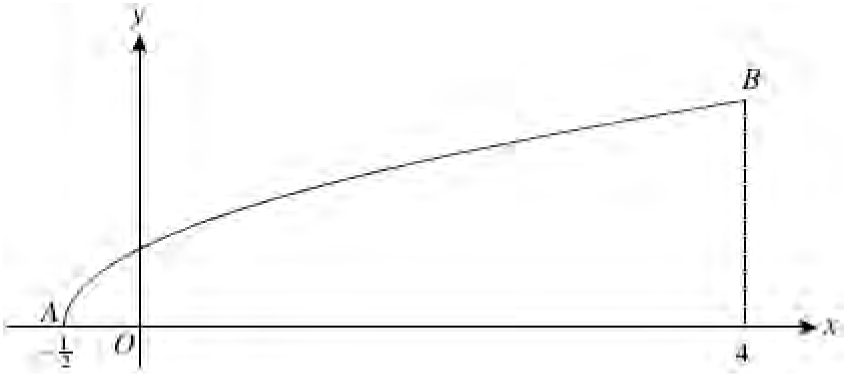
FP2 Polar Coordinates

1. June 2010 qu.9



The diagram shows the curve with equation $y = \sqrt{2x+1}$ between the points $A(-\frac{1}{2}, 0)$ and $B(4, 3)$.

(i) Find the area of the region bounded by the curve, the x-axis and the line $x = 4$. Hence find the area of the region bounded by the curve and the lines OA and OB, where O is the origin. [4]

(ii) Show that the curve between B and A can be expressed in polar coordinates as

$$r = \frac{1}{1-\cos\theta}, \text{ where } \tan^{-1}\left(\frac{3}{4}\right) \leq \theta \leq \pi. \qquad [5]$$

(iii) Deduce from parts (i) and (ii) that $\int_{\tan^{-1}\left(\frac{3}{4}\right)}^{\pi} \operatorname{cosec}^4\left(\frac{1}{2}\theta\right) d\theta = 24.$ [4]

2. Jan 2010 qu. 4

The equation of a curve, in polar coordinates, is

$$r = e^{-2\theta}, \quad \text{for } 0 \leq \theta \leq \pi.$$

(i) Sketch the curve, stating the polar coordinates of the point at which r takes its greatest value. [2]

(ii) The pole is O and points P and Q, with polar coordinates (r_1, θ_1) and (r_2, θ_2) respectively, lie on the curve. Given that $\theta_2 > \theta_1$, show that the area of the region enclosed by the curve and the lines OP and OQ can be expressed as $k(r_1^2 - r_2^2)$, where k is a constant to be found. [5]

3. June 2009 qu.9

(i) It is given that, for non-negative integers n, $I_n = \int_0^{\frac{1}{2}\pi} \sin^n\theta \, d\theta$.

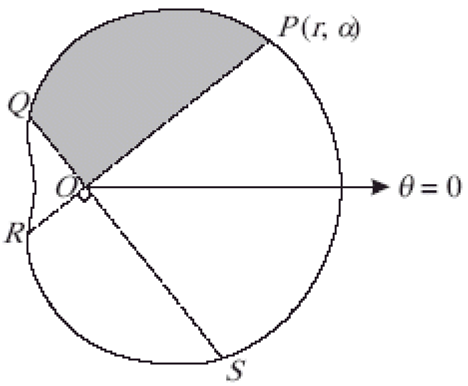
Show that, for $n \geq 2$, $nI_n = (n-1)I_{n-2}$. [4]

(ii) The equation of a curve, in polar coordinates, is $r = \sin^3\theta$, for $0 \leq \theta \leq \pi$.

(a) Find the equations of the tangents at the pole and sketch the curve. [4]

(b) Find the exact area of the region enclosed by the curve. [6]

4. Jan 2009 qu. 7



The diagram shows the curve with equation, in polar coordinates,

$$r = 3 + 2\cos\theta, \qquad \text{for } 0 \leq \theta < 2\pi.$$

The points P, Q, R and S on the curve are such that the straight lines POR and QOS are perpendicular, where O is the pole. The point P has polar coordinates (r, α).

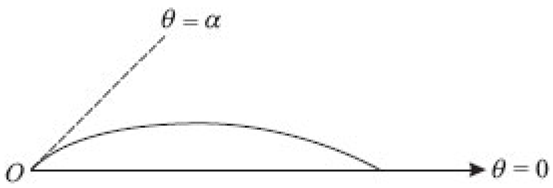
(i) Show that $OP + OQ + OR + OS = k$, where k is a constant to be found. [3]

(ii) Given that $\alpha = \frac{1}{4}\pi$, find the exact area bounded by the curve and the lines OP and OQ (shaded in the diagram). [5]

5. June 2008 qu.8

The equation of a curve, in polar coordinates, is $r = 1 - \sin 2\theta$, for $0 \leq \theta < 2\pi$.

(i)



The diagram shows the part of the curve for which $0 \leq \theta \leq \alpha$, where $\theta = \alpha$ is the equation of the tangent to the curve at O. Find α in terms of π. [2]

(ii) (a) If $f(\theta) = 1 - \sin 2\theta$, show that $f\left(\frac{1}{2}(2k + 1)\pi - \theta\right) = f(\theta)$ for all θ, where k is an integer. [3]

(b) Hence state the equations of the lines of symmetry of the curve

$$r = 1 - \sin 2\theta, \qquad \text{for } 0 \leq \theta < 2\pi.$$ [2]

(iii) Sketch the curve with equation $r = 1 - \sin 2\theta$, for $0 \leq \theta < 2\pi$.

State the maximum value of r and the corresponding values of θ. [4]

6. Jan 2008 qu. 4

The equation of a curve, in polar coordinates, is $r = 1 + 2\sec\theta$, for $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$.

(i) Find the exact area of the region bounded by the curve and the lines $\theta = 0$ and $\theta = \frac{1}{6}\pi$. [5]

(ii) Show that a cartesian equation of the curve is $(x - 2)\sqrt{x^2 + y^2} = x$ [3]

7. June 2007 qu.1

The equation of a curve, in polar coordinates, is $r = 2\sin 3\theta$, for $0 < \theta < \frac{1}{3}\pi$.

Find the exact area of the region enclosed by the curve between $\theta = 0$ and $\theta = \frac{1}{3}\pi$. [4]

8. Jan 2007 qu. 9

The equation of a curve, in polar coordinates, is $r = \sec\theta + \tan\theta$, for $0 \le \theta \le \frac{1}{3}\pi$

(i) Sketch the curve. [2]

(ii) Find the exact area of the region bounded by the curve and the lines $\theta = 0$ and $\theta = \frac{1}{3}\pi$ [6]

(iii) Find a cartesian equation of the curve. [3]

9. June 2006 qu.7

The equation of a curve, in polar coordinates, is $r = \sqrt{3} + \tan\theta$, for $-\frac{1}{3}\pi \le \theta \le \frac{1}{4}\pi$

(i) Find the equation of the tangent at the pole. [2]

(ii) State the greatest value of r and the corresponding value of θ. [2]

(iii) Sketch the curve. [2]

(iv) Find the exact area of the region enclosed by the curve and the lines $\theta = 0$ and $\theta = \frac{1}{4}\pi$ [5]

10. Jan 2006 qu. 8

The equation of a curve, in polar coordinates, is $r = 1 + \cos 2\theta$, for $0 \le \theta < 2\pi$.

(i) State the greatest value of r and the corresponding values of θ. [2]

(ii) Find the equations of the tangents at the pole. [2]

(iii) Find the exact area enclosed by the curve and the lines $\theta = 0$ and $\theta = \frac{1}{2}\pi$ [5]

(iv) Find, in simplified form, the cartesian equation of the curve. [4]