3}\times 3^3\right\} = \dfrac{13}{3}$	A1	6	CSO
	Total		**6**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\int \ln x\,dx = x\ln x - \int x\left(\frac{1}{x}\right)dx$	M1		Integration by parts
	$= x\ln x - x + c$	A1	2	CSO AG
(b)	$\int_0^1 \ln x\,dx = \lim_{a\to 0} \int_a^1 \ln x\,dx$	M1		OE
	$= \lim_{a\to 0} \{0 - 1 - [a\ln a - a]\}$	M1		F(1) – F(a) OE
	But $\lim_{a\to 0} a\ln a = 0$	E1		Accept a general form eg $\lim_{a\to 0} a^k \ln a = 0$
	So $\int_0^1 \ln x\,dx = -1$	A1	4	
	Total		**6**	
5(a)	When $\theta = \pi$, $r = \frac{2}{3+2\cos\pi} = \frac{2}{3+2(-1)} = 2$	B1	1	Correct verification
(b)(i)	$\frac{2}{3+2\cos\theta} = 1 \Rightarrow \cos\theta = -\frac{1}{2}$	M1		Equates r's and attempts to solve.
	Points of intersection $\left(1, \frac{2\pi}{3}\right)$, $\left(1, \frac{4\pi}{3}\right)$	A2,1	3	Condone eg $-2\pi/3$ for $4\pi/3$ A1 if either one point correct or two correct solutions of $\cos\theta = -0.5$
(ii)	Area $OMN = \frac{1}{2}\times1\times1\times\sin\left(\lvert\theta_M - \theta_N\rvert\right)$	M1		**<u>ALT</u>** $MN = 2\times1\times\sin\frac{\pi}{3}$ M1
	$= \frac{1}{2}\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{4}$	A1		Perp. from L to MN $= 2 - 1\cos\frac{\pi}{3} = \frac{3}{2}$ M1A1
	Area $OMLN = 2\times\frac{1}{2}\times1\times2\times\sin\frac{\pi}{3}$	M1		Area $LMN = \frac{1}{2}\times\sqrt{3}\times\frac{3}{2} = \frac{3\sqrt{3}}{4}$ A1
	Area $LMN = \sqrt{3} - \frac{\sqrt{3}}{4} = \frac{3\sqrt{3}}{4}$	A1	4	
(c)	$3r + 2r\cos\theta = 2$	M1		
	$3r + 2x = 2$	B1		$r\cos\theta = x$ stated or used
	$3r = 2 - 2x$	A1		$3r = \pm(2-2x)$
	$9(x^2+y^2) = (2-2x)^2$	M1		$r^2 = x^2 + y^2$ used
	$9y^2 = (2-2x)^2 - 9x^2$	A1	5	CSO ACF for f(x) eg $9y^2 = -5x^2 - 8x + 4$
	Total		**13**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$e^{2x} = 1 + 2x + 2x^2 + \frac{4}{3}x^3 + \ldots$	M1 A1	 2	Clear use of $x \to 2x$ in expansion of e^x ACF
(ii)	$\{f(x)\} = e^{2x}(1+3x)^{-\frac{2}{3}}$ $(1+3x)^{-\frac{2}{3}} = 1 + \left(-\frac{2}{3}\right)(3x) + \frac{\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)(3x)^2}{2} - \frac{40}{3}x^3$ $= 1 - 2x + 5x^2 - \frac{40}{3}x^3$ $\{f(x) \approx\}$ $1 + 2x + 2x^2 + \frac{4x^3}{3} - 2x - 4x^2 - 4x^3 + 5x^2 + 10x^3 - \frac{40x^3}{3}$ $= 1 + 3x^2 - 6x^3$	 M1 A1 m1 A1ft A1	 5	First three terms as $1 + \left(-\frac{2}{3}\right)(3x) + kx^2$ OE Dep on both prev MS Condone one sign or numerical slip in mult. CSO AG A0 if binominal series not used
(b)(i)	$y = \ln(1 + 2\sin x) \Rightarrow \frac{dy}{dx} = \frac{1}{1+2\sin x} \times 2\cos x$ $\frac{d^2y}{dx^2} = \frac{(1+2\sin x)(-2\sin x) - 2\cos x(2\cos x)}{(1+2\sin x)^2} = \frac{-2(\sin x + 2)}{(1+2\sin x)^2}$	M1 A1 M1 A1	 4	Chain rule Quotient rule OE with u and v non constant ACF
(ii)	$y(0) = 0$, $y'(0) = 2$, $y''(0) = -4$ McL Thm.: $\{\ln(1+2\sin x)\} \approx 0 + 2x - 4\left(\frac{x^2}{2}\right) + .. \approx 2x - 2x^2$	M1 A1	 2	 CSO AG
(c)	$\lim_{x \to 0} \frac{1 - f(x)}{x\ln(1+2\sin x)} = \lim_{x \to 0} \frac{-3x^2 + 6x^3}{2x^2 - 2x^3}$ $= \lim_{x \to 0} \frac{-3+6x}{2-2x}$ $= -\frac{3}{2}$	M1 m1 A1	 3	Using expansions Division by x^2 stage before taking limit. CSO
	Total		**16**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = \mathrm{e}^t \ \{= x\}$	B1		OE
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}x} = \mathrm{e}^{-t}\frac{\mathrm{d}y}{\mathrm{d}t}$	M1 A1		Chain rule OE eg $x\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t}$
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{-t}\frac{\mathrm{d}y}{\mathrm{d}t}\right) = \frac{\mathrm{d}t}{\mathrm{d}x}\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{-t}\frac{\mathrm{d}y}{\mathrm{d}t}\right)$	M1		$\frac{\mathrm{d}}{\mathrm{d}x}(\ \ ) = \frac{\mathrm{d}t}{\mathrm{d}x}\frac{\mathrm{d}}{\mathrm{d}t}(\ \ )$ OE
	$= \frac{\mathrm{d}t}{\mathrm{d}x}\left(-\mathrm{e}^{-t}\frac{\mathrm{d}y}{\mathrm{d}t} + \mathrm{e}^{-t}\frac{\mathrm{d}^2y}{\mathrm{d}t^2}\right)$	M1		Product rule OE
	$\ldots = \mathrm{e}^{-t}\left(-\mathrm{e}^{-t}\frac{\mathrm{d}y}{\mathrm{d}t} + \mathrm{e}^{-t}\frac{\mathrm{d}^2y}{\mathrm{d}t^2}\right)$	A1		OE
	$\ldots = x^{-2}\left(-\frac{\mathrm{d}y}{\mathrm{d}t} + \frac{\mathrm{d}^2y}{\mathrm{d}t^2}\right)$			
	$\Rightarrow x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \left(\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - \frac{\mathrm{d}y}{\mathrm{d}t}\right)$	A1	7	CSO AG Completion. Be convinced
(b)	$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 4x\frac{\mathrm{d}y}{\mathrm{d}x} = 10$			
	$\left(\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - \frac{\mathrm{d}y}{\mathrm{d}t}\right) - 4\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right) = 10$	M1		
	$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - 5\frac{\mathrm{d}y}{\mathrm{d}t} = 10$	A1	2	CSO AG Completion. Be convinced
(c)	$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - 5\frac{\mathrm{d}y}{\mathrm{d}t} = 10 \quad (*)$			
	Auxl eqn $m^2 - 5m = 0$	M1		PI
	$m(m-5) = 0$ $m = 0$ and 5	A1		
	CF: $(y_C =) A + B\mathrm{e}^{5t}$	M1		ft wrong values of m provided 2 arb. constants in CF. condone x for t here
	PI: $(y_P =) -2t$	B1		
	GS of (*) $\{y\} = A + B\,\mathrm{e}^{5t} - 2t$	B1ft	5	ft on c's CF + PI, provided PI is non-zero and CF has two arbitrary constants
(d)	$\Rightarrow y = A + Bx^5 - 2\ln x$	M1		
	$y'(x) = 5Bx^4 - 2x^{-1}$	A1ft		Must involve differentiating $a \ln x$ ft slip
	Using boundary conditions to find A & B	M1		
	$B = 2$; $A = -2$; $\{ y = -2 + 2x^5 - 2\ln x \}$	A1;A1ft	5	ft a slip.
	Total		**19**	
	TOTAL		**75**	

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General Certificate of Education

Mathematics 6360

MFP3 Further Pure 3

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2009 examination - June series

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MFP3

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1(a)	$y(3.1) = y(3) + 0.1\sqrt{3^2+2+1}$	M1A1		
	$= 2 + 0.1\times\sqrt{12} = 2.3464(10..)$ $= 2.3464$	A1	3	Condone > 4dp if correct
(b)	$y(3.2) = y(3) + 2(0.1)[f(3.1, y(3.1))]$	M1		
	$\ldots. = 2 + 2(0.1)[\sqrt{(3.1^2+2.3464+1)}]$	A1F		ft on candidate's answer to **(a)**
	$\ldots. = 2 + 0.2\times3.599499.. = 2.719(89..)$ $= 2.720$	A1	3	CAO Must be 2.720
	Total		**6**	
2	IF is $e^{\int -\tan x\, dx}$	M1		Award even if negative sign missing
	$= e^{\ln(\cos x)\ (+c)}$	A1		OE Condone missing c
	$= (k)\cos x$	A1F		ft earlier sign error
	$\cos x\frac{dy}{dx} - y\tan x\cos x = 2\sin x\cos x$			
	$\frac{d}{dx}(y\cos x) = 2\sin x\cos x$	M1		LHS as $\frac{d}{dx}(y \times \text{IF})$ PI
	$y\cos x = \int 2\sin x\cos x\, dx\, dx$	A1F		ft on c's IF provided no exp or logs
	$y\cos x = \int \sin 2x\, dx$	m1		Double angle or substitution OE for integrating $2\sin x\cos x$
	$y\cos x = -\frac{1}{2}\cos 2x(+c)$	A1		ACF
	$2 = -\frac{1}{2} + c$	m1		Boundary condition used to find c
	$c = \frac{5}{2}$			
	$y\cos x = -\frac{1}{2}\cos 2x + \frac{5}{2}$	A1	9	ACF eg $y\cos x - 2 + \sin^2 x$ Apply ISW after ACF
	Total		**9**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	Centre of circle is $M(3, 4)$	B1		PI
	$A(6, 8)$	B1	2	
(b)(i)	$k = OA = 10$	B1		
	$\tan\alpha = \frac{y_A}{x_A} = \frac{4}{3}$	B1	2	SC " $r = 10$ and $\tan\theta = \frac{8}{6}$ " = B1 only
(b)(ii)	$x^2 + y^2 - 6x - 8y + 25 = 25$	B1		If polar form before expansion award the B1 for correct expansions of both $(r\cos\theta - m)^2$ and $(r\sin\theta - n)^2$ where $(m,n) = (3,4)$ or $(m,n) = (4,3)$
	$r^2 - 6r\cos\theta - 8r\sin\theta = 0$	M1M1		1st M1 for use of any one of $x^2 + y^2 = r^2$, $x = r\cos\theta$, $y = r\sin\theta$ 2nd M1 for use of these to convert the form $x^2 + y^2 + ax + by = 0$ correctly to the form $r^2 + ar\cos\theta + br\sin\theta = 0$
	$\{r = 0$, origin$\}$ Circle: $r = 6\cos\theta + 8\sin\theta$	A1	4	NMS Mark as 4 or 0
	ALTn Circle has eqn $r = OA\cos(\alpha - \theta)$ $r = OA\cos\alpha\cos\theta + OA\sin\alpha\sin\theta$ Circle: $r = 6\cos\theta + 8\sin\theta$	 (M2) (m1) (A1)		 OE
	Total		**8**	

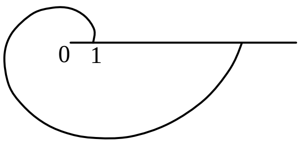
MFP3 (cont)

Q	Solution	Marks	Total	Comments
4	$\int\left(\frac{1}{x}-\frac{4}{4x+1}\right)dx=\ln x-\ln(4x+1)\{+c\}$	B1		OE
	I = $\lim_{a\to\infty}\int_1^a\left(\frac{1}{x}-\frac{4}{4x+1}\right)dx$	M1		∞ replaced by a (OE) and $\lim_{a\to\infty}$
	$=\lim_{a\to\infty}\left[\ln x-\ln(4x+1)\right]_1^a$			
	$=\lim_{a\to\infty}\left[\ln\left(\frac{a}{4a+1}\right)-\ln\frac{1}{5}\right]$	m1		$\ln a-\ln(4a+1)=\ln\left(\frac{a}{4a+1}\right)$ **and** previous M1 scored
	$=\lim_{a\to\infty}\left[\ln\left(\frac{1}{4+\frac{1}{a}}\right)-\ln\frac{1}{5}\right]$	m1		$\ln\left(\frac{a}{4a+1}\right)=\ln\left(\frac{1}{4+\frac{1}{a}}\right)$ **and** previous M1m1 scored
	$=\ln\frac{1}{4}-\ln\frac{1}{5}=\ln\frac{5}{4}$	A1	5	CSO
	Total		**5**	
5(a)	$-k\sin x+2k\cos x+5k\sin x=8\sin x+4\cos x$	M1 A1		Differentiation and subst. into DE
	k = 2	A1	3	
(b)	Auxl eqn $m^2+2m+5=0$			
	$m=\frac{-2\pm\sqrt{4-20}}{2}$	M1		Formula or completing sq. PI
	$m=-1\pm 2\,\mathrm{i}$	A1		
	CF: $\{y_C\}=\mathrm{e}^{-x}(A\sin 2x+B\cos 2x)$	A1F		ft provided m is not real
	GS $\{y\}=\mathrm{e}^{-x}(A\sin 2x+B\cos 2x)+k\sin x$	B1F		ft on CF + PI; must have 2 arb consts
	When $x=0$, $y=1\Rightarrow B=1$	B1F		
	$\frac{dy}{dx}=-\mathrm{e}^{-x}(A\sin 2x+B\cos 2x)$ $+\mathrm{e}^{-x}(2A\cos 2x-2B\sin 2x)+k\cos x$	M1		Product rule
	When $x=0$, $\frac{dy}{dx}=4\Rightarrow 4=-B+2A+k$	A1		PI
	$\Rightarrow A=\frac{3}{2}$			
	$y=\mathrm{e}^{-x}\left(\frac{3}{2}\sin 2x+\cos 2x\right)+2\sin x$	A1	8	CSO
	Total		**11**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$\text{f}(x)=(9+\tan x)^{\frac{1}{2}}$			
	so $\text{f}'(x) = \frac{1}{2}(9+\tan x)^{-\frac{1}{2}}\sec^2 x$	M1 A1		Chain rule
	$\text{f}''(x)= -\frac{1}{4}(9+\tan x)^{-\frac{3}{2}}\sec^4 x$	M1		Product rule, OE
	$+\frac{1}{2}(9+\tan x)^{-\frac{1}{2}}(2\sec^2 x\tan x)$	A1	4	ACF
(a)(ii)	f(0) = 3	B1		
	$\text{f}'(0) = \frac{1}{2}(9)^{-\frac{1}{2}}=\frac{1}{6}$; $\text{f}''(0) = -\frac{1}{4}(9)^{-\frac{3}{2}}=-\frac{1}{108}$	M1		Both attempted and at least one correct ft on c's $\text{f}'(x)$ and $\text{f}''(x)$
	$\text{f}(x) \approx \text{f}(0)+x\,\text{f}'(0)+\frac{1}{2}x^2\,\text{f}''(0)$			
	$(9+\tan x)^{\frac{1}{2}} \approx 3+\frac{x}{6}-\frac{x^2}{216}$	A1	3	CSO AG
(b)	$\frac{\text{f}(x)-3}{\sin 3x}\approx\frac{\frac{x}{6}-\frac{x^2}{216}\dots}{3x-\frac{(3x)^3}{3!}\dots}$	M1		Using series expns.
	$\approx\frac{\frac{1}{6}-\frac{x}{216}\dots}{3-\dots}$	m1		Dividing numerator and denominator by x to get constant term in each
	$\lim_{x\to 0}\left[\frac{\text{f}(x)-3}{\sin 3x}\right] = \frac{1}{18}$	A1	3	
	Total		**10**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
7(a)	Area = $\frac{1}{2}\int\left(1+6e^{-\frac{\theta}{\pi}}\right)^2 d\theta$	M1		Use of $\frac{1}{2}\int r^2\, d\theta$
	$=\frac{1}{2}\int_0^{2\pi}\left(1+12e^{-\frac{\theta}{\pi}}+36e^{-\frac{2\theta}{\pi}}\right) d\theta$	B1		Correct expansion of $\left(1+6e^{-\frac{\theta}{\pi}}\right)^2$
		B1		Correct limits
	$=\frac{1}{2}\left[\theta-12\pi\, e^{-\frac{\theta}{\pi}}-18\pi\, e^{-\frac{2\theta}{\pi}}\right]_0^{2\pi}$	m1		Correct integration of at least two of the three terms 1, $p\, e^{-\frac{\theta}{\pi}}$, $q\, e^{-\frac{2\theta}{\pi}}$
	$=\pi\,(16-6e^{-2}-9e^{-4})$	A1	5	ACF
(b)		B1		Going the correct way round the pole
		B1		Increasing in distance from the pole
	End-points (1, 0) and $(e^2, 2\pi)$	B2,1,0	4	Correct end-points B1 for each pair or for 1 and e^2 shown on graph in correct positions
(c)	$e^{\frac{\theta}{\pi}}=1+6\,e^{-\frac{\theta}{\pi}}$	M1		Elimination of r or θ [$r = 1 + \frac{6}{r}$]
	$\left(e^{\frac{\theta}{\pi}}\right)^2-e^{\frac{\theta}{\pi}}-6=0$	m1		Forming quadratic in $e^{\frac{\theta}{\pi}}$ or in $e^{-\frac{\theta}{\pi}}$ or in r. [$r^2-r-6=0$]
	$\left(e^{\frac{\theta}{\pi}}-3\right)\left(e^{\frac{\theta}{\pi}}+2\right)=0$	m1		OE
	$e^{\frac{\theta}{\pi}}>0$ so $e^{\frac{\theta}{\pi}}=3$	E1		Rejection of negative ‘solution’ PI [$r = 3$]
	Polar coordinates of P are $(3, \pi \ln 3)$	A1	5	
	Total		**14**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
8(a)(i)	$\frac{dx}{dt} = 2t$	B1		PI or for $\frac{dt}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$
	$\frac{dx}{dt}\frac{dy}{dx} = \frac{dy}{dt}$	M1		OE Chain rule $\frac{dy}{dx} = ...$ or $\frac{dy}{dt} = ...$
	$2t\frac{dy}{dx} = \frac{dy}{dt}$ so $2\sqrt{x}\frac{dy}{dx} = \frac{dy}{dt}$	A1	3	AG
(a)(ii)	$\frac{d}{dx}\left(2\sqrt{x}\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{dy}{dt}\right) = \frac{dt}{dx}\frac{d}{dt}\left(\frac{dy}{dt}\right)$	M1		$\frac{d}{dx}(f(t)) = \frac{dt}{dx}\frac{d}{dt}(f(t))$ OE eg $\frac{d}{dt}(g(x)) = \frac{dx}{dt}\frac{d}{dx}(g(x))$
	$2\sqrt{x}\frac{d^2y}{dx^2} + x^{-\frac{1}{2}}\frac{dy}{dx} = \frac{1}{2t}\frac{d^2y}{dt^2}$	M1		Product rule OE
	$4t\sqrt{x}\frac{d^2y}{dx^2} + 2tx^{-\frac{1}{2}}\frac{dy}{dx} = \frac{d^2y}{dt^2}$			
	$\Rightarrow 4x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = \frac{d^2y}{dt^2}$	A1	3	AG Completion
(b)	$4x\frac{d^2y}{dx^2} + 2(1+2\sqrt{x})\frac{dy}{dx} - 3y = 0$			
	$(4x\frac{d^2y}{dx^2} + 2\frac{dy}{dx}) + 2(2\sqrt{x}\frac{dy}{dx}) - 3y = 0$			
	$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = 0$	M1 A1	2	Use of either (a)(i) or (a)(ii) AG Completion
(c)	$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = 0$ (*)			
	Auxl. Eqn. $m^2 + 2m - 3 = 0$			
	$(m+3)(m-1) = 0$	M1		PI
	$m = -3$ and 1	A1		PI
	GS of (*) $\{y\} = Ae^{-3t} + Be^{t}$	M1		$Ae^{-3x} + Be^{x}$ scores M0 here
	$\Rightarrow y = Ae^{-3\sqrt{x}} + Be^{\sqrt{x}}$	A1	4	
	Total		**12**	
	TOTAL		**75**	

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General Certificate of Education

Mathematics 6360

MFP3 Further Pure 3

Mark Scheme

2010 examination - January series

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Further copies of this Mark Scheme are available to download from the AQA Website: www.aqa.org.uk

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP3

Q	Solution	Marks	Total	Comments
1(a)	$y_1 = 2 + 0.1 \times [3\ln(2\times 3+2)]$ =2 + 0.3ln8 = 2.6238(3...)	M1A1		
	$y(3.1)$ = 2.6238 (to 4dp)	A1	3	Condone greater accuracy
(b)	$k_1 = 0.1\times 3\ln 8 = 0.6238(32\ldots)$	B1F		PI ft from (a), 4dp or better
	$k_2 = 0.1\times \mathrm{f}(3.1,\ 2.6238(32...))$	M1		
	… = $0.1\times 3.1\times \ln 8.8238(32..)$ [= 0.6750(1…)	A1F		PI; ft on 0.1×3.1× ln[6.2 + answer(a)]
	$y(3.1) = 2 + \frac{1}{2}[0.6238(3..) + 0.6750(1..)]$	m1		
	= 2.6494(2…) = 2.6494 to 4dp	A1	5	CAO Must be 2.6494
	Total		**8**	
2(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{4+3x}\times 3$	M1		Chain rule
	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -3(4+3x)^{-2}\times 3 = -9(4+3x)^{-2}$	M1A1	3	M1 for quotient (PI) or chain rule used
(b)	$\ln(4+3x) = \ln 4 + y'(0)\,x + y''(0)\frac{1}{2}x^2 + ..$	M1		Clear attempt to use Maclaurin's theorem with numerical values for $y'(0)$ and $y''(0)$
	First three terms: $\ln 4 + \frac{3}{4}x - \frac{9}{32}x^2$	A1F	2	ft on c's answers to (a) provided $y'(0)$ and $y''(0)$ are $\neq 0$. Accept 1.38(6..) for ln4
(c)	$\ln(4-3x) = \ln 4 - \frac{3}{4}x - \frac{9}{32}x^2$	B1F	1	ft $x \to -x$ in c's answer to (b)
(d)	$\ln\left(\frac{4+3x}{4-3x}\right) = \ln(4+3x) - \ln(4-3x)$	M1		
	$\approx \ln 4 + \frac{3}{4}x - \frac{9}{32}x^2 - \ln 4 + \frac{3}{4}x + \frac{9}{32}x^2$			
	$\approx \frac{3}{2}x$	A1	2	CSO AG
	Total		**8**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$u = \frac{dy}{dx} \Rightarrow \frac{du}{dx} = \frac{d^2y}{dx^2}$	M1		
	$x\frac{du}{dx} + 2u = 3x \Rightarrow \frac{du}{dx} + \frac{2}{x}u = 3$	A1	2	CSO AG Substitution into LHS of DE and completion
3(b)	IF is exp ($\int \frac{2}{x}\,dx$)	M1		exp ($\int \frac{k}{x}\,dx$), for $k = \pm 2, \pm 1$ and integration attempted
	$= e^{2\ln x}$; $= x^2$	A1;A1		
	$\frac{d}{dx}(ux^2) = 3x^2$	M1		LHS as differential of $u \times$ IF
	$ux^2 = x^3 + A \Rightarrow u = x + Ax^{-2}$	A1	5	Must have an arbitrary constant
(c)	$\frac{dy}{dx} = x + Ax^{-2}$	M1		and with integration attempted
	$\frac{dy}{dx} = x + Ax^{-2} \Rightarrow y = \frac{1}{2}x^2 - \frac{A}{x} + B$	A1F	2	ft only if IF is M1A0A0
	Total		**9**	
4(a)	$\sin 3x = 3x - \frac{1}{3!}(3x)^3 + = 3x - 4.5x^3 + \ldots$	B1	1	
(b)	$\cos 2x = 1 - \frac{1}{2!}(2x)^2 + \ldots$	B1		
	$\lim_{x \to 0}\left[\frac{3x\cos 2x - \sin 3x}{5x^3}\right] =$ $\lim_{x \to 0} \frac{3x - 6x^3 - 3x + 4.5x^3 + \ldots}{5x^3}$	M1		Using expansions
	$= \lim_{x \to 0} \frac{-1.5 + (o(x^2))\ldots}{5}$	m1		Division by x^3 stage to reach relevant form of quotient before taking limit.
	$= -\frac{3}{10}$	A1	4	CSO OE
	Total		**5**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$y_{\text{PI}} = pxe^{-2x} \Rightarrow \dfrac{dy}{dx} = pe^{-2x} - 2pxe^{-2x}$	M1		Product Rule used
	$\Rightarrow \dfrac{d^2y}{dx^2} = -2pe^{-2x} - 2pe^{-2x} + 4pxe^{-2x}$	A1		
	$-4pe^{-2x} + 4pxe^{-2x} + 3pe^{-2x} - 6pxe^{-2x} + 2pxe^{-2x} = 2e^{-2x}$.	M1		Sub. into DE
	$-pe^{-2x} = 2e^{-2x} \Rightarrow p = -2$	A1F	4	ft one slip in differentiation
5(b)	Aux. eqn. $m^2 + 3m + 2 = 0$ $\Rightarrow m = -1, -2$	B1		
	CF is $Ae^{-x} + Be^{-2x}$	M1		ft on real values of m only
	GS $y = Ae^{-x} + Be^{-2x} - 2xe^{-2x}$.	B1F		Their CF + their PI must have 2 arb consts
	When $x = 0$, $y = 2 \Rightarrow A + B = 2$	B1F		Must be using GS; ft on wrong non-zero values for p and m
	$\dfrac{dy}{dx} = -Ae^{-x} - 2Be^{-2x} - 2e^{-2x} + 4xe^{-2x}$	B1F		Must be using GS; ft on wrong non-zero values for p and m
	When $x = 0$, $\dfrac{dy}{dx} = 0 \Rightarrow -A - 2B - 2 = 0$	B1F		Must be using GS; ft on wrong non-zero values for p and m and slips in finding $y'(x)$
	Solving simultaneously, 2 eqns each in two arbitrary constants	m1		
	$A = 6$, $B = -4$; $y = 6e^{-x} - 4e^{-2x} - 2xe^{-2x}$.	A1	8	CSO
	Total		**12**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
6(a)	The interval of integration is infinite	E1	1	OE
(b)(i)	$x = \frac{1}{y} \Rightarrow$ ‘$\text{d}x = -y^{-2}\,\text{d}y$’			
	$\int \frac{\ln x^2}{x^3}\,\text{d}x \Rightarrow \int (y^3 \ln y^{-2})(-y^{-2})\,\text{d}y$	M1		
	$= \int -y \ln y^{-2}\,\text{d}y = \int 2y \ln y\,\text{d}y$	A1	2	CSO AG
(ii)	$\int 2y \ln y\,\text{d}y = y^2 \ln y - \int y^2 \left(\frac{1}{y}\right)\text{d}y$	M1		$\ldots = ky^2 \ln y \pm \int f(y)\,\text{d}y$ with f(y) not involving the ‘original’ ln y
		A1		
	$\ldots\ldots = y^2 \ln y - \frac{1}{2}y^2 + c$	A1		Condone absence of ‘+ c’
	$\int_0^1 2y \ln y\,\text{d}y = \lim_{a \to 0} \int_a^1 2y \ln y\,\text{d}y$			
	$= \left(0 - \frac{1}{2}\right) - \lim_{a \to 0}\left[a^2 \ln a - \frac{a^2}{2}\right]$	M1		
	$= -\frac{1}{2}$ since $\lim_{a \to 0} a^2 \ln a = 0$	A1	5	CSO Must see clear indication that cand has correctly considered $\lim_{a \to 0} a^k \ln a = 0$
(iii)	So $\int_1^{\infty} \frac{\ln x^2}{x^3}\,\text{d}x = \frac{1}{2}$	B1F	1	ft on minus c’s value as answer to (b)(ii)
	Total		**9**	
7	Aux. eqn. $m^2 + 4 = 0 \Rightarrow m = \pm 2\text{i}$	B1		
	CF is $A\cos 2x + B\sin 2x$	M1		OE. If m is real give M0
		A1F		ft on incorrect complex value for m
	PI: Try $ax^2 + b$	M1		Award even if extra terms, provided the relevant coefficients are shown to be zero.
	$+ c\sin x$	M1		
	$2a - c\sin x + 4ax^2 + 4b + 4c\sin x = 8x^2 + 9\sin x$			
	$a = 2$, $b = -1$,	A1		Dep on relevant M mark
	$c = 3$	A1		Dep on relevant M mark
	$(y =)\ A\cos 2x + B\sin 2x + 2x^2 - 1 + 3\sin x$	B1F	8	Their CF + their PI. Must be exactly two arbitrary constants
	Total		**8**	

Q	Solution	Marks	Total	Comments
8(a)	$4\sin\theta(1-\sin\theta)=1$	M1		Elimination of r or θ { $r = 4[1-(1/r)]$}
	$4\sin^2\theta - 4\sin\theta + 1 = 0$	A1		{ $r^2 - 4r + 4 = 0$ }
	$(2\sin\theta - 1)^2 = 0 \Rightarrow \sin\theta = 0.5$	m1		Valid method to solve quadratic eqn. PI { $(r-2)^2 = 0 \Rightarrow r = 2$ }
	$\theta = \frac{\pi}{6}$, $\theta = \frac{5\pi}{6}$, $r = 2$	A2,1		A1 for any two of the three.
	[$P\left(2,\ \frac{\pi}{6}\right)$ $Q\left(2,\ \frac{5\pi}{6}\right)$]		5	SC: Verification of $P\left(2,\ \frac{\pi}{6}\right)$ scores max of B1 & a further B1 if $Q\left(2,\ \frac{5\pi}{6}\right)$ stated
8(b)	Area triangle OPQ = $\frac{1}{2}\times 2\times r_Q \times \sin POQ$	M1		Any valid method to correct (ft eg on r_Q) expression with just one remaining unknown
	Angle POQ = $\frac{5\pi}{6} - \frac{\pi}{6}$ $\left(=\frac{2\pi}{3}\right)$	m1		Valid method to find remaining unknown either relevant angle or relevant side
	Area triangle $OPQ = 2\sin\frac{2\pi}{3} = \sqrt{3}$	A1		
	Unshaded area bounded by line OP and arc OP = $\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} [4(1-\sin\theta)]^2\ d\theta$	M1		Use of $\frac{1}{2}\int r^2\ d\theta$ for relevant area(s) (condone missing/wrong limits)
	$= 8\int (1 - 2\sin\theta + \sin^2\theta)\ d\theta$	B1		Correct expn of $(1-\sin\theta)^2$
	$= 8\int \left(1 - 2\sin\theta + \frac{1-\cos 2\theta}{2}\right) d\theta$	M1		Attempt to write $\sin^2\theta$ in terms of $\cos 2\theta$
	$= 8\left[\theta + 2\cos\theta + \frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]$ $(+\ c)$	A1F		Correct integration ft wrong coeffs
	$8\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1-\sin\theta)^2\ d\theta =$			
	$8\times\left[\frac{3\theta}{2} + 2\cos\theta - \frac{\sin 2\theta}{4}\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$			
	$= 8\times\{\frac{3\pi}{4} - \left(\frac{3\pi}{12} + 2\cos\frac{\pi}{6} - \frac{1}{4}\sin\frac{2\pi}{6}\right)\}$	m1		$F\left(\frac{\pi}{2}\right) - F\left(\frac{\pi}{6}\right)$ OE for relevant area(s)
	$= 8\times\left(\frac{\pi}{2} - \sqrt{3} + \frac{\sqrt{3}}{8}\right)$ $\{= 4\pi - 7\sqrt{3}\}$	A1F		ft one slip; accept terms in π and $\sqrt{3}$ left unsimplified
	Shaded area = Area of triangle OPQ − $2\times\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} [4(1-\sin\theta)]^2\ d\theta$	M1		OE
	Shaded area = $\sqrt{3} - 16\left(\frac{\pi}{2} - \sqrt{3} + \frac{\sqrt{3}}{8}\right) = 15\sqrt{3} - 8\pi$	A1	11	CSO Accept $m = 15$, $n = -8$
	Total		**16**	
	TOTAL		**75**	

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General Certificate of Education
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Mathematics MFP3

Further Pure 3

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	$y(1.1) = y(1) + 0.1\left[1+3+\sin 1\right]$	M1A1		
	$= 1 + 0.1\times 4.84147. = 1.4841(47..)$ $= 1.4841$ to 4dp	A1	3	Condone > 4dp
(b)	$y(1.2) = y(1) + 2(0.1)\{f[1.1, y(1.1)]\}$	M1		
	$\ldots. = 1 + 2(0.1)\{1.1+3+\sin[1.4841(47..)]\}$	A1F		Ft on cand's answer to **(a)**
	$= 2.019$ to 3dp	A1	3	CAO Must be 2.019 **Note:** If using degrees max mark is 4/6 ie M1A1A0;M1A1FA0
	Total		**6**	
2(a)	$-4k\sin 2x + k\sin 2x = \sin 2x$	M1 A1		Substituting into the differential equation
	$k = -\dfrac{1}{3}$	A1	3	Accept correct PI
(b)	(Aux. eqn $m^2 + 1 = 0$) $m = \pm$ i	B1		PI
	CF: $A\cos x + B\sin x$	M1 A1F		M0 if m is real OE Ft on incorrect complex values for m For the A1F do not accept if left in the form $Ae^{ix} + Be^{-ix}$
	(GS: $y =$) $A\cos x + B\sin x - \dfrac{1}{3}\sin 2x$	B1F	4	c's CF +c's PI but must have 2 constants
	Total		**7**	
3(a)	The interval of integration is infinite	E1	1	OE
(b)	$\int 4xe^{-4x}dx = -xe^{-4x} - \int -e^{-4x}dx$	M1 A1		$kxe^{-4x} - \int ke^{-4x}dx$ for non-zero k
	$= -xe^{-4x} - \dfrac{1}{4}e^{-4x}$ {+c}	A1F	3	Condone absence of +c
(c)	$I = \int_1^{\infty} 4xe^{-4x}\,dx = \lim_{a\to\infty}\int_1^{a} 4xe^{-4x}\,dx$ $\lim_{a\to\infty}\{-ae^{-4a} - \dfrac{1}{4}e^{-4a}\} - \left[-\dfrac{5}{4}e^{-4}\right]$	M1		F(a) – F(1) with an indication of limit '$a \to \infty$'
	$\lim_{a\to\infty} a\,e^{-4a} = 0$	M1		For statement with limit/ limiting process shown
	$I = \dfrac{5}{4}e^{-4}$	A1	3	CSO
	Total		**7**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
4	IF is $\exp\left(\int \frac{3}{x}\,\mathrm{d}x\right)$	M1		and with integration attempted
	$= \mathrm{e}^{3\ln x}$	A1		PI
	$= x^3$	A1		
	$\frac{\mathrm{d}}{\mathrm{d}x}\left[yx^3 \right] = x^3(x^4+3)^{\frac{3}{2}}$	M1 A1		LHS. Use of c's IF. PI
	$\Rightarrow yx^3 = \frac{1}{10}\left(x^4+3 \right)^{\frac{5}{2}} + A$	m1 A1		$k\left(x^4+3 \right)^{\frac{5}{2}}$ Condone missing 'A'
	$\Rightarrow \frac{1}{5} = \frac{1}{10}(4)^{\frac{5}{2}} + A$	m1		Use of boundary conditions in attempt to find constant after intgr. Dep on two M marks, not dep on m
	$\Rightarrow A = -3$; (*) $\Rightarrow yx^3 = \frac{1}{10}\left(x^4+3\right)^{\frac{5}{2}} - 3$	A1	9	ACF. The A1 can be awarded at line (*) provided a correct earlier eqn in y, x and 'A' is seen immediately before boundary conditions are substituted.
	Total		**9**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\cos 4x \approx 1 - \frac{(4x)^2}{2} + \frac{(4x)^4}{4!} \ldots$	M1		Clear attempt to replace x by $4x$ in expansion of $\cos x$…condone missing brackets for the M mark
	$\approx 1 - 8x^2 + \frac{32}{3}x^4 \ldots$	A1	2	
(b)(i)	$\frac{dy}{dx} = \frac{1}{2-e^x} \times (-e^x)$	M1 A1		Chain rule
	$\frac{d^2y}{dx^2} = \frac{(2-e^x)(-e^x) - (-e^x)(-e^x)}{(2-e^x)^2}$	M1 A1		Quotient rule OE ACF
	$= \frac{-2e^x}{(2-e^x)^2}$			
	$\frac{d^3y}{dx^3} = \frac{(2-e^x)^2(-2e^x) - (-2e^x)2(2-e^x)(-e^x)}{(2-e^x)^4}$	m1		All necessary rules attempted (dep on previous 2 M marks)
		A1	6	ACF
(ii)	$y(0) = 0$; $y'(0) = -1$; $y''(0) = -2$; $y'''(0) = -6$	M1		At least three attempted
	$\text{Ln}(2-e^x) \approx y(0) + xy'(0) + \frac{x^2}{2}y''(0) + \frac{x^3}{6}y'''(0) \ldots$			
	$\ldots. \approx -x - x^2 - x^3 \ldots$	A1	2	CSO AG (The previous 7 marks must have been awarded and no double errors seen)
(c)	$\left[\frac{x\ln(2-e^x)}{1-\cos 4x}\right] \approx \frac{-x^2 - x^3 - x^4 \ldots}{8x^2 - \frac{32}{3}x^4}$	M1		Using the expansions
	Limit = $\lim_{x \to 0} \frac{-x^2 - o(x^3)}{8x^2 - o(x^4)}$			The notation $o(x^n)$ can be replaced by a term of the form kx^n
	….. $= \lim_{x \to 0} \frac{-1 - o(x)}{8 - o(x^2)}$	m1		Division by x^2 stage before taking the limit
	…… $= -\frac{1}{8}$	A1	3	CSO
	Total		**13**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$x^2 + y^2 = r^2,\ x = r\cos\theta,\ y = r\sin\theta$	B2,1,0		B1 for one stated or used
	$r^2 = 2r(\cos\theta - \sin\theta)$	M1		
	$x^2 + y^2 = 2(x - y)$	A1	4	ACF
(ii)	$(x-1)^2 + (y+1)^2 = 2$	M1 A1F		
	Centre $(1, -1)$; radius $\sqrt{2}$	A1F	3	
(b)(i)	Area = $\frac{1}{2}\int (4+\sin\theta)^2\,d\theta$	M1		Use of $\frac{1}{2}\int r^2\,d\theta$.
	$= \frac{1}{2}\int_0^{2\pi}(16 + 8\sin\theta + \sin^2\theta)\,d\theta$	B1 B1		Correct expn of $[4+\sin\theta]^2$ Correct limits
	$= \int_0^{2\pi}(8 + 4\sin\theta + 0.25(1-\cos 2\theta))\,d\theta$	M1		Attempt to write $\sin^2\theta$ in terms of $\cos 2\theta$
	$= \left[8\theta - 4\cos\theta + \frac{1}{4}\theta - \frac{1}{8}\sin 2\theta\right]_0^{2\pi}$	A1F		Correct integration ft wrong coefficients
	$= 16.5\pi$	A1	6	CSO
(ii)	For the curves to intersect, the eqn $2(\cos\theta - \sin\theta) = 4 + \sin\theta$ must have a solution.	M1		Equating rs and simplifying to a suitable form
	$2\cos\theta - 3\sin\theta = 4$ $R\cos(\theta+\alpha) = 4$,	M1		OE. Forming a relevant eqn from which valid explanation can be stated directly
	where $R = \sqrt{2^2 + 3^2}$ and $\cos\alpha = \frac{2}{R}$	A1		OE. Correct relevant equation
	$\cos(\theta+\alpha) = \frac{4}{\sqrt{13}} > 1$. Since must have $-1 \le \cos X \le 1$ there are no solutions of the equation $2(\cos\theta - \sin\theta) = 4 + \sin\theta$ so the two curves do not intersect.	E1	4	Accept other valid explanations.
(iii)	Required area = answer (b)(i) $-\pi(\text{radius of } C_1)^2$	M1		
	$= 16.5\pi - 2\pi = 14.5\pi$	A1F	2	Ft on (a)(ii) and (b)(i)
	Total		**19**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$\frac{dx}{dt}\frac{dy}{dx} = \frac{dy}{dt}$	M1		OE Chain rule
	$\frac{1}{2}t^{-\frac{1}{2}}\frac{dy}{dx} = \frac{dy}{dt}$ so $\frac{dy}{dx} = 2t^{\frac{1}{2}}\frac{dy}{dt}$	A1	2	CSO A.G.
(a)(ii)	$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(2t^{\frac{1}{2}}\frac{dy}{dt}\right) = \frac{dt}{dx}\frac{d}{dt}\left(2t^{\frac{1}{2}}\frac{dy}{dt}\right)$	M1		$\frac{d}{dx}(\mathrm{f}(t)) = \frac{dt}{dx}\frac{d}{dt}(\mathrm{f}(t))$ O.E. eg $\frac{d}{dt}(\mathrm{g}(x)) = \frac{dx}{dt}\frac{d}{dx}(\mathrm{g}(x))$
	$\frac{d^2y}{dx^2} = 2t^{\frac{1}{2}}\left[2t^{\frac{1}{2}}\frac{d^2y}{dt^2} + t^{-\frac{1}{2}}\frac{dy}{dt}\right]$	m1		Product rule O.E. used dep on previous M1 being awarded at some stage
	$\frac{d^2y}{dx^2} = 4t\frac{d^2y}{dt^2} + 2\frac{dy}{dt}$	A1	3	CSO A.G.
(b)	$t^{\frac{1}{2}}\left[4t\frac{d^2y}{dt^2} + 2\frac{dy}{dt}\right] - (8t+1)2t^{\frac{1}{2}}\frac{dy}{dt}$ $+ 12t^{\frac{3}{2}}y = 12t^{\frac{5}{2}}$	M1		Subst. using (a)(i), (a)(ii) into given DE to eliminate all x
	$4t^{\frac{3}{2}}\frac{d^2y}{dt^2} - 16t^{\frac{3}{2}}\frac{dy}{dt} + 12t^{\frac{3}{2}}y = 12t^{\frac{5}{2}}$			
	Divide by $4t^{\frac{3}{2}}$ gives $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 3y = 3t$	A1	2	CSO A.G.
(c)	Solving $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 3y = 3t$ (*)			
	Auxl. Eqn. $m^2 - 4m + 3 = 0$ $(m-1)(m-3) = 0$	M1		PI
	$m = 1$ and 3	A1		
	CF $Ae^t + Be^{3t}$	M1		Condone x for t here; ft c's 2 real values for 'm'
	For PI try $y = pt + q$	M1		OE
	$-4p + 3pt + 3q = 3t \Rightarrow p = 1,\ q = \frac{4}{3}$	A1		
	GS of (*) is $y = Ae^t + Be^{3t} + t + \frac{4}{3}$	B1F		CF + PI with 2 arb. constants and both CF and PI functions of t only
	GS of $x\frac{d^2y}{dx^2} - (8x^2+1)\frac{dy}{dx} + 12x^3y = 12x^5$ is $y = Ae^{x^2} + Be^{3x^2} + x^2 + \frac{4}{3}$	A1	7	
	Total		**14**	
	TOTAL		**75**	

Version 1.0

General Certificate of Education (A-level)
January 2011

Mathematics MFP3

(Specification 6360)

Further Pure 3

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Otherwise we require evidence of a correct method for any marks to be awarded.

MFP3

Q	Solution	Marks	Total	Comments
1	$k_1 = 0.1\times\left(3+\sqrt{4}\right)$ (=0.5)	M1		
	k_2 = 0.1 f (3.1, 4.5)	M1		
	$k_2 = 0.1\times\left(3.1+\sqrt{4.5}\right) = 0.522132...$	A1		PI accept 3dp or better
	$y(3.1) = y(3) + \frac{1}{2}[k_1 + k_2]$ $= 4 + 0.5 \times 1.022132....$	m1		Dep on previous two Ms and numerical values for k's
	$y(3.1) = 4.511$	A1	5	Must be 4.511
	Total		**5**	
2(a)	$p\cos x - q\sin x + 5p\sin x + 5q\cos x = 13\cos x$	M1		Differentiation and subst. into DE
	$p + 5q = 13$; $5p - q = 0$	m1		Equating coeffs.
	$p = \frac{1}{2}$; $q = \frac{5}{2}$	A1	3	OE Need both
(b)	Aux. eqn. $m + 5 = 0$	M1		PI. Or solving $y'(x)+5y=0$ as far as $y=$
	$(y_{CF} =)Ae^{-5x}$	A1		OE
	$(y_{GS} =)Ae^{-5x} + \frac{1}{2}\sin x + \frac{5}{2}\cos x$	B1F	3	c's CF + c's PI with exactly one arbitrary constant OE
	Total		**6**	
3(a)	$r + r\cos\theta = 2$	M1		
	$r + x = 2$	B1		$r\cos\theta = x$ stated or used
	$r = 2 - x$	A1		
	$x^2 + y^2 = (2-x)^2$	M1		$r^2 = x^2 + y^2$ used
	$y^2 = 4 - 4x$	A1	5	Must be in the form $y^2 = f(x)$ but accept ACF for f(x).
(b)	Equation of line: $r\cos\theta = \frac{3}{4} \Rightarrow x = \frac{3}{4}$	M1		Use of $r\cos\theta = x$
		A1		$4x=3$ OE
	$y^2 = 4 - 4\left(\frac{3}{4}\right) = 1 \Rightarrow y = \pm 1$; [Pts $\left(\frac{3}{4}, \pm 1\right)$]	M1		
	Distance between pts (0.75, 1) and (0.75,−1) is 2	A1	4	
	<u>**Altn:**</u>			
	At pts of intersection, $r = \frac{5}{4}$ and $\cos\theta = \frac{3}{5}$ OE	(M1A1)		(M1 elimination of either r or θ) (For A condone slight prem approx.)
	Distance $PQ = 2r\sin\theta$	(M1)		Or use of cosine rule or Pythag.
	$= 2\times\frac{5}{4}\times\frac{4}{5} = 2$	(A1)		Must be from exact values.
	Total		**9**	

MFP3(cont)

Q	Solution	Marks	Total	Comments
4	IF is $e^{\int -\frac{2}{x}\,dx}$	M1		Award even if negative sign missing
	$= e^{-2\ln(x)\ (+c)} = e^{\ln(x)^{-2}\ (+c)}$	A1		OE Condone missing c
	$= (k)x^{-2}$	A1F		Ft earlier sign error
	$x^{-2}\dfrac{dy}{dx} - 2x^{-3}y = 2xe^{2x}$			
	$\dfrac{d}{dx}\left(x^{-2}y\right) = 2x\,e^{2x}$	M1		LHS as d/dx(y×IF) PI
	$x^{-2}y = \int 2x\,e^{2x}\,dx$			
	$= \int x\ d(e^{2x}) = x\,e^{2x} - \int e^{2x}\,dx$	M1 A1		Integration by parts in correct dirn
	$x^{-2}y = xe^{2x} - \frac{1}{2}e^{2x}\ (+c)$	A1		ACF
	When $x = 2$, $y = e^4$ so $\frac{1}{4}e^4 = 2e^4 - \frac{1}{2}e^4 + c$	m1		Boundary condition used to find c after integration.
	$c = -\frac{5}{4}e^4$			
	$y = x^3e^{2x} - \frac{1}{2}x^2e^{2x} - \frac{5}{4}x^2e^4$	A1	9	Must be in the form $y = f(x)$
	Total		**9**	

MFP3(cont)

Q	Solution	Marks	Total	Comments
5(a)	$\frac{12x+8-12x-3}{(4x+1)(3x+2)}=\frac{5}{(4x+1)(3x+2)}$	B1	1	Accept $C = 5$
(b)	$\int\frac{10}{(4x+1)(3x+2)}\mathrm{d}x=2\int\left(\frac{4}{4x+1}-\frac{3}{3x+2}\right)\mathrm{d}x$	M1		
	$= 2\left[\ln(4x+1)-\ln(3x+2)\right]$ $(+c)$	A1		OE
	$\mathrm{I} = \lim_{a\to\infty}\int_1^a\left(\frac{10}{(4x+1)(3x+2)}\right)\mathrm{d}x$	M1		∞ replaced by a and $\lim_{a\to\infty}$ (OE)
	$=2\lim_{a\to\infty}\left[\ln(4a+1)-\ln(3a+2)\right]-(\ln 5-\ln 5)$			
	$=2\lim_{a\to\infty}\left[\ln\left(\frac{4a+1}{3a+2}\right)\right]=2\lim_{a\to\infty}\left[\ln\left(\frac{4+\frac{1}{a}}{3+\frac{2}{a}}\right)\right]$	m1,m1		Limiting process shown. Dependent on the previous M1M1
	$= 2\ln\frac{4}{3}=\ln\frac{16}{9}$	A1	6	CSO
	Total		**7**	

MFP3(cont)

Q	Solution	Marks	Total	Comments
6	Area = $\frac{1}{2}\int\left(2\sin 2\theta\sqrt{\cos\theta}\right)^2\ \mathrm{d}\theta$	M1		Use of $\frac{1}{2}\int r^2\ \mathrm{d}\theta$
	$=\frac{1}{2}\int_0^{\frac{\pi}{2}}\left(4\cos\theta\ \sin^2 2\theta\right)\ \mathrm{d}\theta$	B1		$r^2=4\cos\theta\sin^2 2\theta$ or better
		B1		Correct limits
	$=\frac{1}{2}\int_0^{\frac{\pi}{2}}\left(16\sin^2\theta\ \cos^3\theta\right)\ \mathrm{d}\theta$	M1		$\sin^2 2\theta=k\sin^2\theta\cos^2\theta$ (k>0)
	$=\int_0^{\frac{\pi}{2}}\left(8\sin^2\theta\ (1\text{-}\sin^2\theta)\right)\ \mathrm{d}\sin\theta$	m1		Substitution or another valid method to integrate $\sin^2\theta\cos^3\theta$
	$=\left[\frac{8\sin^3\theta}{3}-\frac{8\sin^5\theta}{5}\right]_0^{\frac{\pi}{2}}$	A1F		Correct integration of $p\sin^2\theta\cos^3\theta$
	$=\left(\frac{8}{3}-\frac{8}{5}\right)-0=\frac{16}{15}$	A1	7	CSO AG
	<u>Alternatives for the last four marks</u>			
	Area = $\int_0^{\frac{\pi}{2}}\left(\cos\theta-\cos 4\theta\cos\theta\right)\ \mathrm{d}\theta$	(M1)		$2\cos\theta\sin^2 2\theta=\lambda\cos\theta+\mu\cos 4\theta\cos\theta$ $(\lambda,\mu\neq 0)$
	$\int\left(\cos 4\theta\cos\theta\right)\ \mathrm{d}\theta$	(m1)		Integration by parts twice or use of $\cos 4\theta\cos\theta=\frac{1}{2}\left(\cos 5\theta+\cos 3\theta\right)$
	$=-\frac{1}{15}(\cos 4\theta\sin\theta-4\sin 4\theta\cos\theta)$	(A1F)		Correct integration of $p\cos 4\theta\cos\theta$ [eg $p\left[\frac{1}{10}\sin 5\theta+\frac{1}{6}\sin 3\theta\right]$]
	Area = (1−0) + $\frac{1}{15}[(1-0)-(0)]=\frac{16}{15}$	(A1)		CSO AG $\{1-\frac{1}{10}+\frac{1}{6}=\frac{16}{15}\}$
	Total		**7**	

MFP3(cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$\cos x + \sin x = 1 + x - \frac{1}{2}x^2 - \frac{1}{6}x^3$	B1	1	Accept coeffs unsimplified, even 3! for 6.
(ii)	$\ln(1+3x) = 3x - \frac{1}{2}(3x)^2 + \frac{1}{3}(3x)^3 = 3x - \frac{9}{2}x^2 + 9x^3$	B1	1	Accept coeffs unsimplified
(b)(i)	$y = e^{\tan x}, \quad \frac{dy}{dx} = \sec^2 x\, e^{\tan x}$	M1 A1		Chain rule ACF eg $y\sec^2 x$
	$\frac{d^2y}{dx^2} = 2\sec^2 x \tan x\, e^{\tan x} + \sec^4 x\, e^{\tan x}$	m1 A1		Product rule OE ACF
	$= \sec^2 x\, e^{\tan x}(2\tan x + \sec^2 x)$			
	$= \frac{dy}{dx}(2\tan x + 1 + \tan^2 x)$			
	$\frac{d^2y}{dx^2} = (1+\tan x)^2 \frac{dy}{dx}$	A1	5	AG Completion; CSO any valid method.
(ii)	$\frac{d^3y}{dx^3} = 2(1+\tan x)\sec^2 x \frac{dy}{dx} + (1+\tan x)^2 \frac{d^2y}{dx^2}$	M1		
	When $x = 0$, $\frac{d^3y}{dx^3} = 2(1)(1)(1)+(1)(1) = 3$	A1	2	CSO
(iii)	$y(0) = 1$; $y'(0) = 1$; $y''(0) = 1$; $y'''(0) = 3$; $y(x) \approx y(0) + x\,y'(0) + \frac{1}{2}x^2 y''(0) + \frac{1}{3!}x^3 y'''(0)$	M1		
	$e^{\tan x} \approx 1 + x + \frac{1}{2}x^2 + \frac{1}{2}x^3$	A1	2	CSO AG
(c)	$\lim_{x\to 0}\left[\frac{e^{\tan x} - (\cos x + \sin x)}{x\ln(1+3x)}\right]$			
	$= \lim_{x\to 0} \frac{1 + x + \frac{x^2}{2} + \frac{x^3}{2} - 1 - x + \frac{x^2}{2} + \frac{x^3}{6}}{x\left(3x - \frac{9}{2}x^2 + \ldots\right)}$	M1		Using series expns.
	$= \lim_{x\to 0}\left[\frac{x^2 + \frac{2}{3}x^3 + ..}{3x^2 - \frac{9}{2}x^3 \ldots}\right] = \lim_{x\to 0}\left[\frac{1 + \frac{2}{3}x + ..}{3 - \frac{9}{2}x \ldots}\right]$	m1		Dividing numerator and denominator by x^2 to get constant terms. OE following a slip.
	$= \frac{1}{3}$	A1	3	
	Total		**14**	

MFP3(cont)

Q	Solution	Marks	Total	Comments
8(a)	$\frac{dx}{dt}\frac{dy}{dx} = \frac{dy}{dt}$	M1		Chain rule
	$e^t \frac{dy}{dx} = \frac{dy}{dt} \Rightarrow x\frac{dy}{dx} = \frac{dy}{dt}$	A1	2	CSO AG
(b)	$\frac{d}{dt}\left(x\frac{dy}{dx}\right) = \frac{d^2y}{dt^2}$; $\frac{dx}{dt}\frac{d}{dx}\left(x\frac{dy}{dx}\right) = \frac{d^2y}{dt^2}$	M1		OE $\frac{d}{dx}\left(x\frac{dy}{dx}\right) = \frac{dt}{dx}\frac{d^2y}{dt^2}$
	$\frac{dx}{dt}\left(\frac{dy}{dx} + x\frac{d^2y}{dx^2}\right) = \frac{d^2y}{dt^2}$	m1		Product rule (dep on previous M)
	$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = \frac{d^2y}{dt^2}$	A1		OE
	$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 2\ln x$ becomes			
	$\frac{d^2y}{dt^2} - x\frac{dy}{dx} - 3x\frac{dy}{dx} + 4y = 2\ln x$			
	$\Rightarrow \frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 4y = 2\ln e^t$ (using (a))	m1		
	$\Rightarrow \frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 4y = 2t$	A1	5	CSO AG
(c)	Auxl eqn $m^2 - 4m + 4 = 0$	M1		PI
	$(m-2)^2 = 0$, $m = 2$	A1		PI
	CF: $(y_C =)$ $(At + B)e^{2t}$	M1		Ft wrong value of m provided equal roots and 2 arb. constants in CF. Condone x for t here
	PI Try $(y_P =)$ $at + b$	M1		If extras, coeffs. must be shown to be 0.
	$-4a + 4at + 4b = 2t \Rightarrow a = b = \frac{1}{2}$	A1		Correct PI. Condone x for t here
	GS $\{y\} = (At + B)e^{2t} + 0.5(t + 1)$	B1F	6	Ft on c's CF + PI, provided PI is non-zero and CF has two arbitrary constants and RHS is fn of t only
(d)	$\Rightarrow y = (A\ln x + B)x^2 + 0.5(\ln x + 1)$	M1		
	$y = 1.5$ when $x = 1 \Rightarrow B = 1$	A1F		Ft one earlier slip
	$y'(x) = (A\ln x + B)\,2x + Ax + 0.5x^{-1}$	m1		Product rule
	$y'(1) = 0.5 \Rightarrow A = -2$	A1F		Ft one earlier slip
	$y = (1 - 2\ln x)x^2 + \frac{1}{2}(\ln x + 1)$	A1	5	ACF
	Total		**18**	
	TOTAL		**75**	

Version 1.0

General Certificate of Education (A-level)
June 2011

Mathematics **MFP3**

(Specification 6360)

Further Pure 3

Final

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M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

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Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP3

Q	Solution	Marks	Total	Comments
1	$k_1 = 0.2\times[2+\ln(1+1)]$	M1		PI. May be seen within given formula
	= 0.5386(29…) (= *)			Accept 3sf rounded or truncated or better as evidence of the M1 line
	$k_2 = 0.2\times f(2.2,\ 1+*...)$			
	… = 0.2 ×[2.2 + ln (1+1.5386…)]	M1		0.2×[2.2+ln(1+1+c's k_1)]. PI May be seen within given formula
	…. = 0.6263(248….)	A1		4dp or better. PI by later work
	$y(2.2) = y(2)+\frac{1}{2}[k_1+k_2]$ = 1 + 0.5×[0.5386… + 0.6263…]	m1		Dep on previous two Ms but ft on c's numerical values (or numerical expressions) for *k*'s following evaluation of these.
	= 1+ 0.5×1.16495….			
	(= 1.582477…) = 1.5825 to 4dp	A1	5	CAO Must be 1.5825
				SC For those scoring M1M0 who have k_2=0.5261(78..), and final answer 1.5324 (ie 4 dp) for y(2.2) award a total of 2 marks [M1B1]
	Total		**5**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
2(a)	PI: $y_{PI} = p + qx\mathrm{e}^{-2x}$			
	$y'_{PI} = q\mathrm{e}^{-2x} - 2qx\mathrm{e}^{-2x}$	M1		Product rule used
	$y''_{PI} = -4q\mathrm{e}^{-2x} + 4qx\mathrm{e}^{-2x}$			
	$-4q\mathrm{e}^{-2x} + 4qx\mathrm{e}^{-2x} + q\mathrm{e}^{-2x} - 2qx\mathrm{e}^{-2x}$	M1		Subst. into DE
	$-2p - 2qx\mathrm{e}^{-2x} = 4 - 9\mathrm{e}^{-2x}$			
	$-3q = -9$ and $-2p = 4$	m1		Equating coefficients
	$-3q = -9$ so $q = 3$;	A1		
	$-2p = 4$ so $p = -2$;	B1	5	
	[$y_{PI} = 3x\mathrm{e}^{-2x} - 2$]			
(b)	Aux. eqn. $m^2 + m - 2 = 0$			
	$(m-1)(m+2) = 0$	M1		Factorising or using quadratic formula OE PI by correct two values of '*m*' seen/used
	$y_{CF} = A\mathrm{e}^{x} + B\mathrm{e}^{-2x}$	A1		
	$y_{GS} = A\mathrm{e}^{x} + B\mathrm{e}^{-2x} + 3x\mathrm{e}^{-2x} - 2$	B1F	3	(y_{GS}) = c's CF + c's PI, provided 2 arbitrary constants
(c)	$x = 0,\ y = 4 \Rightarrow 4 = A + B - 2$	B1F		Only ft if exponentials in GS
	$\frac{\mathrm{d}y}{\mathrm{d}x} = A\mathrm{e}^{x} - 2B\mathrm{e}^{-2x} + 3\mathrm{e}^{-2x} - 6x\mathrm{e}^{-2x}$			
	As $x \to \infty$, ($\mathrm{e}^{-2x} \to 0$ and) $x\mathrm{e}^{-2x} \to 0$	E1		
	As $x \to \infty$, $\frac{\mathrm{d}y}{\mathrm{d}x} \to 0$ so $A = 0$	B1		
	When $A = 0$, $4 = 0 + B - 2 \Rightarrow B = 6$			
	$y = 6\mathrm{e}^{-2x} + 3x\mathrm{e}^{-2x} - 2$	B1	4	$y = 6\mathrm{e}^{-2x} + 3x\mathrm{e}^{-2x} - 2$ OE
	Total		**12**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\int x^2 \ln x \,\mathrm{d}x = \frac{x^3}{3}\ln x - \int \frac{x^3}{3}\left(\frac{1}{x}\right)\mathrm{d}x$	M1		$... = kx^3 \ln x \pm \int \mathrm{f}(x)$, with f(x) not involving the 'original' ln x
		A1		
	$\ldots\ldots = \frac{x^3}{3}\ln x - \frac{x^3}{9}$ (+ c)	A1	3	Condone absence of '+ c'
(b)	Integrand is not defined at $x = 0$	E1	1	OE
(c)	$\int_0^{\mathrm{e}} x^2 \ln x \,\mathrm{d}x = \left\{\lim_{a\to 0} \int_a^{\mathrm{e}} x^2 \ln x \,\mathrm{d}x\right\}$			OE
	$= \left(\frac{\mathrm{e}^3}{3}\ln \mathrm{e} - \frac{\mathrm{e}^3}{9}\right) - \lim_{a\to 0}\left[\frac{a^3}{3}\ln a - \frac{a^3}{9}\right]$	M1		F(e) − $\lim_{a\to 0}$ [F(a)]
	But $\lim_{a\to 0} a^3 \ln a = 0$	E1		Accept a general form eg $\lim_{x\to 0} x^k \ln x = 0$
	So $\int_0^{\mathrm{e}} x^2 \ln x \,\mathrm{d}x = \frac{2\mathrm{e}^3}{9}$	A1	3	CSO
	Total		**7**	
4	$\frac{\mathrm{d}y}{\mathrm{d}x} + (\cot x)y = \sin 2x$			
	IF is exp ($\int \cot x \,\mathrm{d}x$)	M1		and with integration attempted
	$= \mathrm{e}^{\ln(\sin x)\ (+c)}$	A1		OE Condone missing '+c'
	= (k) sin x	A1		'IF = sin x' scores M1A1A1
	$\sin x\frac{\mathrm{d}y}{\mathrm{d}x} + (\cos x)y = \sin 2x \sin x$			
	$\frac{\mathrm{d}}{\mathrm{d}x}[y\sin x] = \sin 2x \sin x$	M1		LHS as differential of y×IF PI
	$y \sin x = \int \sin 2x \sin x \,\mathrm{d}x$	A1F		Ft on c's IF provided no exp. or logs
	$\Rightarrow y \sin x = \int 2\sin^2 x \cos x \,\mathrm{d}x$	B1		sin2x = 2sin$x$cos$x$ used
	$\Rightarrow y \sin x = \int 2\sin^2 x \,\mathrm{d}(\sin x)$	m1		dep on both Ms Use of relevant substitution to stage $\int 2s^2 \mathrm{d}s$ or further or by inspection to $k\sin^3 x$
	$y \sin x = \frac{2}{3}\sin^3 x$ (+c)	A1		ACF
	$\frac{1}{2}\sin\frac{\pi}{6} = \frac{2}{3}\sin^3\frac{\pi}{6} + c$	m1		dep on both Ms Boundary condition used in attempt to find value of c after integration
	$c = \frac{1}{6}$ so $y \sin x = \frac{2}{3}\sin^3 x + \frac{1}{6}$	A1	10	CSO – no errors seen – accept equivalent forms
	Total		**10**	

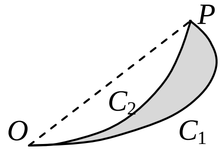
MFP3 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\frac{dy}{dx} = \frac{2\sec^2 x}{1+2\tan x}$	M1 A1		Chain rule ACF for $y'(x)$
	$\frac{d^2y}{dx^2} = \frac{(1+2\tan x)(4\sec^2 x\tan x) - 2\sec^2 x(2\sec^2 x)}{(1+2\tan x)^2}$	M1 A1	 4	Quotient rule OE in which both u and v are not const. or applied to a correct form of y' ACF for $y''(x)$
(b)	McC. Thm: $y(0) + x\,y'(0) + \frac{x^2}{2}\,y''(0)$ $(y(0) = 0)$; $y'(0) = 2$; $y''(0) = -4$	M1		Attempt to evaluate at least $y'(0)$ and $y''(0)$. PI
	$\ln(1+2\tan x) \approx 2x - 2x^2$	A1	2	Dep on previous 5 marks
(c)	$\ln(1-x) = -x - \frac{1}{2}x^2 \ldots.$	B1		Ignore higher power terms
	$\left[\frac{\ln(1+2\tan x)}{\ln(1-x)}\right] \approx \frac{2x - 2x^2\ldots}{-x - \frac{1}{2}x^2\ldots}$	M1		Expansions used
	$= \frac{2 - 2x\ldots}{-1 - \frac{1}{2}x\ldots}$	m1		Dividing num. and den. by x to get constant term in each and non-const term in at least num. or den.
	So $\lim_{x \to 0}\left[\frac{\ln(1+2\tan x)}{\ln(1-x)}\right] = \frac{2}{-1} = -2$	A1F	4	ft c's answer to (b) provided answer (b) is in the form $\pm px \pm qx^2\ldots$ and B1 awarded
	Total		**10**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$u=\dfrac{\mathrm{d}y}{\mathrm{d}x}-2x \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x}=\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}-2$	M1 A1		Differentiating subst wrt x, $\geq$ two terms correct
	DE becomes			
	$(x^3+1)(\dfrac{\mathrm{d}u}{\mathrm{d}x}+2)-3x^2(u+2x)=2-4x^3$	M1		Substitute into LHS of DE as far as no ys
	$(x^3+1)\dfrac{\mathrm{d}u}{\mathrm{d}x}+2x^3+2-3x^2u-6x^3=2-4x^3$			
	DE becomes $(x^3+1)\dfrac{\mathrm{d}u}{\mathrm{d}x}=3x^2u$	A1	4	CSO AG
(b)	$\int \dfrac{1}{u}\,\mathrm{d}u=\int \dfrac{3x^2}{x^3+1}\,\mathrm{d}x$	M1		Separate variables OE PI
	$\ln u=\ln\left(x^3+1\right)+\ln A$	A1;A1		$\ln u$; $\ln(x^3+1)$
		A1F		Applying law of logs to correctly combine two log terms or better
	$u=A\left(x^3+1\right)$	A1		OE RHS
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=A\left(x^3+1\right)+2x$	m1		u = f(x) to $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\pm\mathrm{f}(x)\pm 2x$
	$y=A\left(\dfrac{x^4}{4}+x\right)+x^2+B$	m1 A1	8	Solution with two arbitrary constants and both previous M and m scored OE RHS
	Total		**12**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$r=2\sin\theta \Rightarrow \quad r^2=2r\sin\theta$	M1		
	$x^2+y^2=2y$	A2,1	3	OE (A1) either for $r^2=x^2+y^2$ or for $r\sin\theta=y$ **SC** If M0 give B1 for $r^2=x^2+y^2$ or for $r\sin\theta=y$ used
(b)(i)	$2\sin\theta=\tan\theta$ $2\sin\theta\cos\theta=\sin\theta$	M1		Equating rs
	$\sin\theta(2\cos\theta-1)=0$	m1		Both solutions have to be considered if not in factorised form
	$\sin\theta=0\Rightarrow\theta=0\,;\cos\theta=\frac{1}{2}\Rightarrow\theta=\frac{\pi}{3}$			Alternative: $\sin 2\theta=\sin\theta\Rightarrow\theta=0,\ \frac{\pi}{3}$
	$\theta=0\Rightarrow r=0$ ie pole O (0,0)	B1		Indep. Can just verify using both eqns +statement.
	$\theta=\frac{\pi}{3}\Rightarrow r=\sqrt{3}\quad\left(P\left(\sqrt{3},\frac{\pi}{3}\right)\right)$	A1	4	CSO
(ii)	At A, $\theta=\frac{\pi}{4}$, $r=2\sin\frac{\pi}{4}=\sqrt{2}$ At B, $\theta=\frac{\pi}{4}$, $r=\tan\frac{\pi}{4}=1$	M1		Substitute $\theta=\frac{\pi}{4}$ into the equations of both curves.
	Since $\sqrt{2}>1$, A is further away (from the pole than B.)	E1	2	CSO
(iii)				
	Area bounded by line OP and curve C_1 $=\frac{1}{2}\int_0^{\frac{\pi}{3}} 4\sin^2\theta\ \mathrm{d}\theta$	M1		Use of $\frac{1}{2}\int r^2\ \mathrm{d}\theta$; ignore limits here
	$=\int\ (1-\cos 2\theta)\ \mathrm{d}\theta$	m1		Attempt to write $\sin^2\theta$ in terms of $\cos 2\theta$ only
	$=\left[\theta-\frac{1}{2}\sin 2\theta\right]$	A1		Ignore limits here
	$=\left(\frac{\pi}{3}-\frac{1}{2}\times\frac{\sqrt{3}}{2}\right)-0=\frac{\pi}{3}-\frac{\sqrt{3}}{4}$	A1		PI
	Area bounded by line OP and curve C_2 $=\frac{1}{2}\int_0^{\frac{\pi}{3}} \tan^2\theta\ \mathrm{d}\theta$	M1		Use of $\frac{1}{2}\int r^2\ \mathrm{d}\theta$; ignore limits here
	$=\frac{1}{2}\int\ (\sec^2\theta-1)\ \mathrm{d}\theta$	m1		Using $\tan^2\theta=\pm\sec^2\theta\ \pm 1$ PI
	$=\frac{1}{2}[\tan\theta-\theta]$	A1		Ignore limits here
	$=\frac{1}{2}\left(\sqrt{3}-\frac{\pi}{3}\right)-0=\frac{\sqrt{3}}{2}-\frac{\pi}{6}$	A1		PI
	Required area = $\left(\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right)-\left(\frac{\sqrt{3}}{2}-\frac{\pi}{6}\right)$	M1		Can award earlier eg if we see $\frac{1}{2}\int_0^{\frac{\pi}{3}} 4\sin^2\theta\ \mathrm{d}\theta-\frac{1}{2}\int_0^{\frac{\pi}{3}} \tan^2\theta\ \mathrm{d}\theta$
	$=\frac{1}{2}\pi-\frac{3}{4}\sqrt{3}\quad\left(a=\frac{1}{2},\quad b=-\frac{3}{4}\right)$	A1	10	CSO
	Total		**19**	
	TOTAL		**75**	

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Q	Solution	Marks	Total	Comments
1(a)	$y(1.1) = y(1) + 0.1\left[\frac{2-1}{4+1}\right]$	M1A1		
	$= 2 + 0.02 = 2.02$	A1	3	
(b)	$y(1.2) = y(1) + 2(0.1)\{f[1.1, y(1.1)]\}$	M1		
	$= 2 + 2(0.1)\left[\frac{2.02-1.1}{2.02^2+1.1}\right]$	A1F		ft on c's answer to (a)
	$= 2.035518\ldots = 2.036$ to 3dp	A1	3	CAO Must be 2.036
	Total		**6**	
2	$\sqrt{4+x} = 2\left(1+\frac{x}{4}\right)^{\frac{1}{2}} = 2\left[1+\frac{1}{2}\left(\frac{x}{4}\right)+O(x^2)\right]$	M1		Attempt to use binomial theorem OE The notation $O(x^n)$ can be replaced by a term of the form kx^n
	$\left[\frac{\sqrt{4+x}-2}{x+x^2}\right] = \left[\frac{\frac{x}{4}+O(x^2)}{x+x^2}\right] = \left[\frac{\frac{1}{4}+O(x)}{1+x}\right]$	m1		Division by x stage before taking the limit
	$\lim_{x\to 0}\left[\frac{\sqrt{4+x}-2}{x+x^2}\right] = \frac{1}{4}$	A1	3	CSO NMS 0/3
	Total		**3**	
3	$m^2 + 2m + 10 = 0$	M1		PI
	$m = -1 \pm 3\text{i}$	A1		
	Complementary function is $(y =)\ \text{e}^{-x}(A\cos 3x + B\sin 3x)$	A1F		OE Ft on incorrect **complex value** of m
	Particular integral: try $y = k\text{e}^x$	M1		
	$k + 2k + 10k = 26 \Rightarrow k = 2$	A1		
	(GS $y =$) $\text{e}^{-x}(A\cos 3x + B\sin 3x) + 2\text{e}^x$	B1F		c's CF+ c's non-zero PI but must have 2 arb consts
	$x = 0, y = 5 \Rightarrow 5 = A + 2$ so $A = 3$	B1F		ft c's k ie $A = 5 - k, k \neq 0$
	$\frac{\text{d}y}{\text{d}x} =$ $\text{e}^{-x}(-3A\sin 3x + 3B\cos 3x - A\cos 3x - B\sin 3x) + 2\text{e}^x$	M1		Attempt to differentiate c's **GS** (ie CF + PI)
	$11 = 3B - A + 2 \qquad (B = 4)$	A1		
	$y = \text{e}^{-x}(3\cos 3x + 4\sin 3x) + 2\text{e}^x$	A1	10	CSO
	Total		**10**	

Q	Solution	Marks	Total	Comments
4(a)	IF is $\exp\left(\int \frac{2}{x}\,\mathrm{d}x\right)$	M1		and with integration attempted
	$= e^{2\ln x}$	A1		PI
	$= x^2$	A1		
	$\frac{\mathrm{d}}{\mathrm{d}x}\left[yx^2\right] = x^2 \ln x$	M1		LHS; PI
	$\Rightarrow yx^2 = \int (\ln x)\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x^3}{3}\right)$	M1		Attempt integration by parts in correct direction to integrate $x^p \ln x$
	$= \frac{x^3}{3}\ln x - \int \frac{x^2}{3}\,\mathrm{d}x$	A1		RHS
	$yx^2 = \frac{x^3}{3}\ln x - \frac{x^3}{9} + A$			
	$\{\, y = \frac{x}{3}\ln x - \frac{x}{9} + Ax^{-2} \,\}$	A1	7	
(b)	Now, as $x \to 0$, $x^k \ln x \to 0$	E1		Must be stated explicitly for a value of $k > 0$
	As $x \to 0$, $y \to 0 \Rightarrow A = 0$	B1		Const of int = 0 must be convincing
	$yx^2 = \frac{x^3}{3}\ln x - \frac{x^3}{9}$			
	When $x = 1$, $y = -\frac{1}{9}$	B1F	3	ft on one slip but must have made a realistic attempt to find A
	Total		**10**	

Q	Solution	Marks	Total	Comments
5(a)	The interval of integration is infinite	E1	1	OE
(b)	$u = x^2 \text{ e}^{-4x} + 3 \Rightarrow \text{d}u = (2x\text{e}^{-4x} - 4x^2\text{e}^{-4x})\text{ d}x$	M1		du/dx or 'better'
	$\int \frac{x(1-2x)}{x^2+3\text{e}^{4x}}\text{ d}x = \int \frac{1}{2} \times \frac{2x(1-2x)\text{e}^{-4x}}{x^2\text{e}^{-4x}+3}\text{ d}x$			
	$= \frac{1}{2} \times \int \frac{1}{u}\text{ d}u$	A1		
	$= \frac{1}{2}\ln u + c = \frac{1}{2}\ln\left(x^2\text{e}^{-4x}+3\right) \{+c\}$	A1	3	OE Condone missing c. Accept later substitution back if explicit
(c)	$\text{I} = \int_{\frac{1}{2}}^{\infty} \frac{x(1-2x)}{x^2+3\text{e}^{4x}}\text{ d}x$			
	$= \lim_{a\to\infty} \int_{\frac{1}{2}}^{a} \frac{x(1-2x)}{x^2+3\text{e}^{4x}}\text{ d}x$	M1		
	$= \lim_{a\to\infty} \frac{1}{2}\{\ln\left(a^2\text{e}^{-4a}+3\right) - \ln(\frac{\text{e}^{-2}}{4}+3)\}$	M1		Uses part (b) and F(a) – F(1/2)
	$= \frac{1}{2}\ln\{\lim_{a\to\infty}\left(a^2\text{e}^{-4a}+3\right)\} - \frac{1}{2}\ln(\frac{\text{e}^{-2}}{4}+3)$			
	Now $\lim_{a\to\infty}\left(a^2\text{e}^{-4a}\right) = 0$	E1		Stated explicitly (could be in general form)
	$\text{I} = \frac{1}{2}\ln 3 - \frac{1}{2}\ln(\frac{\text{e}^{-2}}{4}+3)$	A1	4	CSO ACF
	Total		**8**	

Q	Solution	Marks	Total	Comments
6(a)	$y = \ln\cos 2x \Rightarrow y'(x) = \dfrac{1}{\cos 2x}(-2\sin 2x)$	M1 A1		Chain rule
	$y''(x) = -4\sec^2 2x$	m1		$\lambda\sec^2 2x$ OE
	$y'''(x) = -8\sec 2x\,(2\sec 2x\tan 2x)$	M1		$K\sec^2 2x\tan 2x$ OE
	$\{y'''(x) = -16\tan 2x\,(\sec^2 2x)\}$			
	$y''''(x) = -16[2\sec^2 2x(\sec^2 2x) +$ $\tan 2x(2\sec 2x\,(2\sec 2x\tan 2x))]$	M1 A1	6	Product rule OE ACF
(b)	$y(0) = 0, y'(0) = 0, y''(0) = -4, y'''(0) = 0,$ $y''''(0) = -32$	B1F		ft c's derivatives
	$\ln\cos 2x \approx 0 + 0 + \dfrac{x^2}{2}(-4) + 0 + \dfrac{x^4}{4!}(-32)$	M1		
	$\approx -2x^2 - \dfrac{4}{3}x^4$	A1	3	CSO throughout parts (a) and (b) AG
(c)	$\ln(\sec^2 2x) = -2\ln(\cos 2x)$	M1		PI
	$\approx 4x^2 + \dfrac{8}{3}x^4$	A1	2	
	Total		**11**	

Q	Solution	Marks	Total	Comments
7(a)	$u = xy$			
	$\frac{du}{dx} = y + x\frac{dy}{dx}$	M1 A1		Product rule OE OE
	$\frac{d^2u}{dx^2} = \frac{dy}{dx} + (\frac{dy}{dx} + x\frac{d^2y}{dx^2})$	A1		OE
	$x\frac{d^2y}{dx^2} + 2(3x+1)\frac{dy}{dx} + 3y(3x+2) = 18x$			
	$(x\frac{d^2y}{dx^2} + 2\frac{dy}{dx}) + 6(x\frac{dy}{dx} + y) + 9xy = 18x$			
	$\frac{d^2u}{dx^2} + 6\frac{du}{dx} + 9u = 18x$	A1	4	CSO AG Be convinced
(b)	$\frac{d^2u}{dx^2} + 6\frac{du}{dx} + 9u = 18x$			
	CF: Aux eqn $m^2 + 6m + 9 = 0$	M1		PI
	$(m+3)^2 = 0$ so $m = -3$	A1		PI
	CF: $(u =)\ e^{-3x}(Ax + B)$	A1F		
	PI: Try $(u =)\ px + q$ $0 + 6p + 9(px + q) = 18x$	M1		PI. Must be more than just stated
	$9p = 18, \quad 6p + 9q = 0$	m1		
	$p = 2;\ \ q = -\frac{12}{9}$	A1		Both
	$u = e^{-3x}(Ax + B) + 2x - \frac{4}{3}$	B1F		c's CF + c's PI but must have 2 constants, also must be in the form $u = f(x)$
	$xy = e^{-3x}(Ax + B) + 2x - \frac{4}{3}$			
	$y = \frac{1}{x}\{e^{-3x}(Ax + B) + 2x - \frac{4}{3}\}$	A1	8	
	Total		**12**	

Q	Solution	Marks	Total	Comments
8(a)	Area = $\frac{1}{2}\int(3+2\cos\theta)^2\,\mathrm{d}\theta$	M1		Use of $\frac{1}{2}\int r^2\,\mathrm{d}\theta$ or $\int_0^{\pi} r^2\,\mathrm{d}\theta$
	$= \frac{1}{2}\int_0^{2\pi}(9+12\cos\theta+4\cos^2\theta)\,\mathrm{d}\theta$	B1 B1		Correct expn of $[3+2\cos\theta]^2$ Correct limits
	$= \int_0^{2\pi}(4.5+6\cos\theta+(1+\cos 2\theta))\,\mathrm{d}\theta$	M1		Attempt to write $\cos^2\theta$ in terms of $\cos 2\theta$
	$= \left[4.5\theta+6\sin\theta+\theta+\frac{1}{2}\sin 2\theta\right]_0^{2\pi}$	A1F		Correct integration ft wrong coefficients
	$= 11\pi$	A1	6	CSO
(b)(i)	$x^2+y^2-8x+16=16$	M1		Use of **any two** of $x = r\cos\theta$, $y = r\sin\theta$, $x^2+y^2=r^2$
	$r^2-8r\cos\theta+16=16 \Rightarrow r=8\cos\theta$	A1		
	At intersection, $8\cos\theta = 3+2\cos\theta$ $\Rightarrow \cos\theta = \frac{3}{6}$	M1		Equating rs or equating $\cos\theta$s with a further step to solve eqn. (OE eg $4r = 12 + r \Rightarrow 4r - r = 12$)
	Points $\left(4, \frac{\pi}{3}\right)$ and $\left(4, \frac{5\pi}{3}\right)$	A1		OE
	$AB = 2\times\left(4\sin\frac{\pi}{3}\right)$	M1		Valid method to find AB, ft c's r and θ values
	$= 4\sqrt{3}$	A1	6	OE surd
(ii)	Let M=centre of circle then $\angle AMB = \frac{2\pi}{3}$	B1		Accept equiv eg $\angle AMO = \frac{\pi}{3}$
	Length of arc AOB of circle = $4\times\frac{2\pi}{3}$	M1		Use of arc = 4 × ($\angle AMB$ in rads)
	Perimeter of segment $AOB = \frac{8\pi}{3}+4\sqrt{3}$	A1	3	
	Total		**15**	
	Alternative to (b)(i): Writing $r = 3+2\cos\theta$ in cartesian form (M1A1) Finding cartesian coordinates of points A and B ie $(2, \pm 2\sqrt{2})$ (M1A1) Finding length AB (M1A1)			
	TOTAL		**75**	

Version 1.0

General Certificate of Education (A-level)
June 2012

Mathematics MFP3

(Specification 6360)

Further Pure 3

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√ or ft or F	follow through from previous incorrect result
CAO	correct answer only CSO
	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP3 : June 2012

Q	Solution	Marks	Total	Comments
1	$k_1 = 0.25 \times \left(\sqrt{2 \times 2} + \sqrt{9}\right)$ (=1.25)	M1		PI. May see within given formula
	k_2 = 0.25 f (2.25, 9 +1.25)			Either k_2 =0.25 f (2.25, 10.25) stated/used or
	$k_2 = 0.25 \times \left(\sqrt{2 \times 2.25} + \sqrt{9 + 1.25}\right)$	M1		$k_2 = 0.25 \times \left(\sqrt{2 \times 2.25} + \sqrt{9 + \text{c's } k_1}\right)$ PI. May see within given formula
	k_2 = 1.33(072…)	A1		k_2 = 1.33(072…) 2 dp or better PI by later work
	$y\,(2.25) = y\,(2) + \frac{1}{2}[k_1 + k_2]$			
	= 9 + 0.5 [1.25 + 1.33 (072…)] = 9 + 0.5×2.58 (072…)	m1		Dep on previous two Ms and y (2) = 9 and numerical values for k's
	y (2.25) = 10.29036… = 10.29 (to 2 dp)	A1	5	CAO Must be 10.29
	Total		**5**	
2(a)	$\sin 2x = 2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} \ldots$ $= 2x - \frac{4}{3}x^3 + \frac{4}{15}x^5$	B1	1	Accept ACF even if unsimplified
(b)	$\lim_{x \to 0} \left[\frac{2x - \sin 2x}{x^2 \ln(1 + kx)}\right]$			
	$= \lim_{x \to 0} \frac{2x - (2x - \frac{4}{3}x^3 + \frac{4}{15}x^5 \ldots)}{x^2\left(kx - \frac{(kx)^2}{2} + \ldots\right)}$	M1 B1		Using series expansions. Expansion of ln (1 + kx) = kx (−…)
	$= \lim_{x \to 0} \left[\frac{\frac{4}{3}x^3 - \frac{4}{15}x^5 + ..}{kx^3 - \frac{k^2}{2}x^4}\right]$			
	$= \lim_{x \to 0} \left[\frac{\frac{4}{3} - O(x^2)}{k - O(x)}\right]$	m1		Dividing numerator and0 denominator by x^3 to get constant term in each. Must be at least a total of 3 terms divided by x^3
	$\frac{4}{3k} = 16 \Rightarrow k = \frac{1}{12}$	A1	4	OE exact value. Dep on numerator being of form 4/3(OE) + λx^2... ($\lambda \neq 0$) and denominator being of form $k + \mu x$.. ($\mu \neq 0$) before limit taken
	Total		**5**	

Q	Solution	Marks	Total	Comments
3	Area = $\frac{1}{2}\int \left(2\sqrt{1+\tan\theta}\right)^2\ (\mathrm{d}\theta)$	M1		Use of $\frac{1}{2}\int r^2\ (\mathrm{d}\theta)$
	$= \frac{1}{2}\int_{-\frac{\pi}{4}}^{0} 4(1+\tan\theta)\ \mathrm{d}\theta$	B1		Correct limits. If any contradiction use the limits at the substitution stage
	$= 2[\theta + \ln\sec\theta\ ]_{-\frac{\pi}{4}}^{0}$	B1		$\int k(1+\tan\theta)(\mathrm{d}\theta) = k(\theta + \ln\sec\theta)$ ACF ft on c's k
	$= 2\left\{0 - \left[-\frac{\pi}{4} + \ln\sec\left(-\frac{\pi}{4}\right)\right]\right\}$			
	$= 2\left(\frac{\pi}{4} - \ln\sqrt{2}\right) = \frac{\pi}{2} - 2\ln\sqrt{2} = \frac{\pi}{2} - \ln 2$	A1	4	CSO AG
	Total		**4**	
4(a)	IF is $e^{\int \frac{4}{2x+1}\mathrm{d}x}$	M1		PI
	$e^{2\ln(2x+1)\ (+c)} = e^{\ln(2x+1)^2\ (+c)}$	A1		Either O.E. Condone missing '+ c'
	$= (A)(2x+1)^2$	A1F		Ft on earlier $e^{\lambda \ln(2x+1)}$, condone missing 'A'
	$(2x+1)^2\frac{\mathrm{d}y}{\mathrm{d}x} + 4(2x+1)y = 4(2x+1)^7$			
	$\frac{\mathrm{d}}{\mathrm{d}x}[(2x+1)^2 y] = 4\ (2x+1)^7$	M1		LHS as d/dx (y × c's IF) PI **and** also RHS of form $p\ (2x+1)^q$
	$(2x+1)^2 y = \int 4(2x+1)^7\ \mathrm{d}x$	A1		
	$(2x+1)^2 y = \frac{1}{4}(2x+1)^8\ (+c)$	B1F		Correct integration of $p\ (2x+1)^q$ to $\frac{p(2x+1)^{q+1}}{2(q+1)}$ (+ c) ft for q>2 only
	(GS): $y = \frac{1}{4}(2x+1)^6 + c(2x+1)^{-2}$	A1	7	Must be in the form $y = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is ACF
(b)	$y = \frac{1}{4}(2x+1)^6 + c(2x+1)^{-2}$			
	When $x = 0$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	M1		Using boundary condition $x = 0$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and c's GS in (a) towards obtaining a value for c
	$\Rightarrow y = 1\ \left[\frac{\mathrm{d}y}{\mathrm{d}x} = 3(2x+1)^5 - 4c(2x+1)^{-3}\right]$	B1		Either $y = 1$ or correct expression for dy/dx in terms of x only
	$\Rightarrow c = \frac{3}{4}$ so $y = \frac{1}{4}(2x+1)^6 + \frac{3}{4}(2x+1)^{-2}$	A1	3	CSO
	Total		**10**	

Q	Solution	Marks	Total	Comments
5(a)	$\int x^2 e^{-x} dx = -x^2 e^{-x} - \int -2x e^{-x} dx$	M1 A1		$kx^2 e^{-x} - \int 2kx\, e^{-x}$ (dx) for $k = \pm 1$
	$= -x^2 e^{-x} + 2\{-x e^{-x} - \int - e^{-x} dx\}$	m1		$\int x\, e^{-x} dx = \lambda x e^{-x} - \int \lambda e^{-x}$ (dx) for $\lambda = \pm 1$ in 2nd application of integration by parts
	$= -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} (+c)$	A1	4	Condone absence of $+ c$
(b)	$I = \int_0^\infty x^2 e^{-x}\, dx = \lim_{a \to \infty} \int_0^a x^2 e^{-x}\, dx$ $\lim_{a \to \infty} \{-a^2 e^{-a} - 2a e^{-a} - 2e^{-a}\} - [-2]$	M1		F(a) – F(0) with an indication of limit '$a \to \infty$' and F(x) containing at least one $x^n e^{-x}$, $n > 0$ term
	$\lim_{a \to \infty} a^k e^{-a} = 0$, (k>0)	E1		For general statement or specific statement for either $k = 1$ or $k = 2$
	$\int_0^\infty x^2 e^{-x}\, dx = 2$	A1	2	CSO
	Total			
6(a)	$y = \ln(1 + \sin x)$, $\dfrac{dy}{dx} = \dfrac{1}{1 + \sin x} \times (\cos x)$	M1 A1	2	Chain rule OE ACF eg $e^{-y} \cos x$
(b)	$\left(\dfrac{d^2 y}{dx^2} =\right) \dfrac{(1 + \sin x)(-\sin x) - \cos x(\cos x)}{(1 + \sin x)^2}$	M1 A1		Quotient rule OE, with u and v non constant ACF
	$\dfrac{d^2 y}{dx^2} = \dfrac{-\sin x - 1}{(1 + \sin x)^2} = \dfrac{-1}{1 + \sin x} = \dfrac{-1}{e^y} = -e^{-y}$	A1	3	CSO AG Completion must be convincing
(c)	$\dfrac{d^3 y}{dx^3} = e^{-y} \dfrac{dy}{dx}$	B1		ACF for $\dfrac{d^3 y}{dx^3}$
	$\dfrac{d^4 y}{dx^4} = -e^{-y} \left(\dfrac{dy}{dx}\right)^2 + e^{-y} \dfrac{d^2 y}{dx^2}$	M1		Product rule OE and chain rule
	$\dfrac{d^4 y}{dx^4} = -e^{-y} \left(\dfrac{dy}{dx}\right)^2 - (e^{-y})^2$	A1	3	OE in terms of e^{-y} and $\dfrac{dy}{dx}$ only
(d)	$y(0) = 0$; $y'(0) = 1$; $y''(0) = -1$;	B1F		Ft only for $y'(0)$; other two values must be correct
	$y(x) \approx$ $y(0) + x y'(0) + \dfrac{x^2}{2} y''(0) + \dfrac{x^3}{3!} y'''(0) + \dfrac{x^4}{4!} y^{(iv)}(0)$ $y'''(0) = 1$; $y^{(iv)}(0) = -2$	M1		Maclaurin's theorem applied with numerical values for $y'(0)$, $y''(0)$, $y'''(0)$ and $y^{(iv)}(0)$. M0 if missing an expression for any one of the 1st, 3rd or 4th derivatives
	$\ln(1 + \sin x) \approx x - \dfrac{1}{2} x^2 + \dfrac{1}{6} x^3 - \dfrac{1}{12} x^4 \ldots$	A1	3	A0 if FIW
	Total		**11**	

Q	Solution	Marks	Total	Comments
7(a)	$\frac{dx}{dt}\frac{dy}{dx}=\frac{dy}{dt}$	M1		OE Relevant chain rule eg $\frac{dy}{dx}=\frac{dt}{dx}\frac{dy}{dt}$
	$e^t\frac{dy}{dx}=\frac{dy}{dt}\Rightarrow x\frac{dy}{dx}=\frac{dy}{dt}$	A1		OE eg $\frac{dy}{dx}=e^{-t}\frac{dy}{dt}$
	$\frac{d}{dt}\left(x\frac{dy}{dx}\right)=\frac{d^2y}{dt^2}$; $\frac{dx}{dt}\frac{d}{dx}\left(x\frac{dy}{dx}\right)=\frac{d^2y}{dt^2}$	M1		OE. Valid 1st stage to differentiate $x\,y'(x)$ oe with respect to t or to differentiate $x^{-1}y'(t)$ oe with respect to x.
	$\frac{dx}{dt}\left(\frac{dy}{dx}+x\frac{d^2y}{dx^2}\right)=\frac{d^2y}{dt^2}$	m1		Product rule (dep on previous M)
	$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}=\frac{d^2y}{dt^2}$	A1		OE eg $\frac{d^2y}{dx^2}=e^{-t}\left[-e^{-t}\frac{dy}{dt}+e^{-t}\frac{d^2y}{dx^2}\right]$ {Note: e^{-t} could be replaced by $1/x$}
	$x^2\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+6y=3+20\sin(\ln x)$ becomes $\frac{d^2y}{dt^2}-x\frac{dy}{dx}-4x\frac{dy}{dx}+6y=3+20\sin(\ln x)$			
	$\Rightarrow\frac{d^2y}{dt^2}-5\frac{dy}{dt}+6y=3+20\sin(\ln e^t)$	m1		Substitution to reach a 'one-step away' stage for LHS. Dep on previous M M m
	$\Rightarrow\frac{d^2y}{dt^2}-5\frac{dy}{dt}+6y=3+20\sin t$	A1	7	CSO AG
(b)	Auxl eqn $m^2-5m+6=0$	M1		PI
	$(m-2)(m-3)=0$, $m=2, 3$	A1		
	CF: $(y_C=)\ Ae^{2t}+Be^{3t}$	A1F		Ft wrong values of m provided 2 real roots, and 2 arb. constants in CF. Condone x for t here
	P.Int. Try $(y_P=)\ a+b\sin t+c\cos t$	M1		Condone 'a' missing here
	$(y'(t)=)\ b\cos t-c\sin t$	A1		
	$(y''(t)=)\ -b\sin t-c\cos t$	A1F		ft can be consistent sign error(s)
	Substitute into DE gives	M1		Substitution and comparing coefficients at least once
	$a=0.5$	B1		
	$5c+5b=20$ and $5c-5b=0$	A1		OE
	$b=c=2$	A1		
	GS $(y=)\ Ae^{2t}+Be^{3t}+2\sin t+2\cos t+\frac{1}{2}$	B1F	11	Ft on c's CF + PI, provided PI is non-zero and CF has two arbitrary constants and RHS is fn of t only
(c)	$y=Ax^2+Bx^3+2\sin(\ln x)+2\cos(\ln x)+0.5$	B1	1	CAO
	Total		**19**	

Q	Solution	Marks	Total	Comments
8(a)	$xy = 8 \Rightarrow\ r\cos\theta\ r\sin\theta\ = 8$	M1		
	$\frac{1}{2}r^2\sin 2\theta = 8$	m1		Use of $\sin 2\theta = 2\sin\theta\cos\theta$
	$r^2 = \frac{16}{\sin 2\theta} = 16\,\text{cosec}\,2\theta$	A1	3	AG Completion
(b)(i)	(At N, r is a minimum $\Rightarrow \sin 2\theta = 1$) $N\left(4, \frac{\pi}{4}\right)$	B1B1	2	B1 for each correct coordinate.
(ii)	At pts of intersection, $\left(4\sqrt{2}\right)^2 = 16\,\text{cosec}\,2\theta$	M1		
	$\sin 2\theta = \frac{1}{2}$	A1		PI by $\text{cosec}\,2\theta = 2$ and a correct exact or 3SF value for 2θ or θ
	$2\theta = \frac{\pi}{6}, \ \frac{5\pi}{6}$	A1		PI OE exact values
	$\left(4\sqrt{2}, \ \frac{\pi}{12}\right)\ \left(4\sqrt{2}, \ \frac{5\pi}{12}\right)$	A1	4	Both required, written in correct order
(iii)	$\angle POQ = \frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3}$ **or** $\angle PON = \frac{\pi}{6} (= \angle QON)$	B1F		Ft on c's $\theta_P, \theta_Q, \theta_N$ as appropriate OE
	$PN^2 = \left(4\sqrt{2}\right)^2 + (r_N)^2 - 2\left(4\sqrt{2}\right) r_N \cos\left(\frac{1}{2}POQ\right)$ **or** $PT = 4\sqrt{2}\sin\left(\frac{1}{2}POQ\right)$ **or** $PT = \frac{1}{2}\times 4\sqrt{2}$ **or** $NT = 4\sqrt{2}\cos\left(\frac{1}{2}POQ\right) - r_N$	M1		Finding the lengths of **two** unequal sides of ΔPNQ or ΔPNT or ΔQNT, where T is the point at which ON produced meets PQ. Any valid equivalent methods eg finding $\tan\angle OPN$ or finding $\sin\angle ONP$.
	$PN = \sqrt{\left(48 - 16\sqrt{6}\right)}$ [=2.96(7855…)] = NQ **or** $PT = 2\sqrt{2}$ [=2.82(8427…)] **or** $PQ = 4\sqrt{2}$ **or** $NT = 2\sqrt{6} - 4$ [=0.898(979…)]	A1		Two correct unequal lengths of sides of ΔPNQ or ΔPNT or ΔQNT PI OE eg $\tan\angle OPN = 1/\left(2\sqrt{2} - \sqrt{3}\right)$ or $\sin\angle ONP = 2\sqrt{2}/(\sqrt{48 - 16\sqrt{6}})$
	$\tan\frac{\alpha}{2} = \frac{PT}{NT} = \frac{2\sqrt{2}}{2\sqrt{6} - 4}$ [=3.14626…] OE **or** $\frac{\alpha}{2} = \frac{\pi}{2} - \left[\frac{\pi}{3} - \tan^{-1}\left(\frac{1}{2\sqrt{2} - \sqrt{3}}\right)\right]$ **or** $32 = 2PN^2(1 - \cos\alpha) \Rightarrow 1 - \cos\alpha = \frac{1}{3 - \sqrt{6}}$	m1		Valid method to reach an eqn in α (or in $\frac{\alpha}{2}$) only; dep on prev M but not on prev A. Alternative choosing eg obtuse ONP then $\frac{\alpha}{2} = \pi - 1.87(85...)$
	$\frac{\alpha}{2} = 1.263056...$; $\alpha = 2.5261...2.53$ to 3sf	A1	5	2.53… Condone >3sf.
	Total		**14**	
	TOTAL		**75**	

Version

General Certificate of Education (A-level)
January 2013

Mathematics **MFP3**

(Specification 6360)

Further Pure 3

Final

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	$y(3.2) = y(3) + 0.2\sqrt{2\times 3+5}$	M1		
	$= 5 + 0.2 \times \sqrt{11}$	A1		
	$= 5.66332\ldots = 5.6633$ to 4dp	A1	3	Condone >4dp
(b)	$y(3.4) = y(3) + 2(0.2)\{\text{f}\,[3.2, y(3.2)]\}$	M1		
	$\ldots = 5 + 2(0.2)\sqrt{2\times 3.2+5.6633...}$	A1F		Ft on cand's answer to **(a)**
	$(= 5 + (0.4)\sqrt{12.0633...}\,)$			
	$= 6.389$ to 3dp	A1	3	CAO Must be 6.389
	Total		**6**	
2				Ignore higher powers beyond x^2 throughout this question
(a)	$e^{3x} = 1 + 3x + 4.5x^2$	B1	1	
(b)	$(1+2x)^{-3/2} = 1 - 3x + \frac{15}{2}x^2$	M1		$(1+2x)^{-3/2} = 1\pm 3x + kx^2$ or $1 + kx \pm 7.5x^2$ OE
		A1		$1-3x+7.5x^2$ OE (simplified PI)
	$e^{3x}(1+2x)^{-3/2} = (1 + 3x + 4.5x^2)(1-3x+7.5x^2)$	M1		Product of c's two expansions with an attempt to multiply out to find x^2 term
	x^2 term(s): $7.5x^2 - 9x^2 + 4.5x^2 = 3x^2$.	A1	4	
	Total		**5**	

Q	Solution	Marks	Total	Comments
3	PI: $y_{PI} = kx^2\mathrm{e}^x$	M1		
	$y'_{PI} = 2kx\mathrm{e}^x + kx^2\mathrm{e}^x$ $y''_{PI} = 2k\mathrm{e}^x + 4kx\mathrm{e}^x + kx^2\mathrm{e}^x$	m1		Product rule used in finding both derivatives
	$2k\mathrm{e}^x + 4kx\mathrm{e}^x + kx^2\mathrm{e}^x - 4kx\mathrm{e}^x - 2kx^2\mathrm{e}^x + kx^2\mathrm{e}^x = 6\mathrm{e}^x$	m1		Subst. into DE
	$2k = 6$; $k = 3$; $y_{PI} = 3x^2\mathrm{e}^x$	A1		CSO
	(GS: $y =$) $\mathrm{e}^x(Ax+B) + 3x^2\mathrm{e}^x$	B1F	5	$\mathrm{e}^x(Ax+B) + kx^2\mathrm{e}^x$, ft c's k.
	Total		**5**	
4(a)	Integrand is not defined at $x = 0$	E1	1	OE
(b)	$\int x^4 \ln x \, \mathrm{d}x = \frac{x^5}{5}\ln x - \int \frac{x^5}{5}\left(\frac{1}{x}\right)\mathrm{d}x$	M1 A1		$... = kx^5 \ln x \pm \int \mathrm{f}(x)$, with f($x$) not involving the 'original' $\ln x$
	$\ldots\ldots = \frac{x^5}{5}\ln x - \frac{x^5}{25}$ $(+c)$	A1		
	$\int_0^1 x^4 \ln x \, \mathrm{d}x = \{\lim_{a \to 0} \int_a^1 x^4 \ln x \, \mathrm{d}x\}$ $= -\frac{1}{25} - \lim_{a \to 0}\left[\frac{a^5}{5}\ln a - \frac{a^5}{25}\right]$	M1		Limit 0 replaced by a limiting process and F(1)−F(a) OE
	But $\lim_{a \to 0} a^5 \ln a = 0$	E1		Accept $\lim_{x \to 0} x^k \ln x = 0$ for any $k>0$
	So $\int_0^1 x^4 \ln x \, \mathrm{d}x = -\frac{1}{25}$	A1	6	Dep on M and A marks all scored
	Total		**7**	

Q	Solution	Marks	Total	Comments
5	$\frac{dy}{dx}+\frac{\sec^2 x}{\tan x}y=\tan x$			
(a)	IF is $\exp\left(\int \frac{\sec^2 x}{\tan x}\,dx\right)$	M1		and with integration attempted
	$=e^{\ln(\tan x)}=\tan x$	A1	2	AG Be convinced
(b)	$\tan x\frac{dy}{dx}+(\sec^2 x)y=\tan^2 x$ $\frac{d}{dx}\left[y\tan x\right]=\tan^2 x$	M1		LHS as differential of $y\times$IF PI
	$y\tan x=\int \tan^2 x\,dx$	A1		
	$\Rightarrow y\tan x=\int(\sec^2 x-1)\,dx$	m1		Using $\tan^2 x=\pm\sec^2 x\pm 1$ PI or other valid methods to integrate $\tan^2 x$
	$y\tan x=\tan x-x$ $(+c)$	A1		Correct integration of $\tan^2 x$; condone absence of $+c$.
	$3\tan\frac{\pi}{4}=\tan\frac{\pi}{4}-\frac{\pi}{4}+c$	m1		Boundary condition used in attempt to find value of c
	$c=2+\frac{\pi}{4}$ so $y\tan x=\tan x-x+2+\frac{\pi}{4}$ $y=1+(2-x+\frac{\pi}{4})\cot x$	A1	6	ACF
	Total		**8**	

Q	Solution	Marks	Total	Comments
6(a)(i)	$y = \ln\left(e^{3x}\cos x\right) = \ln e^{3x} + \ln\cos x = 3x + \ln\cos x$	B1		
	$\frac{dy}{dx} = 3 + \frac{1}{\cos x} \times (-\sin x)$	M1		Chain rule for derivative of ln cos$x$
	$\frac{dy}{dx} = 3 - \tan x$	A1	3	CSO AG
(ii)	$\frac{d^2y}{dx^2} = -\sec^2 x$; $\frac{d^3y}{dx^3} = -2\sec x(\sec x\tan x)$	B1; M1		M1 for d/dx\{ $[f(x)]^2$ \}= $2f(x)f'(x)$
	$\frac{d^4y}{dx^4} = -4\sec x(\sec x\tan x)\tan x - 2\sec^4 x$	A1	3	ACF
(b)	Maclaurin's Thm: $y(0)+x\,y'(0)+\frac{x^2}{2!}\,y''(0)+\frac{x^2}{3!}\,y'''(0)+\frac{x^4}{4!}\,y^{(iv)}(0)$ $y(0) = \ln 1 = 0$; $y'(0) = 3$; $y''(0) = -1$; $y'''(0) = 0$; $y^{(iv)}(0) = -2$	M1		Mac. Thm with attempt to evaluate at least two derivatives at x=0
	$\ln\left(e^{3x}\cos x\right) = 0 + 3x + \frac{-1}{2!}x^2 + \frac{0}{3!}x^3 + \frac{-2}{4!}x^4 \ldots$	A1F		At least 3 of 5 terms correctly obtained. Ft one miscopy in (a)
	$= 3x - \frac{1}{2}x^2 - \frac{1}{12}x^4$	A1	3	CSO AG Be convinced
(c)	$\{\ln(1+px)\} = px - \frac{1}{2}p^2x^2$	B1	1	accept $(px)^2$ for p^2x^2; ignore higher powers;
(d)(i)	$\left[\frac{1}{x^2}\{\ln\left(e^{3x}\cos x\right) - \ln(1+px)\}\right] =$			
	$\left[\frac{1}{x^2}\left\{3x - \frac{1}{2}x^2 - O(x^4) - \left(px - \frac{1}{2}p^2x^2 + O(x^3)\right)\right\}\right]$	M1		Law of logs and expansions used;
	For $\lim_{x \to 0}\left[\frac{1}{x^2}\ln\left(\frac{e^{3x}\cos x}{1+px}\right)\right]$ to exist, $p = 3$	A1		p=3 convincingly found
(ii)	…… $= \lim_{x \to 0}\left[\left(\frac{3-p}{x}\right) - \frac{1}{2} + \frac{p^2}{2} - O(x)\right]$	m1		Divide throughout by x^2 before taking limit. (m1 can be awarded before or after the A1 above)
	Value of limit $= -\frac{1}{2} + \frac{p^2}{2} = 4.$	A1	4	Must be convincingly obtained
	Total		**14**	

Q	Solution	Marks	Total	Comments
7(a)	Solving $\dfrac{d^2y}{dt^2} - 6\dfrac{dy}{dt} + 10y = e^{2t}$ (*)			
	Auxl. Eqn. $m^2 - 6m + 10 = 0$ $(m-3)^2 + 1 = 0$	M1		PI Completing sq or using quadratic formula to find m.
	$m = 3 \pm i$	A1		
	CF (y_{CF} =) $e^{3t}(A\cos t + B\sin t)$	M1		OE Condone x for t here; ft c's 2 **non-real** values for 'm'.
	For PI try (y_{PI} =) ke^{2t}	M1		Condone x for t here
	$4k - 12k + 10k = 1 \Rightarrow k = \dfrac{1}{2}$	A1		
	GS of (*) is (y_{GS} =) $e^{3t}(A\cos t + B\sin t) + \dfrac{1}{2}e^{2t}$	B1F	6	CF +PI with 2 arb. constants and both CF and PI functions of t only
(b)	$\dfrac{dy}{dx} = \dfrac{dt}{dx}\dfrac{dy}{dt}$	M1		OE Chain rule
	$\dfrac{dy}{dx} = 2x\dfrac{dy}{dt}$	A1		OE
	$\dfrac{d^2y}{dx^2} = \dfrac{d}{dx}\left(2x\dfrac{dy}{dt}\right) = (2x)\dfrac{dt}{dx}\dfrac{d}{dt}\left(\dfrac{dy}{dt}\right) + 2\dfrac{dy}{dt}$ $= (2x)(2x)\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt}$	M1		$\dfrac{d}{dx}(\,f(t)\,) = \dfrac{dt}{dx}\dfrac{d}{dt}(\,f(t)\,)$ OE eg $\dfrac{d}{dt}(\,g(x)\,) = \dfrac{dx}{dt}\dfrac{d}{dx}(\,g(x)\,)$
		m1		Product rule OE used dep on previous M1 being awarded at some stage
	$\dfrac{d^2y}{dx^2} = 4t\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt}$	A1	5	CSO A.G.
(c)	$t^{\frac{1}{2}}\left[4t\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt}\right] - (12t+1)2t^{\frac{1}{2}}\dfrac{dy}{dt} + 40t^{\frac{3}{2}}y = 4t^{\frac{3}{2}}e^{2t}$	M1		Subst. using (b) into given DE to eliminate all x
	$4t^{\frac{3}{2}}\{\dfrac{d^2y}{dt^2} - 6\dfrac{dy}{dt} + 10y\} = 4t^{\frac{3}{2}}e^{2t}$			
	$t \neq 0$ so divide by $4t^{\frac{3}{2}}$ gives $\dfrac{d^2y}{dt^2} - 6\dfrac{dy}{dt} + 10y = e^{2t}$ (*)	A1	2	CSO A.G.
(d)	$y = e^{3x^2}(A\cos x^2 + B\sin x^2) + \dfrac{1}{2}e^{2x^2}$	B1	1	OE Must include y=
	Total		**14**	

Q	Solution	Marks	Total	Comments
8(a)(i)	$r = \sin\frac{2\pi}{3}\sqrt{(2+\frac{1}{2}\cos\frac{\pi}{3})} = \frac{\sqrt{3}}{2}\times\sqrt{\frac{9}{4}} = \frac{3\sqrt{3}}{4}$	M1; A1	2	
(ii)	$x = ON = (3\sqrt{3})/8$			
	Polar eqn of PN is $r\cos\theta = ON$	M1		
	$r = \frac{3\sqrt{3}}{8}\sec\theta$	A1	2	AG Be convinced
(iii)	Area $\Delta\, ONP = 0.5 \times r_N \times r_P \times \sin(\pi/3)$	M1		OE With correct or ft from (a)(i) (ii), values for r_P and r_N .
	$= \frac{1}{2}\times\frac{3\sqrt{3}}{8}\times\frac{3\sqrt{3}}{4}\times\frac{\sqrt{3}}{2} = \frac{27\sqrt{3}}{128}$	A1	2	Be convinced
(b)(i)	$\int \sin^n\theta\cos\theta\,\mathrm{d}\theta = \int u^n\,\mathrm{d}u$	M1		PI
	$= \frac{\sin^{n+1}\theta}{n+1}$ $(+c)$	A1	2	
(ii)	Area of shaded region bounded by line OP			
	and arc $OP = \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin^2 2\theta\left(2+\frac{1}{2}\cos\theta\right)\mathrm{d}\theta$	M1		Use of $\frac{1}{2}\int r^2\,\mathrm{d}\theta$
		B1		Correct limits
	$\frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}(1-\cos 4\theta)\,\mathrm{d}\theta + \frac{1}{4}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 4\sin^2\theta\cos^2\theta\cos\theta\,\mathrm{d}\theta$	M1		$2\sin^2 2\theta = \pm 1 \pm \cos 4\theta$
		B1		$\sin^2 2\theta\ \cos\theta = 4\sin^2\theta\cos^2\theta\ \cos\theta$
	$= \left[\frac{\theta}{2} - \frac{\sin 4\theta}{8}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}}(\sin^2\theta - \sin^4\theta)\cos\theta\,\mathrm{d}\theta$	A1		Correct integration of $0.5(1-\cos 4\theta)$
		m1		Writing 2nd integrand in a suitable form to be able to use (b)(i) OE PI
	$= \left[\frac{\theta}{2} - \frac{\sin 4\theta}{8} + \frac{\sin^3\theta}{3} - \frac{\sin^5\theta}{5}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$	A1		Last two terms OE
	$= \frac{\pi}{12} - \frac{21\sqrt{3}}{160} + \frac{2}{15}$	A1	8	CSO
	Total		**16**	
	TOTAL		**75**	

Version 1.0

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Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1	$k_1 = 0.2\times(2-1)\sqrt{2+1}$ $(= 0.2\sqrt{3})$	M1		PI. May be seen within given formula.
	$= 0.346(410\ldots)$ $(= *)$			Accept 3dp or better as evidence of the M1 line.
	$k_2 = 0.2\times \mathrm{f}(2.2, 1+*\ldots)$			
	$= 0.2\times(2.2-1.346\ldots)\sqrt{2.2+1.346\ldots}$	M1		$0.2\times(2.2-1-\text{c's } k_1)\sqrt{(2.2+1+\text{c's } k_1)}$ PI May be seen within given formula.
	$\ldots. = 0.321(4946\ldots.)$	A1		3dp or better. PI by later work
	$y(2.2) = y(2) + \frac{1}{2}[k_1 + k_2]$			
	$= 1 + 0.5\times[0.3464\ldots + 0.3214\ldots]$	m1		Dep on previous two Ms but ft on c's numerical values for k_1 and k_2 following evaluation of these.
	$= 1 + 0.5\times0.667904\ldots.$			
	$(= 1.33395\ldots) = 1.334$ to 3dp	A1	5	CAO Must be 1.334 **SC** Any <u>consistent</u> use of a MR/MC of printed f(x,y) expression in applying IEF, mark as SC2 for a correct ft final 3dp value otherwise SC0.
	Total		**5**	
2	$(x+8)^2 + (y-6)^2 = 100$			
	$x^2 + y^2 + 16x - 12y + 64 + 36$ $(= 100)$	B1		OE If polar form before expn of brackets award the B1 for correct expansions of both $(r\cos\theta - m)^2$ and $(r\sin\theta - n)^2$ where $(m,n) = (-8, 6)$ or $(m,n) = (6, -8)$
	$r^2 + 16r\cos\theta - 12r\sin\theta = 0$	M1M1		1st M1 for replacement using any one of $\{[x^2 + y^2 = r^2,\ x = r\cos\theta,\ y = r\sin\theta](*)\}$ 2nd M1 for use of (*) to convert the form $x^2+y^2+ax+by=0$ correctly to the form $r^2+ar\cos\theta+br\sin\theta=0$ or better
	{r=0, origin} Circle: $r = 12\sin\theta - 16\cos\theta$	A1	4	
	Total		**4**	

Q	Solution	Marks	Total	Comments
3(a)	$\frac{d^2y}{dx^2}+2\frac{dy}{dx}-3y=3x-8e^{-3x}$			
	P. Integral : $y_{PI}=a+bx+cxe^{-3x}$			
	$y'_{PI}=b+ce^{-3x}-3cxe^{-3x}$	M1		Product rule used at least once giving terms in the form $\pm pe^{-3x}\pm qxe^{-3x}$
	$y''_{PI}=-6ce^{-3x}+9cxe^{-3x}$			
	$-6ce^{-3x}+9cxe^{-3x}+2b+2ce^{-3x}-6cxe^{-3x}$ $-3a-3bx-3cxe^{-3x}=3x-8e^{-3x}$	M1		Substitution into LHS of DE
	$-3b=3$; $2b-3a=0$; $-4c=-8$	m1		Dep on 2nd M only Equating coeffs to obtain at least two of these correct eqns; PI by correct values for at least two constants
	$b=-1$; $c=2$; $a=-\frac{2}{3}$	A2,1,0	5	Dep on M1M1m1 all awarded A1 if any two correct; A2 if all three correct but do not award the 2nd A mark if terms in xe^{-3x} were incorrect in the M1 line
	$[y_{PI}=-\frac{2}{3}-x+2xe^{-3x}]$			
(b)	Aux. eqn. $m^2+2m-3=0$			
	$(m+3)(m-1)=0$	M1		Factorising or using quadratic formula OE PI by correct two values of 'm' seen/used
	$(y_{CF}=)Ae^{-3x}+Be^{x}$	A1		
	$(y_{GS}=)Ae^{-3x}+Be^{x}-\frac{2}{3}-x+2xe^{-3x}$	B1F	3	c's CF + c's PI with 2 arbitrary constants, non-zero values for a,b and c and no trig or ln terms in c's CF
(c)	$x=0,\ y=1\ \Rightarrow 1=A+B-\frac{2}{3}$	B1F		Only ft if previous B1F has been awarded
	$\frac{dy}{dx}=-3Ae^{-3x}+Be^{x}-1+2e^{-3x}-6xe^{-3x}$			
	As $x\to\infty$, ($e^{-3x}\to 0$ and) $xe^{-3x}\to 0$	E1		Must treat xe^{-3x} separately
	(As $x\to\infty$, $\frac{dy}{dx}\to -1$ so) $B=0$	B1		B=0, where B is the coefficient of e^{x}.
	When $B=0$, $1=A-\frac{2}{3}\Rightarrow A=\frac{5}{3}$			
	$y=\frac{5}{3}e^{-3x}-\frac{2}{3}-x+2xe^{-3x}$	A1	4	
	Total		**12**	

Q	Solution	Marks	Total	Comments
4	$\int \left(\frac{2x}{x^2+4} - \frac{4}{2x+3}\right) dx = \ln(x^2+4)$	B1		
	$-2\ln(2x+3)$ $\{+c\}$	B1		OE
	(I =) $\lim_{a\to\infty} \int_0^a \left(\frac{2x}{x^2+4} - \frac{4}{2x+3}\right) dx$	M1		∞ replaced by a (OE) and $\lim_{a\to\infty}$ seen or taken at any stage
	$= \lim_{a\to\infty} \left[\ln(x^2+4) - 2\ln(2x+3)\right]_0^a$			**Remaining marks are dep** on getting $p\ln(x^2+4)+q\ln(2x+3)$ after integration, where p and q are non-zero constants
	$= \lim_{a\to\infty} \left[\ln(a^2+4) - 2\ln(2a+3)\right] - (\ln 4 - 2\ln 3)$ $= \lim_{a\to\infty} \left[\ln\left(\frac{a^2+4}{(2a+3)^2}\right)\right] - (\ln 4 - \ln 9)$	M1		Dealing with the 0 limit correctly and using $\ln P - \ln Q = \ln(P/Q)$ at least once at any stage either before or after using F( )−F(0). OE
	$= \lim_{a\to\infty} \left[\ln\left(\frac{1+\frac{4}{a^2}}{4+\frac{12}{a}+\frac{9}{a^2}}\right)\right] - (\ln 4 - \ln 9)$	M1		Writing F(a) OE in a suitable form when considering $a\to\infty$. OE
	$I = \int_0^\infty \left(\frac{2x}{x^2+4} - \frac{4}{2x+3}\right) dx = \ln\frac{1}{4} - \ln\frac{4}{9} = \ln\frac{9}{16}$	A1	6	CSO
	Total		**6**	

Q	Solution	Marks	Total	Comments
5(a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln(\ln x)\right] = \frac{1}{\ln x}\times\frac{1}{x}$	B1	1	ACF
(b)(i)	$\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{1}{x\ln x}y=9x^2$			
	An IF is exp $\{\int[1/(x\ln x)]\ (\mathrm{d}x)\}$	M1		…. and with integration attempted
	$= \mathrm{e}^{\ln(\ln x)} = \ln x$	A1	2	AG Must see $\mathrm{e}^{\ln(\ln x)}$ before $\ln x$
(ii)	$\ln x\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{1}{x}y=9x^2\ln x$			
	$\frac{\mathrm{d}}{\mathrm{d}x}\left[y\ln x\right] = 9x^2\ \ln x$	M1		LHS as differential of $y\times\ln x$ PI
	$y\ \ln x=\int 9x^2\ln x\,\mathrm{d}x$	A1		
	$\Rightarrow y\ \ln x=\int \ln x\ \ \mathrm{d}\,[3x^3]$			
	$=3x^3\ln x-\int 3x^3\left(\frac{1}{x}\right)\mathrm{d}x$	m1		$\int kx^2\ln x\,(\mathrm{d}x) = px^3\ln x-\int px^3\left(\frac{1}{x}\right)(\mathrm{d}x)$ or better
	$y\ \ln x = 3x^3\ln x - x^3\ (+c)$	A1		ACF Condone missing '$+c$'
	When $x=\mathrm{e}$, $y=4\mathrm{e}^3$, $4\mathrm{e}^3=3\mathrm{e}^3-\mathrm{e}^3+c$	m1		Dep on previous M1m1. Boundary condition used in attempt to find value of 'c' after integration is completed
	$c=2\mathrm{e}^3$			
	$\Rightarrow\ y\ \ln x=3x^3\ln x-x^3+2\mathrm{e}^3$ $y=3x^3-\frac{(x^3-2\mathrm{e}^3)}{\ln x}$	A1	6	ACF
	Total		**9**	

Q	Solution	Marks	Total	Comments
6(a)	$y=(4+\sin x)^{1/2}$ so $y^2 = 4+\sin x$			
	$2y\frac{dy}{dx}=\cos x$	M1		$\frac{d}{dx}(y^2)=2y\frac{dy}{dx}$
	$y\frac{dy}{dx}=\frac{1}{2}\cos x$	A1	2	
(a)	**Altn** $\frac{dy}{dx}=\frac{1}{2}(4+\sin x)^{-1/2}(\cos x)$	(M1)		Chain rule
	$y\frac{dy}{dx}=\frac{1}{2}\cos x$	(A1)	(2)	
(b)	$y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=-\frac{1}{2}\sin x$	M1		Correct differentiation of $y\frac{dy}{dx}$
	When $x=0$, $y=2$, $\frac{dy}{dx}=\frac{1}{4}$, $2\frac{d^2y}{dx^2}+\left(\frac{1}{4}\right)^2=0$	A1F		Ft on RHS of M1 line as $k\sin x$
	$y\frac{d^3y}{dx^3}+\frac{dy}{dx}\frac{d^2y}{dx^2}+2\frac{dy}{dx}\frac{d^2y}{dx^2}=-\frac{1}{2}\cos x$	m1 A1		Correct LHS
	When x=0, $2\frac{d^3y}{dx^3}+3\left(\frac{1}{4}\right)\left(-\frac{1}{32}\right)=-\frac{1}{2}\Rightarrow\frac{d^3y}{dx^3}=-\frac{61}{256}$	A1	5	CSO
(b)	**Altn** $\frac{d^2y}{dx^2}=-\frac{1}{4}(4+\sin x)^{-3/2}(\cos^2 x)+\frac{1}{2}(4+\sin x)^{-1/2}(-\sin x)$	(M1)		Sign and numerical coeffs errors only.
		(A1)		ACF
	$\frac{d^3y}{dx^3}=\frac{3}{8}(4+\sin x)^{-2.5}(\cos^3 x)-\frac{1}{4}(4+\sin x)^{-1.5}(-2\cos x\sin x)$	(m1)		Sign and numerical coeffs errors only.
	$-\frac{1}{4}(4+\sin x)^{-1.5}(\cos x)(-\sin x)-\frac{1}{2}(4+\sin x)^{-0.5}\cos x$	(A1)		ACF
	When $x=0$, $\frac{d^3y}{dx^3}=\frac{3}{8}\times\frac{1}{32}-\frac{1}{2}\times\left(\frac{1}{2}\right)=-\frac{61}{256}$	(A1)	(5)	CSO
(c)	McC. Thm: $y(0)+x\,y'(0)+\frac{x^2}{2}\,y''(0)+\frac{x^3}{3!}\,y'''(0)$	M1		Maclaurin's theorem used with c's numerical values for $y(0)$, $y'(0)$, $y''(0)$ and $y'''(0)$, all found with at least three being non-zero.
	$(4+\sin x)^{1/2}\approx 2+\frac{1}{4}x-\frac{1}{64}x^2-\frac{61}{1536}x^3$ …..	A1	2	CSO Previous 6 marks must have been scored
	Total		**9**	

Q	Solution	Marks	Total	Comments
7(a)	$\sin^2 x\frac{d^2y}{dx^2}-2\sin x\cos x\frac{dy}{dx}+2y=2\sin^4 x\cos x$			
	$y=u\sin x$			
	$\frac{dy}{dx}=\frac{du}{dx}\sin x+u\cos x$	M1		Both derivatives attempted and product rule used at least twice.
	$\frac{d^2y}{dx^2}=\frac{d^2u}{dx^2}\sin x+\frac{du}{dx}\cos x+\frac{du}{dx}\cos x-u\sin x$	A1		Both correct
	$\frac{d^2u}{dx^2}\sin^3 x+2\frac{du}{dx}\cos x\sin^2 x-u\sin^3 x-2\frac{du}{dx}\sin^2 x\cos x$ $-2u\sin x\cos^2 x+2u\sin x=2\sin^4 x\cos x$	m1		Substitution into original DE
	$\frac{d^2u}{dx^2}\sin^3 x+u\sin x\,[-\sin^2 x-2\cos^2 x+2]=2\sin^4 x\cos x$			
	$\frac{d^2u}{dx^2}\sin^3 x+u\sin x\,[-\sin^2 x+2\sin^2 x]=2\sin^4 x\cos x$	A1		Need to see clear use of the trig identity
	(Divide throughout by $\sin^3 x$,) $\frac{d^2u}{dx^2}+u=2\sin x\cos x$			
	$\Rightarrow\ \frac{d^2u}{dx^2}+u=\sin 2x$	A1	5	AG Completion, be convinced
(b)	For $\frac{d^2u}{dx^2}+u=\sin 2x$, aux eqn, $m^2+1=0\Rightarrow m=\pm i$	M1		PI
	CF: $(u=)\ A\sin x+B\cos x$	A1		OE
	For PI try $(u=)\ p\sin 2x$	M1		Condone extra terms provided their coefficients are shown to be zero
	$-4p\sin 2x+p\sin 2x=\sin 2x\ \Rightarrow\ p=-\frac{1}{3}$	A1		Correct Particular integral
	GS for $u=A\sin x+B\cos x-\frac{1}{3}\sin 2x$	B1F		u=g(x), where g(x)= c's (CF+PI) with two arb. constants, PI≠0 and all real. Can be implied by next line.
	GS: $y=A\sin^2 x+B\sin x\cos x-\frac{1}{3}\sin 2x\sin x$	A1	6	y=f(x) with ACF for f(x)
	Total		**11**	

Q	Solution	Marks	Total	Comments
8(a)	At intersections of r=2 and $r = 3 + 2\sin\theta$ $2 = 3 + 2\sin\theta$	M1		Elimation of r
	$\sin\theta = -\frac{1}{2}$, $\Rightarrow$ $\theta = \frac{7}{6}\pi$, $\theta = \frac{11}{6}\pi$	A1		Any one correct solution of $\sin\theta = -\frac{1}{2}$
	(P=) $\left(2, \frac{7\pi}{6}\right)$, ($Q$=) $\left(2, \frac{11\pi}{6}\right)$	A1	3	$\left(2, \frac{7\pi}{6}\right)$ and $\left(2, \frac{11\pi}{6}\right)$
(b)(i)	Angle between OA and initial line = $\frac{\pi}{6}$	B1F		If not correct, ft on $\theta_P - \pi$
	When $\theta = \frac{\pi}{6}$, $r = 3 + 2\sin\frac{\pi}{6} = 4$; $A\left(4, \frac{\pi}{6}\right)$	B1	2	
(ii)	$OA = 4$, $OQ = 2$ Angle AOQ = $\pi - (\theta_Q - \theta_P) = \frac{\pi}{3}$	B1F		If not correct, ft on $\pi - (\theta_Q - \theta_P)$. OE eg Cartesian coords of A and Q both attempted and at least one correct ft.
	$AQ^2 = 4^2 + 2^2 - 2(4)(2)\cos AOQ$ (=12)	M1		Valid method to find AQ (or AQ^2). Ft on c's r_A for OA
	$AQ = \sqrt{12}$	A1	3	ACF but must be exact surd form.
(iii)	Since $4^2 = 2^2 + (\sqrt{12})^2$ so 90°	E1		Justifying why (angle OQA=) 90° OE
	angle OQA=90° $\Rightarrow$ AQ is a tangent	E1	2	Must have convincingly shown that OQA = 90°
(c)	Area of minor sector OPQ of circle $= \frac{1}{2}(2)^2[\theta_Q - \theta_P]$	M1		$\frac{1}{2}(2)^2[\theta_Q - \theta_P]$
	$= \frac{4\pi}{3}$	A1		PI by combined $-\frac{7\pi}{3}$ OE term later.
	Area of minor region OPQ of curve = $\frac{1}{2}\int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} (4\sin^2\theta + 12\sin\theta + 9)\ d\theta$	M1		Use of $\frac{1}{2}\int r^2\ d\theta$ or use of $\int_{\theta_P}^{3\pi/2} r^2\ d\theta$ OE
		B1		$r^2 = 4\sin^2\theta + 12\sin\theta + 9$
	$= \frac{1}{2}\int (2 - 2\cos 2\theta + 12\sin\theta + 9)\, d\theta$	M1		Use of $\cos 2\theta = \pm1 \pm 2\sin^2\theta$ with $k\int r^2\ (d\theta)$
	$= \frac{1}{2}[2\theta - \sin 2\theta - 12\cos\theta + 9\theta]$ =	A1F		Ft wrong non zero coefficients, ie for correct integration of $a + b\cos 2\theta + c\sin\theta$
	$\left[\frac{121\pi}{12} + \frac{\sqrt{3}}{4} - \frac{6\sqrt{3}}{2}\right] - \left[\frac{77\pi}{12} - \frac{\sqrt{3}}{4} + \frac{6\sqrt{3}}{2}\right]$ $\{= \frac{11\pi}{3} - \frac{11\sqrt{3}}{2}\}$	A1		OE eg $\left[\frac{33\pi}{2}\right] - \left[\frac{77\pi}{6} - \frac{\sqrt{3}}{2} + 6\sqrt{3}\right]$ eg $\left[\frac{121\pi}{6} + \frac{\sqrt{3}}{2} - 6\sqrt{3}\right] - \left[\frac{33\pi}{2}\right]$
	Area of shaded region = $\frac{4\pi}{3} - \{\frac{11\pi}{3} - \frac{11\sqrt{3}}{2}\}$	M1		$\frac{1}{2}(2)^2[\theta_Q - \theta_P] - \frac{1}{2}\int_{\theta_P}^{\theta_Q} (3 + 2\sin\theta)^2\ d\theta$
	$= \frac{11}{2}\sqrt{3} - \frac{7}{3}\pi = \frac{1}{6}(33\sqrt{3} - 14\pi)$	A1	9	CSO $\frac{1}{6}(33\sqrt{3} - 14\pi)$. (m = 33, n = −14)
	Total		**19**	
	TOTAL		**75**	

A-LEVEL
MATHEMATICS
Further Pure 3 – MFP3
Mark scheme

6360
June 2014

Version/Stage: v1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Mark	Total	Comment
1	**DO NOT ALLOW ANY MISREADS IN THIS QUESTION**			
	$k_1 = 0.4\left[\frac{\ln(6+3)}{\ln 3}\right]$ (=0.8)	M1		PI. May be seen within given formula
	$k_2 = 0.4 \times f(6.4, 3+k_1)$ $= 0.4 \times \frac{\ln(6.4+3.8)}{\ln 3.8}$	M1		$0.4 \times \frac{\ln(6+0.4+3+\text{c's } k_1)}{\ln(3+\text{c's } k_1)}$ PI. May be seen within given formula
	$k_2 = 0.4 \times 1.7396\ldots = 0.6958(459\ldots)$	A1		0.696 or better. PI by later work
	$y(6.4) = y(6) + \frac{1}{2}[k_1 + k_2]$ $= 3 + \frac{1}{2}[0.8 + 0.6958(459...)]$	m1		$3 + \frac{1}{2}[\text{c's } k_1 + \text{c's } k_2]$ but dependent on previous two Ms scored. PI by 3.748 or 3.7479….
	$(= 3.747922975\ldots) = 3.748$ (to 3dp)	A1	5	CAO Must be 3.748
	Total		**5**	

Q	Solution	Mark	Total	Comment
2(a)	$y = a + b\sin 2x + c\cos 2x$ $\frac{dy}{dx} = 2b\cos 2x - 2c\sin 2x$	B1		Correct expression for $\frac{dy}{dx}$
	$2b\cos 2x - 2c\sin 2x + 4(a + b\sin 2x + c\cos 2x)$ $(= 20 - 20\cos 2x)$	M1		Differentiation and substitution into LHS of DE
	$4a = 20;\ 4b - 2c = 0;\ 2b + 4c = -20$	m1		Equating coefficients OE to form 3 equations at least two correct. PI by next line
	$a = 5,\ b = -2,\ c = -4$	A1	4	
(b)	Aux. eqn. $m + 4 = 0$	M1		PI Or solving $y'(x)+4y=0$ as far as $y=Ae^{\pm 4x}$ OE
	$(y_{CF} =)\ Ae^{-4x}$	A1		OE
	$(y_{GS} =)\ Ae^{-4x} + 5 - 2\sin 2x - 4\cos 2x$	B1F		c's CF + c's PI with exactly one arbitrary constant
	When $x=0$, $y=4 \Rightarrow A = 3$ $y = 3e^{-4x} + 5 - 2\sin 2x - 4\cos 2x$	A1	4	$y = 3e^{-4x} + 5 - 2\sin 2x - 4\cos 2x$ ACF
	Total		**8**	

Q	Solution	Mark	Total	Comment
3	$4r - 3x = 4$	M1		$x = r\cos\theta$ used
	$4r = 3x + 4$	A1		$4r = 3x + 4$
	$16r^2 = (3x+4)^2$			
	$16(x^2 + y^2) = (3x+4)^2$	M1		$x^2 + y^2 = r^2$ used
	$y^2 = \dfrac{16 + 24x - 7x^2}{16}$	A1	4	Must be in form $y^2 = f(x)$ but accept ACF for f(x) eg $y^2 = \dfrac{(4+7x)(4-x)}{16}$
	Total		**4**	
	Accept $y^2 = \dfrac{(3x+4)^2 - 16x^2}{16}$ and apply ISW if incorrect simplification after seeing this form.			

Q	Solution	Mark	Total	Comment
4	Aux eqn $m^2 - 2m - 3 = 0$			
	$(m-3)(m+1) = 0$	M1		Correctly factorising or using quadratic formula OE for relevant Aux eqn. PI by correct two values of 'm' seen/used.
	$(y_{CF} =)\ Ae^{-x} + Be^{3x}$	A1		
	Try $(y_{PI} =)\ axe^{-x}$	M1		
	$(y'_{PI} =)\ ae^{-x} - axe^{-x}$ $(y''_{PI} =)\ -2ae^{-x} + axe^{-x}$	M1		Product rule OE used to differentiate xe^{-x} in at least one derivative, giving terms in the form $\pm e^{-x} \pm xe^{-x}$
	$-2ae^{-x} + axe^{-x} - 2(ae^{-x} - axe^{-x}) - 3axe^{-x}$ $(=2e^{-x})$	m1		Subst. into LHS of DE
	$\Rightarrow -4a = 2 \quad \Rightarrow a = -\dfrac{1}{2}$	A1		A0 if terms in xe^{-x} were incorrect in m1 line
	$(y_{GS} =)\ Ae^{-x} + Be^{3x} - \dfrac{1}{2}xe^{-x}$	B1F		$(y_{GS} =)$ c's CF + c's PI, must have exactly two arbitrary constants
	As $x \to \infty$, $xe^{-x} \to 0$ (and $e^{-x} \to 0$)	E1		As $x \to \infty$, $xe^{-x} \to 0$ OE. Must be treating xe^{-x} term separately
	$y \to 0$ so $B=0$	B1		$B = 0$, where B is the coefficient of e^{3x}
	$(y'(x) = -Ae^{-x} - 0.5e^{-x} + 0.5xe^{-x})$ $(y'(0) = -3 \Rightarrow -3 = -A - 0.5 \Rightarrow A = 2.5)$			
	$y = \dfrac{5}{2}e^{-x} - \dfrac{1}{2}xe^{-x}$	B1	10	$y = \dfrac{5}{2}e^{-x} - \dfrac{1}{2}xe^{-x}$ OE
	Total		**10**	

Q	Solution	Mark	Total	Comment
5(a)	$\ldots = x\left(\frac{1}{8}\sin 8x\right) - \int \frac{1}{8}\sin 8x\,(\mathrm{d}x)$	M1		$kx\sin 8x - \int k\sin 8x\,(\mathrm{d}x)$, with $k = 1, -1, 8, -8, 1/8$ or $-1/8$
		A1		$x\left(\frac{1}{8}\sin 8x\right) - \int \frac{1}{8}\sin 8x\,(\mathrm{d}x)$
	$= x\left(\frac{1}{8}\sin 8x\right) + \frac{1}{64}\cos 8x\ (+c)$	A1	3	
(b)	$\left[\frac{1}{x}\sin 2x\right] = \frac{2x + O(x^3)}{x}$	M1		$\sin 2x \approx 2x$ Ignore higher powers of x. PI by answer 2.
	$\ldots = \lim_{x\to 0}\left[2 + O(x^2)\right] = 2$	A1	2	CSO Must see correct intermediate step
(c)	$2\cot 2x$ and $1/x$ are not defined at $x=0$	E1	1	Only need to use one of the two terms. Condone 'Integrand not defined at lower limit' OE
(d)	$\left(\int \left(2\cot 2x - x^{-1} + x\cos 8x\right)\,\mathrm{d}x =\right)$			
	$\ln\sin 2x - \ln x + x\left(\frac{1}{8}\sin 8x\right) + \frac{1}{64}\cos 8x$	B1F		Ft c's answer to part (a) ie $\ln\sin 2x - \ln x$ + c's answer to part (a)
	$\int_0^{\frac{\pi}{4}} (\ldots)\,\mathrm{d}x = \lim_{a\to 0}\int_a^{\frac{\pi}{4}} (\ldots.)\,\mathrm{d}x$	M1		Limit 0 replaced by a (OE) and $\lim_{a\to 0}$ seen or taken at any stage with no remaining lim relating to $\pi/4$.
	$\int_0^{\frac{\pi}{4}} (\ldots)\,\mathrm{d}x = \left[\frac{x\sin 8x}{8} + \frac{\cos 8x}{64}\right]_0^{\pi/4} + \ln 1 - \ln(\pi/4) - \lim_{a\to 0}\left[\ln\left(\frac{\sin 2a}{a}\right)\right]$			$\lim_{a\to 0}\left[\ln\left(\frac{\sin 2a}{a}\right)\right]$
	$= \frac{1}{64} - \frac{1}{64} - \ln\left(\frac{\pi}{4}\right) - \lim_{a\to 0}\left[\ln\left(\frac{\sin 2a}{a}\right)\right]$	M1		$F(\pi/4) - F(0)$, with $\ln[(\sin 2x)/x]$ a term in $F(x)$, and at least all non ln terms evaluated
	$= -\ln\left(\frac{\pi}{4}\right) - \ln 2 = -\ln\left(\frac{\pi}{2}\right)$	A1	4	OE single term in exact form, eg $\ln\left(\frac{2}{\pi}\right)$.
	Total		**10**	
(a)	Example: $u=x$, $v'=\cos 8x$; $u'=1$, $v = \frac{1}{8}\sin 8x$ and $\ldots = uv - \int v\,u'$ all seen and substitution into $uv - \int v\,u'$ with no more than one miscopy, award the M1			

Q	Solution	Mark	Total	Comment
6(a)	IF is $e^{\int -\frac{2x}{x^2+4}\,dx}$	M1		PI With or without the negative sign
	$= e^{-\ln(x^2+4)\ (+c)} = e^{\ln(x^2+4)^{-1}\ (+c)}$	A1		Either O.E. Condone missing '+c'
	$= (A)(x^2+4)^{-1}$	A1F		Ft on earlier $e^{\lambda \ln(x^2+4)}$, condone missing A
	$\frac{1}{(x^2+4)}\frac{du}{dx} - \frac{2x}{(x^2+4)^2}u = 3$			
	$\frac{d}{dx}[(x^2+4)^{-1}u] = 3$	M1		LHS as d/dx(u×c's IF) PI
	$(x^2+4)^{-1}u = 3x\ (+C)$	A1		Condone missing '+C' here.
	(GS): $u = (3x+C)(x^2+4)$	A1	6	Must be in the form u = f(x), where f(x) is ACF
(b)	$u = x^2\frac{dy}{dx}$ so $\frac{du}{dx} = x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx}$	M1		$\frac{du}{dx} = \pm x^2\frac{d^2y}{dx^2} \pm px\frac{dy}{dx}$, $p \neq 0$
		A1		
	$x^2(x^2+4)\frac{d^2y}{dx^2} + 8x\frac{dy}{dx} =$ $= (x^2+4)[\frac{du}{dx} - 2x\frac{dy}{dx}] + 8x\frac{dy}{dx}$ $= (x^2+4)\frac{du}{dx} - 2x^3\frac{dy}{dx}$ $= (x^2+4)\frac{du}{dx} - 2xu$	m1		Substitution into LHS of DE and correct ft simplification as far as no y's present.
	Given DE becomes: $(x^2+4)\frac{du}{dx} - 2xu = 3(x^2+4)^2$ $\Rightarrow \frac{du}{dx} - \frac{2x}{x^2+4}u = 3(x^2+4)$	A1	4	CSO AG
(c)	From (**a**), $u = (3x+C)(x^2+4)$ So $\frac{dy}{dx} = \frac{(3x+C)(x^2+4)}{x^2}$	M1		$\frac{dy}{dx} = \frac{\text{c's f}(x)\text{ answer to part (a)}}{x^2}$ stated or used
	$\frac{dy}{dx} = \frac{12}{x} + \frac{4C}{x^2} + 3x + C$			
	$y = 12\ln x - \frac{4C}{x} + \frac{3x^2}{2} + Cx + D$	A1	2	OE
	Total		**12**	
(b)	Altn: $\frac{d^2y}{dx^2} = \frac{\pm x^2\frac{du}{dx} \pm pxu}{(x^2)^2}$, $p \neq 0$ (M1) $\frac{d^2y}{dx^2} = \frac{x^2\frac{du}{dx} - 2xu}{(x^2)^2}$ (A1)			

Q	Solution	Mark	Total	Comment
7(a)(i)	$y = \ln(\cos x + \sin x)$, $\frac{dy}{dx} = \frac{-\sin x + \cos x}{\cos x + \sin x}$	M1 A1		Chain rule OE (sign errors only) ACF eg $e^y y'(x) = \cos x - \sin x$
	$y'' = \frac{-(\cos x + \sin x)^2 - (-\sin x + \cos x)^2}{(\cos x + \sin x)^2}$	m1		Quotient rule (sign errors only) OE eg $e^y [y']^2 + e^y y'' = \pm\cos x \pm \sin x$
	$= \frac{-2(\cos^2 x + \sin^2 x)}{(\cos x + \sin x)^2} = \frac{-2}{1 + 2\cos x \sin x}$			
	$\frac{d^2 y}{dx^2} = -\frac{2}{1 + \sin 2x}$	A1	4	CSO AG Completion must be convincing
(a)(ii)	$\frac{d^3 y}{dx^3} = 4(1 + \sin 2x)^{-2} \cos 2x$	B1	1	ACF for $\frac{d^3 y}{dx^3}$
(b)(i)	$y(0) = 0$; $y'(0) = 1$; $y''(0) = -2$; $y'''(0) = 4$	B1F		Ft only for $y'(0)$ and $y'''(0)$
	$y(x) \approx y(0) + xy'(0) + \frac{x^2}{2} y''(0) + \frac{x^3}{3!} y'''(0)$	M1		Maclaurin's theorem applied with numerical vals. for $y'(0)$, $y''(0)$ and $y'''(0)$. M0 if cand is missing an expression OE for the 1st or 3rd derivatives
	$y(x) \approx x - \frac{2}{2}x^2 + \frac{4}{6}x^3 = x - x^2 + \frac{2}{3}x^3$	A1	3	CSO AG Dep on all previous 7 marks awarded with no errors seen.
(b)(ii)	$\ln(\cos x - \sin x) \approx -x - x^2 - \frac{2}{3}x^3$	B1	1	$-x - x^2 - \frac{2}{3}x^3$
(c)	$\ln\left(\frac{\cos 2x}{e^{3x-1}}\right) = \ln \cos 2x - (3x - 1)$	B1		
	$\ln(\cos 2x) = \ln[(\cos x + \sin x)(\cos x - \sin x)]$ $= \ln(\cos x + \sin x) + \ln(\cos x - \sin x)$	B1		
	$\ln\left(\frac{\cos 2x}{e^{3x-1}}\right) \approx$			
	$\approx x - x^2 + \frac{2}{3}x^3 - x - x^2 - \frac{2}{3}x^3 - 3x + 1$	M1		
	$\approx 1 - 3x - 2x^2$	A1	4	CSO Must have used 'Hence'.
	Total		**13**	
(a)(i)	For guidance, working towards AG may include $y'' = -1 - [y']^2$			

Q	Solution	Mark	Total	Comment
8(a)	(Area=) $\frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (1-\tan^2\theta)^2 \sec^2\theta\,(\mathrm{d}\theta)$	M1		Use of $\frac{1}{2}\int r^2\,(\mathrm{d}\theta)$ or use of $\int_0^{\frac{\pi}{4}} r^2(\mathrm{d}\theta)$ OE
	(or) $\int_0^{\frac{\pi}{4}} (1-\tan^2\theta)^2 \sec^2\theta\,(\mathrm{d}\theta)$	B1		Correct limits
	Let $u=\tan\theta$ so (Area)= $\int_{(0)}^{(1)} (1-u^2)^2\,\mathrm{d}u$	M1		Valid method to integrate $\tan^n\theta\sec^2\theta$, n=2 or 4, could be by inspection.
	(Area) = $\left[u-\frac{2u^3}{3}+\frac{u^5}{5}\right]_0^1$	A1		Correct integration of $k(1-\tan^2\theta)^2\sec^2\theta$ OE; ignore limits at this stage
	$=\left(1-\frac{2}{3}+\frac{1}{5}\right)\quad(-0)\ =\frac{8}{15}$	A1	5	CSO AG
(b) (i)	$(1-\tan^2\theta)\sec\theta=\frac{1}{2}\sec^3\theta$	M1		Elimination of r or θ. $[r=2(2r)^{\frac{1}{3}}-2r]$
	$1-\tan^2\theta=\frac{1}{2}(1+\tan^2\theta)$	m1		Using $1+\tan^2\theta=\sec^2\theta$ OE to reach a correct equation in one 'unknown'.
	$\tan^2\theta=\frac{1}{3};\quad \theta=\pm\frac{\pi}{6};\quad r=\frac{4}{3\sqrt{3}}$			
	Coordinates $\left(\frac{4}{3\sqrt{3}},\frac{\pi}{6}\right)$ $\left(\frac{4}{3\sqrt{3}},-\frac{\pi}{6}\right)$	A1	3	
(b) (ii)	$\frac{4}{3\sqrt{3}}\sin\alpha=(1)\sin\left(\pi-\frac{\pi}{6}-\alpha\right)$ OE	B1F		OE eg $AP=\sqrt{\frac{7}{27}}$ or eg $\sin\alpha=\sqrt{\frac{27}{28}}$.
	$\frac{4}{3\sqrt{3}}\sin\alpha=\sin\frac{\pi}{6}\cos\alpha+\cos\frac{\pi}{6}\sin\alpha$	B1		Or $\cos\alpha=-\frac{1}{\sqrt{28}}$ $\left(=-\frac{\sqrt{7}}{14}\right)$
	$\tan\alpha=\dfrac{-1/2}{\frac{\sqrt{3}}{2}-\frac{4}{3\sqrt{3}}}$	M1		OE Valid method to reach an exact numerical expression for $\tan\alpha$.
	$\tan\alpha=-3\sqrt{3}\quad(k=-3)$	A1	4	
	<u>Altn for the two B marks</u> $ON=\frac{4}{3\sqrt{3}}\cos\frac{\pi}{6};\ AN=\frac{4}{3\sqrt{3}}\sin\frac{\pi}{6};$ OP=1	(B1F)		OE Any two correct ft . PI eg NP=1/3 (N is foot of perp from A or B to OP)
	$\tan OPA=\frac{2}{\sqrt{3}}$	(B1)		$\tan OPA=\frac{2}{\sqrt{3}}$ OE or $\tan PAN=\frac{\sqrt{3}}{2}$ OE [Then (M1)(A1) as above]
(b)(iii)	Since $\tan\alpha$ is negative, α is obtuse so point A lies inside the circle. (If A was on the circle α would be a right angle.)	E1F	1	Ft c's sign of k.
	Total		**13**	
	TOTAL		**75**	
Altn (a)	Converts to Cartesian eqn. $y^2=x^2(1-x)$ (M1A1); sets up a correct integral with correct limits for the area using the sym of the curve (B1); valid method to integrate $x(1-x)^{\frac{1}{2}}$ (M1); 8/15 obtained convincingly (A1)			
(b)(ii) alt	Altn expressions for M1: $\tan\alpha=-\tan\left(\frac{\pi}{6}+OPA\right)=\dfrac{-\frac{1}{\sqrt{3}}-\frac{2}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}\frac{2}{\sqrt{3}}}$; $\tan\alpha=\tan\left(\frac{\pi}{3}+PAN\right)=\dfrac{\sqrt{3}+\frac{\sqrt{3}}{2}}{1-\sqrt{3}\frac{\sqrt{3}}{2}}$			

A-LEVEL

Mathematics

Further Pure3 – MFP3

Mark scheme

6360

June 2015

Version/Stage: Final Mark Scheme V1

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
	DO NOT ALLOW ANY MISREADS IN THIS QUESTION			
(a)	$y(2.05) = y(2) + 0.05\left(\frac{2+5^2}{2}\right)$	**M1**		
	$= 5 + 0.05 \times 13.5$ $= 5.675$	**A1**	**2**	OE
(b)	$y(2.1) = y(2) + 2 \times 0.05\,\mathrm{f}[2.05, y(2.05)]$	**M1**		
	$= 5 + 2 \times 0.05 \times \left(\frac{2.05 + 5.675^2}{2.05}\right)$	**A1F**		PI Ft on c's **(a)** answer.
	$= 6.67$ to 3 sf	**A1**	**3**	CAO Must be 6.67
	Total		**5**	
	(b) For the PI if line missing, check to see if evaluation matches $5.1 + \frac{2}{41} \times [\text{answer (a)}]^2$ to at least 3sf			

Q2	Solution	Mark	Total	Comment
	I.F. $e^{\int \tan x\, dx}$	**M1**		
	$= e^{\ln \sec x}$	**A1**		OE eg $e^{-\ln \cos x}$
	$= \sec x$	**A1F**		OE Only ft sign error in integrating tan *x*.
	$\sec x \frac{dy}{dx} + \sec x(\tan x)y = \tan^3 x \sec^2 x$			
	$\frac{d}{dx}[y \sec x] = \tan^3 x \sec^2 x$	**M1**		LHS as $\frac{d}{dx}[y \times \text{candidate's IF}]$ PI
	$y \sec x = \int \tan^3 x \sec^2 x\ (dx)$	**A1**		
	$y \sec x = \int t^3\ dt$	**m1**		PI OE eg $y \sec x = \int \left(\frac{1}{u^3} - \frac{1}{u^5}\right) du$, where $u = \cos x$
	$y \sec x = \frac{1}{4}\tan^4 x\ (+c)$	**A1**		
	$2\sec\frac{\pi}{3} = \frac{1}{4}\tan^4\frac{\pi}{3} + c;\ \ 4 = \frac{9}{4} + c$	**m1**		Dep on prev MMm. Correct boundary condition applied to obtain an eqn in *c* with correct exact value for either $\sec\frac{\pi}{3}$ or $\tan^4\frac{\pi}{3}$ used
	$y \sec x = \frac{1}{4}\tan^4 x + \frac{7}{4}$			
	$y = \frac{\cos x}{4}(7 + \tan^4 x)$	**A1**	**9**	ACF
	Total		**9**	
	Condone answer left in a 'correct' form different to *y*= f(*x*), eg $4y\sec x = \tan^4 x + 7$.			

Q3	Solution	Mark	Total	Comment
(a)(i)	$\ln(1+2x)=2x-\frac{(2x)^2}{2}+\frac{(2x)^3}{3}-\frac{(2x)^4}{4}\ldots$ $=2x-2x^2+\frac{8}{3}x^3-4x^4\ldots$	B1	1	ACF Condone correct unsimplified
(a)(ii)	$\ln[(1+2x)(1-2x)]=\ln(1+2x)+\ln(1-2x)$	M1		$\ln(1+2x)+\ln(1-2x)$ PI {or $\ln(1-4x^2)=-4x^2-\frac{(-4x^2)^2}{2}\ldots$} PI
	$=-4x^2-8x^4\ldots\ldots$	A1		CSO Must be simplified
	Expansion valid for $-\frac{1}{2}<x<\frac{1}{2}$	B1	3	Condone $\lvert x\rvert<\frac{1}{2}$
(b)	$x\sqrt{9+x}=3x\left[1+\frac{x}{18}+O(x^2)\right]$	B1		Correct first two terms in expn. of $\sqrt{9+x}$
	$\left[\frac{3x-x\sqrt{9+x}}{\ln[(1+2x)(1-2x)]}\right]=\left[\frac{3x-3x-\frac{3x^2}{18}\ldots}{-4x^2-8x^4\ldots}\right]$	M1		Series expansions used in both numerator and denominator.
	$\lim_{x\to 0}\left[\frac{3x-x\sqrt{9+x}}{\ln[(1+2x)(1-2x)]}\right]$			
	$=\lim_{x\to 0}\left[\frac{-\frac{1}{6}+O(x)}{-4+O(x^2)}\right]$	m1		Dividing numerator and denominator by x^2 to get constant term in each, leading to a finite limit. Must be at least a total of 3 'terms' divided by x^2
	$=\frac{1}{24}$	A1	4	$=\frac{1}{24}$ NOT $\to\frac{1}{24}$
	Total		**8**	

Q4	Solution	Mark	Total	Comment
(a)	The interval of integration is infinite	E1	1	OE
(b)	$\int (x-2)\mathrm{e}^{-2x}\,\mathrm{d}x$			
	$u = x-2,\ \frac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{e}^{-2x},\ \frac{\mathrm{d}u}{\mathrm{d}x} = 1,\ v = -0.5\mathrm{e}^{-2x}$	M1		$\frac{\mathrm{d}u}{\mathrm{d}x} = 1,\ \ v = k\,\mathrm{e}^{-2x}$ with $k = \pm 0.5, \pm 2$
	….. = $-\frac{1}{2}(x-2)\mathrm{e}^{-2x} - \int -\frac{1}{2}\mathrm{e}^{-2x}\,\mathrm{d}x$	A1		$-\frac{1}{2}(x-2)\mathrm{e}^{-2x} - \int -\frac{1}{2}\mathrm{e}^{-2x}\,(\mathrm{d}x)$ OE
	$= -\frac{1}{2}(x-2)\mathrm{e}^{-2x} - \frac{1}{4}\mathrm{e}^{-2x}\ (+c)$	A1		
	$\int_2^{\infty}(x-2)\,\mathrm{e}^{-2x}\,\mathrm{d}x = \lim_{a\to\infty}\int_2^{a}(x-2)\,\mathrm{e}^{-2x}\,\mathrm{d}x$	M1		Evidence of limit ∞ having been replaced by a (OE) at any stage and $\lim_{a\to\infty}$ seen or taken at any stage with no remaining lim relating to 2.
	$\lim_{a\to\infty}\left[-\frac{1}{2}(a-2)\mathrm{e}^{-2a} - \frac{1}{4}\mathrm{e}^{-2a}\right] - \left(-\frac{1}{4}\mathrm{e}^{-4}\right)$			
	Now $\lim_{a\to\infty} a^p\,\mathrm{e}^{-2a} = 0,\ \ (p>0)$	E1		General statement or specific statement with $p = 1$ stated explicitly. Each must include the 2 in the exponential.
	$\int_2^{\infty}(x-2)\mathrm{e}^{-2x}\,\mathrm{d}x = \frac{1}{4}\mathrm{e}^{-4}$	A1	6	No errors seen in $\mathrm{F}(a) - \mathrm{F}(2)$. (M1E0A1 is possible)
	Total		**7**	

Q5	Solution	Mark	Total	Comment
(a)	Aux eqn $m^2+6m+9=0$ $(m+3)^2=0$	M1		Factorising or using quadratic formula OE on correct aux eqn. PI by correct value of 'm' seen/used.
	$(y_{CF}=)\ (Ax+B)\mathrm{e}^{-3x}$	A1		
	Try $(y_{PI}=)\ a\sin 3x+b\cos 3x$ $(y'_{PI}=)\ 3a\cos 3x-3b\sin 3x$ $(y''_{PI}=)\ -9a\sin 3x-9b\cos 3x$	M1		$a\sin 3x+b\cos 3x$ or Altn. $k\cos 3x$
	$-9a\sin 3x-9b\cos 3x+6(3a\cos 3x-3b\sin 3x)$ $+9(a\sin 3x+b\cos 3x)=36\sin 3x$	m1		Substitution into DE, dep on previous M and differentiations being in form $p\cos 3x+q\sin 3x$ or Altn. $-3k\sin 3x$ and $-9k\cos 3x$
	$-18b=36 \qquad 18a=0$	A1		Seen or used
	$y_{PI}=-2\cos 3x$	A1		Correct y_{PI} seen or used
	$(y_{GS}=)\ (Ax+B)\mathrm{e}^{-3x}-2\cos 3x$	B1F	7	$(y_{GS}=)$ c's CF + c's PI, must have exactly two arbitrary constants
(b)(i)	$\mathrm{f}''(0)+6\mathrm{f}'(0)+9\mathrm{f}(0)=36\sin 0$ $\mathrm{f}''(0)+6(0)+9(0)=0 \quad \Rightarrow \mathrm{f}''(0)=0$	E1	1	AG Convincingly shown with no errors.
(b)(ii)	$\mathrm{f}'''(0)=108\cos 0-0-0=108$ $\mathrm{f}^{(iv)}(0)=0-6\,\mathrm{f}'''(0)-0=-648$	B1		$\mathrm{f}'''(0)=108$ and $\mathrm{f}^{(iv)}(0)=-648$ seen or used
	$\mathrm{f}(x)\approx 0+x(0)+\frac{x^2}{2}(0)+\frac{x^3}{3!}\mathrm{f}'''(0)+\frac{x^4}{4!}\mathrm{f}^{(iv)}(0)\ldots$ $\mathrm{f}(x)\approx \frac{x^3}{3!}(108)+\frac{x^4}{4!}(-648)\ldots$	M1		$\mathrm{f}(x)\approx \frac{x^3}{3!}\mathrm{f}'''(0)+\frac{x^4}{4!}\mathrm{f}^{(iv)}(0)$ used with c's non-zero values for $\mathrm{f}'''(0)$ and $\mathrm{f}^{(iv)}(0)$
	$=18x^3-27x^4$	A1	3	$18x^3-27x^4$ Ignore any extra higher powers of x terms
	Altn: Use of answer to part **(a)** $\mathrm{f}(x)=(6x+2)\mathrm{e}^{-3x}-2\cos 3x$	[B1]		
	$=$	[M1]		Correct series for e^{-3x} (at least from x^2 terms up to x^4 terms inclusive) and $\cos 3x$ (at least x^2 terms and x^4 terms) substituted and also product of $(px+q)$ term with e^{-3x} series attempted where p and q are numbers.
	$=(2-2)+(6-6)x+(9-18+9)x^2+(27-9)x^3+$ $+(6.75-27-6.75)x^4$ $=18x^3-27x^4$	[A1]	[3]	
	Total		**11**	
	If using **(a)** to answer **(b)(i)**, for guidance, $\mathrm{f}''(x)=54x\mathrm{e}^{-3x}-18\mathrm{e}^{-3x}+18\cos 3x$			

Q6	Solution	Mark	Total	Comment
(a)	$\frac{dx}{dt}\frac{dy}{dx}=\frac{dy}{dt}$	M1		OE Relevant chain rule eg $\frac{dy}{dx}=\frac{dt}{dx}\frac{dy}{dt}$
	$2e^{2t}\frac{dy}{dx}=\frac{dy}{dt} \Rightarrow 2x\frac{dy}{dx}=\frac{dy}{dt}$	A1		OE eg $\frac{dy}{dx}=\frac{1}{2}e^{-2t}\frac{dy}{dt}$
	$\frac{d}{dt}\left(2x\frac{dy}{dx}\right)=\frac{d^2y}{dt^2}$; $\frac{dx}{dt}\frac{d}{dx}\left(2x\frac{dy}{dx}\right)=\frac{d^2y}{dt^2}$	M1		OE.Valid 1st stage to differentiate $xy'(x)$ oe wrt t or to differentiate $x^{-1}y'(t)$ oe wrt x.
	$\frac{dx}{dt}\left(2\frac{dy}{dx}+2x\frac{d^2y}{dx^2}\right)=\frac{d^2y}{dt^2}$	m1		Product rule OE (dep on MM) to obtain an eqn involving both second derivatives
	$4x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}=\frac{d^2y}{dt^2}$	A1		OE eg $\frac{d^2y}{dx^2}=\frac{1}{2}e^{-2t}\left[-e^{-2t}\frac{dy}{dt}+\frac{1}{2}e^{-2t}\frac{d^2y}{dt^2}\right]$ {Note: e^{-t} could be replaced by $\frac{1}{\sqrt{x}}$ }
	$4\sqrt{x^5}\frac{d^2y}{dx^2}+2\sqrt{x}\,y=\sqrt{x}\,(\ln x)^2+5$ becomes $\frac{d^2y}{dt^2}-4x\frac{dy}{dx}+2y=(\ln x)^2+\frac{5}{\sqrt{x}}$	A1		Or better
	$\Rightarrow \frac{d^2y}{dt^2}-2\frac{dy}{dt}+2y=(2t)^2+\frac{5}{e^t}$			
	$\Rightarrow \frac{d^2y}{dt^2}-2\frac{dy}{dt}+2y=4t^2+5e^{-t}$	A1	7	AG Be convinced
(b)	Auxl eqn $m^2-2m+2=0 \qquad (m-1)^2+1=0$	M1		$(m-1)^2+k$ or using quadratic formula on correct aux eqn. PI by correct values of 'm' seen/used.
	$m=1\pm i$	A1		
	CF: $(y_C=)\ e^t(A\cos t+B\sin t)$	B1F		Ft on $m=p\pm qi$, $p, q\neq 0$ and 2 arb. constants in CF. Condone x for t here
	P.Int. Try $(y_P=)\ a+bt+ct^2+de^{-t}$	M1		
	$(y'(t)=)\ b+2ct-de^{-t}$; $(y''(t)=)\ 2c+de^{-t}$ Substitute into DE gives $2c+de^{-t}-2(b+2ct-de^{-t})+$ $+2(a+bt+ct^2+de^{-t})=4t^2+5e^{-t}$	M1		Substitution and comparing coeffs at least once
	$d=1;\ c=2$	B1		Need both
	$2b-4c=0$ and $2c-2b+2a=0$	A1		OE PI by c's b=2×c's c and c's a=c's c provided c's $c\neq 0$
	$b=4$ and $a=2$	A1		Need both
	GS (y=) $e^t(A\cos t+B\sin t)+2+4t+2t^2+e^{-t}$	B1F		Ft on c's CF + PI, provided PI is non-zero and CF has two arbitrary constants and RHS is fn of t only
	$y=\sqrt{x}\left[A\cos(\ln\sqrt{x})+B\sin(\ln\sqrt{x})\right]+2+$ $+2\ln x+\frac{1}{2}(\ln x)^2+\frac{1}{\sqrt{x}}$	A1	10	y=f(x) with ACF for f(x)
	Total		17	

Q7	Solution	Mark	Total	Comment
(a)	Area = $\frac{1}{2}\int_{(-\frac{\pi}{2})}^{(\frac{\pi}{2})}(1+\cos 2\theta)^2\,(\mathrm{d}\theta)$	M1		Use of $\frac{1}{2}\int r^2\,(\mathrm{d}\theta)$ or $\int_0^{\frac{\pi}{2}} r^2\,(\mathrm{d}\theta)$
	$=\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(1+2\cos 2\theta+\cos^2 2\theta)\mathrm{d}\theta$	B1		Correct expn of $[1+\cos 2\theta]^2$ and correct limits
	$=\frac{1}{2}\int(1+2\cos 2\theta+0.5+0.5\cos 4\theta)\,\mathrm{d}\theta$	M1		$2\cos^2 2\theta=\pm1\pm\cos 4\theta$ used with $k\int r^2\,\mathrm{d}\theta$
	$=\frac{1}{2}\left[\theta+\sin 2\theta+0.5\theta+\frac{1}{8}\sin 4\theta\right]_{-\pi/2}^{\pi/2}$	A1F		Correct integration ft wrong coefficients
	$=\frac{3}{4}\pi$	A1	5	CSO
(b)(i)	$1+\sin\theta=1+\cos 2\theta$ $1+\sin\theta=1+1-2\sin^2\theta$	M1		Equating rs (or equating $\sin\theta$s) followed (or preceded) by $\cos 2\theta=\pm(1\pm2\sin^2\theta)$
	$(2\sin\theta-1)(\sin\theta+1)$ (=0)	A1		Or $r(2r-3)$ (=0), each PI by correct 2 roots
	$\sin\theta=-1$ gives the pole, O	E1		Or $r=0$ gives the pt O. OE eg finds 2nd pair of coords (0, - π/2) and chooses (3/2, π/6)
	At A, $\sin\theta=0.5$ so $A\left(\frac{3}{2},\frac{\pi}{6}\right)$	A1	4	$r=1.5$, $\theta=\frac{\pi}{6}$
(b)(ii)	Eqn of line thro' A parallel to initial line is $r\sin\theta=\frac{3}{4}$	B1F		PI Ft on $r\sin\theta=r_A\sin\theta_A$
	At B, $r=2-2\sin^2\theta=2-2\left(\frac{9}{16r^2}\right)$	M1		Solving $r\sin\theta=k$ and $r=1+\cos 2\theta$ to reach a cubic eqn in r or in $\sin\theta$
	$16r^3=32r^2-18$	A1		Correct cubic eqn in r (or in $\sin\theta$ eg $8\sin^3\theta=8\sin\theta-3$)
	$(2r-3)(4r^2-2r-3)=0$	A1		Or $(2\sin\theta-1)(4\sin^2\theta+2\sin\theta-3)=0$
	Since $r_A=1.5$ and $r_B>0$, $OB=r_B=\frac{2+\sqrt{4+48}}{8}=\frac{1}{4}(\sqrt{13}+1)$	A2,1,0	6	A.G. Note: A2 requires correct surd for OB and also correct justifications for ignoring the other two roots of the cubic eqn. Max of A1 if justification absent
(b)(iii)	$AB=\pm(r_A\cos\theta_A-r_B\cos\theta_B)$	M1		OE method to find AB or AB^2. eg $AB=\frac{OB\sin(\theta_B-\theta_A)}{\sin\theta_A}$ OE single 'eqn' or $AB^2=r_A{}^2+OB^2-2r_AOB\cos(\theta_B-\theta_A)$ or $OB^2=r_A{}^2+AB^2-2r_AAB\cos\theta_A$
	$\cos\theta_B=\sqrt{\frac{r_B}{2}}\left(=\sqrt{\frac{\sqrt{13}+1}{8}}\right)$=(0.758(7..))	m1		OE eg solving correct quadratic eg $\sin\theta_B=\frac{3}{\sqrt{13}+1}$ or $\theta_B=0.709(41...)$
	$AB=0.425$ (to 3sf)	A1	3	0.425 Condone >3sf (0.425428….)
	Total		**18**	
	TOTAL		**75**	
(b)(ii)	$(2\sin\theta-1)(4\sin^2\theta+2\sin\theta-3)=0$ $\sin\theta=0.5$ (pt A), eg $\sin\theta<-1$ impossible, so $\sin\theta=\frac{-2+\sqrt{52}}{8}$			