FP2 Differentiation

1. June 2010 qu.1

It is given that $f(x) = \tan^{-1} 2x$ and $g(x) = p \tan^{-1} x$, where p is a constant.

Find the value of p for which $f'\left(\frac{1}{2}\right) = g'\left(\frac{1}{2}\right)$. [4]

2. June 2010 qu.2

It is given that $f(x) = \tan^{-1}(1 + x)$.

(i) Find $f(0)$ and $f'(0)$, and show that $f''(0) = -\frac{1}{2}$. [4]

(ii) Hence find the Maclaurin series for $f(x)$ up to and including the term in x^2. [2]

3. Jan 2010 qu.1

(i) Given that $y = \tanh^{-1} x$, for $-1 < x < 1$, prove that $y = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$. [3]

(ii) It is given that $f(x) = a \cosh x - b \sinh x$, where a and b are positive constants.

(a) Given that $b \geq a$, show that the curve with equation $y = f(x)$ has no stationary points. [3]

(b) In the case where $a > 1$ and $b = 1$, show that $f(x)$ has a minimum value of $\sqrt{a^2 - 1}$. [6]

4. June 2009 qu.3

(i) Given that $f(x) = e^{\sin x}$, find $f'(0)$ and $f''(0)$. [4]

(ii) Hence find the first three terms of the Maclaurin series for $f(x)$. [2]

5. Jan 2009 qu. 1

(i) Write down and simplify the first three terms of the Maclaurin series for e^{2x}. [2]

(ii) Hence show that the Maclaurin series for $\ln(e^{2x} + e^{-2x})$

begins $\ln a + bx^2$, where a and b are constants to be found. [4]

6. Jan 2009 qu. 3

(i) Prove that the derivative of $\sin^{-1} x$ is $\frac{1}{\sqrt{1-x^2}}$. [3]

(ii) Given that $\sin^{-1} 2x + \sin^{-1} y = \frac{1}{2}\pi$, find the exact value of $\frac{dy}{dx}$ when $x = \frac{1}{4}$. [4]

7. June 2008 qu. 7

It is given that $f(x) = \tanh^{-1}\left(\frac{1-x}{2+x}\right)$, for $x > -\frac{1}{2}$.

(i) Show that $f'(x) = -\frac{1}{1+2x}$, and find $f''(x)$. [6]

(ii) Show that the first three terms of the Maclaurin series for $f(x)$ can be written as $\ln a + bx + cx^2$, for constants a, b and c to be found. [4]

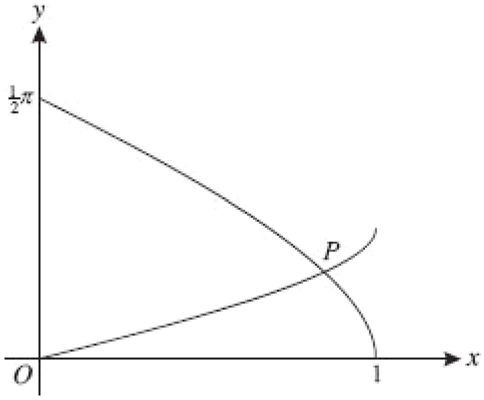
8. Jan 2008 qu. 1

It is given that $f(x) = \ln(1 + \cos x)$.

(i) Find the exact values of $f(0)$, $f'(0)$ and $f''(0)$. [4]

(ii) Hence find the first two non-zero terms of the Maclaurin series for $f(x)$. [2]

9. Jan 2008 qu. 2

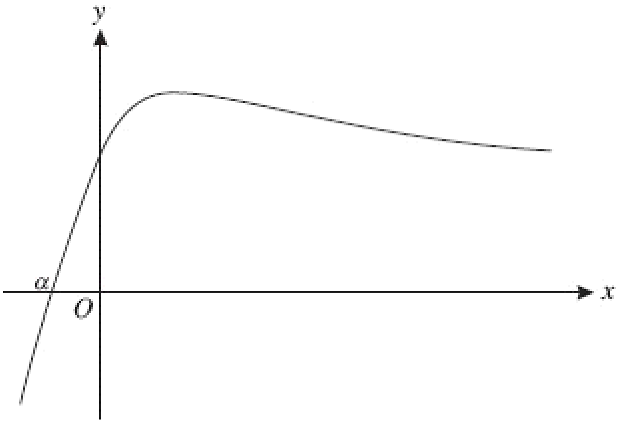


The diagram shows parts of the curves with equations $y = \cos^{-1} x$ and $y = \frac{1}{2}\sin^{-1} x$ and their point of intersection P.

(i) Verify that the coordinates of P are $\left(\frac{1}{2}\sqrt{3}, \frac{1}{6}\pi\right)$ [2]

(ii) Find the gradient of each curve at P. [3]

10. Jan 2008 qu. 5



The diagram shows the curve with equation $y = xe^{-x} + 1$. The curve crosses the x-axis at $x = \alpha$.

(i) Use differentiation to show that the x-coordinate of the stationary point is 1. [2]

α is to be found using the Newton-Raphson method, with $f(x) = xe^{-x} + 1$.

(ii) Explain why this method will not converge to α if an initial approximation x_1 is chosen such that $x_1 > 1$. [2]

(iii) Use this method, with a first approximation $x_1 = 0$, to find the next three approximations x_2, x_3 and x_4. Find α, correct to 3 decimal places. [5]

11. Jan 2008 qu. 9

(i) Prove that $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2 - 1}}$ [3]

(ii) Hence, or otherwise, find $\int \frac{1}{\sqrt{4x^2 - 1}} dx$. [2]

(iii) By means of a suitable substitution, find $\int \sqrt{4x^2 - 1}\, dx$. [6]

12. June 2007 qu. 2

(i) Given that $f(x) = \sin\left(2x + \frac{\pi}{4}\right)$, show that $f(x) = \frac{1}{2}\sqrt{2}\,(\sin 2x + \cos 2x)$ [2]

(ii) Hence find the first four terms of the Maclaurin series for $f(x)$. [You may use appropriate results given in the List of Formulae.] [3]

13. June 2007 qu. 4

(i) Given that $y = x\sqrt{1 - x^2} - \cos^{-1} x$, find $\frac{dy}{dx}$ in a simplified form. [4]

(ii) Hence, or otherwise, find the exact value of $\int_0^1 2\sqrt{1 - x^2}\, dx$. [3]

14. Jan 2007 qu. 1

It is given that $f(x) = \ln(3 + x)$.

(i) Find the exact values of (0) and f'(0), and show that $f''(0) = -\frac{1}{9}$. [3]

(ii) Hence write down the first three terms of the Maclaurin series for $f(x)$, given that

$-3 < x \leq 3$. [2]

15. June 2006 qu.1

Find the first three non-zero terms of the Maclaurin series for $(1 + x)\sin x$,

simplifying the coefficients. [3]

16. June 2006 qu.2

(i) Given that $y = \tan^{-1} x$, prove that $\frac{dy}{dx} = \frac{1}{1+x^2}$. [3]

(ii) Verify that $y = \tan^{-1} x$ satisfies the equation $(1+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0$. [3]

17. Jan 2006 qu.1

(i) Write down and simplify the first three non-zero terms of the Maclaurin series for

$\ln(1 + 3x)$. [3]

(ii) Hence find the first three non-zero terms of the Maclaurin series for $e^x \ln(1 + 3x)$,

simplifying the coefficients. [3]

18. June 2010 qu.3

Given that the first three terms of the Maclaurin series for $(1 + \sin x)e^{2x}$ are identical to the first three terms of the binomial series for $(1 + ax)^n$, find the values of the constants a and n. (You may use appropriate results given in the List of Formulae (MF1).)

[6]