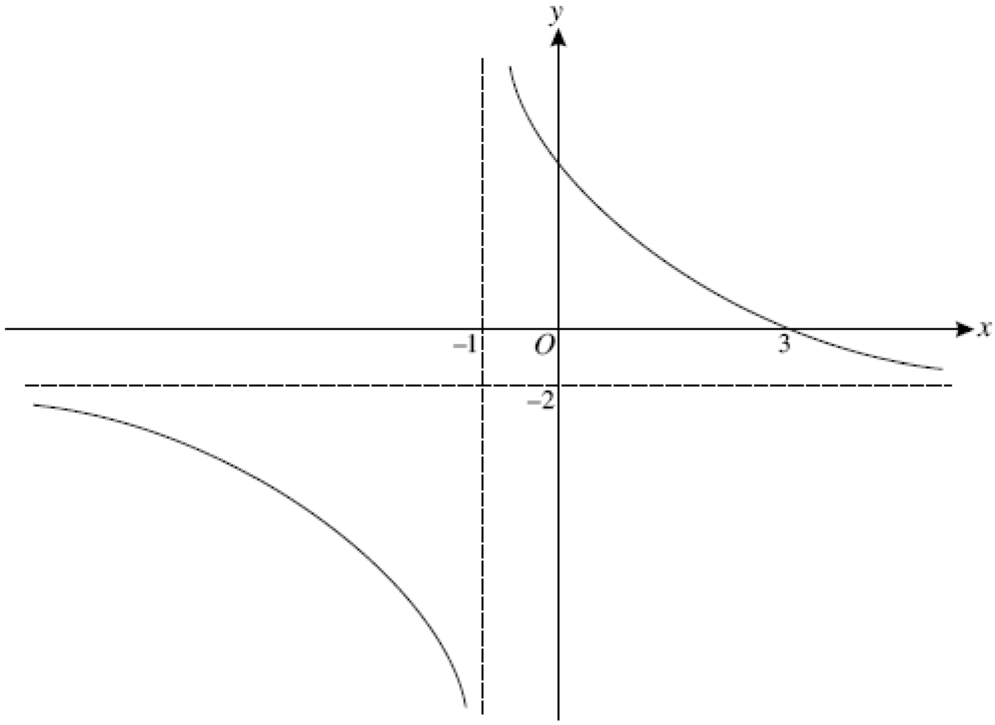
FP2 Graphs

1. June 2010 qu. 4



The diagram shows the curve with equation $y = \dfrac{ax+b}{x+c}$, where a, b and c are constants.

(i) Given that the asymptotes of the curve are $x = -1$ and $y = -2$ and that the curve passes through (3, 0), find the values of a, b and c. [3]

(ii) Sketch the curve with equation $y^2 = \dfrac{ax+b}{x+c}$,

for the values of a, b and c found in part (i). State the coordinates of any points where the curve crosses the axes, and give the equations of any asymptotes. [4]

2. Jan 2010 qu.8

The equation of a curve is $y = \dfrac{kx}{(x-1)^2}$, where k is a positive constant.

(i) Write down the equations of the asymptotes of the curve. [2]

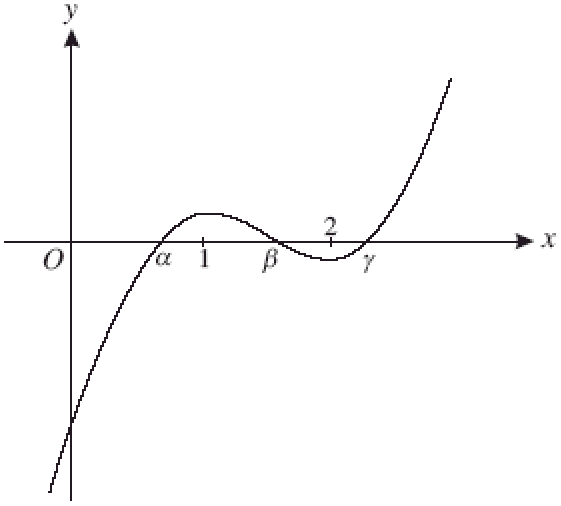
(ii) Show that $y \geq -\dfrac{1}{4}k$. [4]

(iii) Show that the x-coordinate of the stationary point of the curve is independent of k, and sketch the curve. [4]

3. June 2009 qu. 2

Given that $y = \dfrac{x^2 + x + 1}{(x-1)^2}$, prove that $y \geq \dfrac{1}{4}$ for all $x \neq 1$. [4]

4. Jan 2009 qu.5



The diagram shows the curve with equation $y = f(x)$, where $f(x) = 2x^3 - 9x^2 + 12x - 4.36$.

The curve has turning points at $x = 1$ and $x = 2$ and crosses the x-axis at $x = \alpha$, $x = \beta$ and $x = \gamma$, where $0 < \alpha < \beta < \gamma$.

(i) The Newton-Raphson method is to be used to find the roots of the equation $f(x) = 0$, with $x_1 = k$.

(a) To which root, if any, would successive approximations converge in each of the cases $k < 0$ and $k = 1$? [2]

(b) What happens if $1 < k < 2$? [2]

(ii) Sketch the curve with equation $y^2 = f(x)$. State the coordinates of the points where the curve crosses the x-axis and the coordinates of any turning points. [4]

5. Jan 2009 qu.9

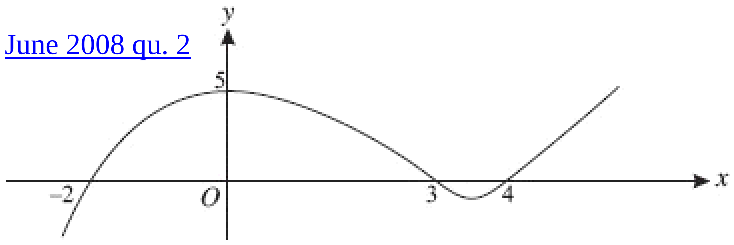
A curve has equation $y = \dfrac{4x - 3a}{2(x^2 + a^2)}$, where a is a positive constant.

(i) Explain why the curve has no asymptotes parallel to the y-axis. [2]

(ii) Find, in terms of a, the set of values of y for which there are no points on the curve. [5]

(iii) Find the exact value of $\displaystyle\int_a^{2a} \frac{4x - 3a}{2(x^2 + a^2)}\,dx$, showing that it is independent of a. [5]

6. June 2008 qu. 2



The diagram shows the curve $y = \mathrm{f}(x)$. The curve has a maximum point at (0, 5) and crosses the x-axis at (−2, 0), (3, 0) and (4, 0). Sketch the curve $y^2 = \mathrm{f}(x)$, showing clearly the coordinates of any turning points and of any points where this curve crosses the axes. [5]

7. June 2008 qu. 4

(i) Sketch, on the same diagram, the curves with equations $y = \operatorname{sech} x$ and $y = x^2$. [3]

(ii) By using the definition of sech x in terms of e^x and e^{-x}, show that the x-coordinates of the points at which these curves meet are solutions of the equation $x^2 = \dfrac{2e^x}{e^{2x}+1}$. [3]

(iii) The iteration $x_{n+1} = \sqrt{\dfrac{2e^{x_n}}{e^{2x_n}+1}}$

can be used to find the positive root of the equation in part (ii). With initial value $x_1 = 1$, the approximations $x_2 = 0.8050$, $x_3 = 0.8633$, $x_4 = 0.8463$ and $x_5 = 0.8513$ are obtained, correct to 4 decimal places. State with a reason whether, in this case, the iteration produces a 'staircase' or a 'cobweb' diagram. [2]

8. Jan 2008 qu.6

The equation of a curve is $y = \dfrac{2x^2 - 11x - 6}{x-1}$.

(i) Find the equations of the asymptotes of the curve. [3]

(ii) Show that y takes all real values. [5]

9. June 2007 qu. 9

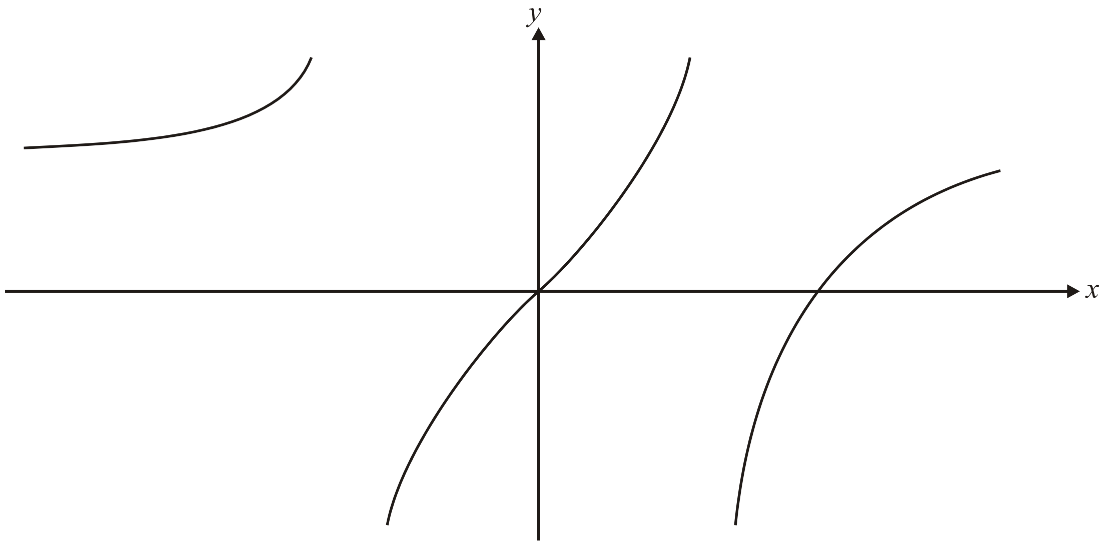
It is given that the equation of a curve is $y = \dfrac{x^2 - 2ax}{x-a}$, where a is a positive constant.

(i) Find the equations of the asymptotes of the curve. [4]

(ii) Show that y takes all real values. [4]

(iii) Sketch the curve $y = \dfrac{x^2 - 2ax}{x-a}$ [3]

10. Jan 2007 qu.6



The diagram shows the curve with equation $y = \dfrac{2x^2 - 3ax}{x^2 - a^2}$, where a is a positive constant.

(i) Find the equations of the asymptotes of the curve. [3]

(ii) Sketch the curve with equation $y^2 = \dfrac{2x^2 - 3ax}{x^2 - a^2}$.

State the coordinates of any points where the curve crosses the axes, and give the equations of any asymptotes. [5]

11. June 2006 qu. 3

The equation of a curve is $y = \dfrac{x+1}{x^2+3}$.

(i) State the equation of the asymptote of the curve. [1]

(ii) Show that $-\dfrac{1}{6} \le y \le \dfrac{1}{2}$. [5]

12. Jan 2009 qu.5

(i) Find the equations of the asymptotes of the curve with equation $y = \dfrac{x^2+3x+3}{x+2}$ [3]

(ii) Show that y cannot take values between –3 and 1. [5]