

Edexcel Maths FP2

Topic Questions from Papers

Polar Coordinates

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Question 4 continued

Lined area for writing the answer to Question 4 continued.



5.

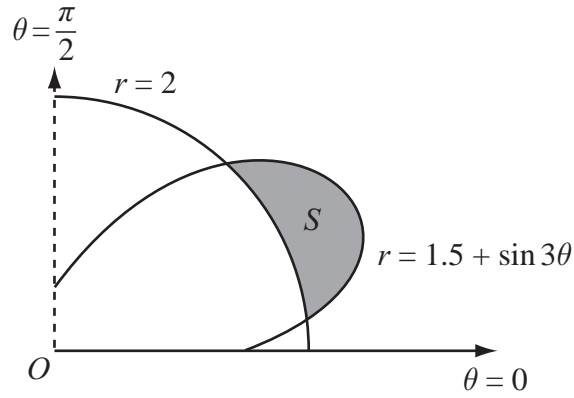


Figure 1

Figure 1 shows the curves given by the polar equations

$$r = 2, \quad 0 \leq \theta \leq \frac{\pi}{2},$$

and $r = 1.5 + \sin 3\theta, \quad 0 \leq \theta \leq \frac{\pi}{2}.$

- (a) Find the coordinates of the points where the curves intersect. (3)

The region *S*, between the curves, for which $r > 2$ and for which $r < (1.5 + \sin 3\theta)$, is shown shaded in Figure 1.

- (b) Find, by integration, the area of the shaded region *S*, giving your answer in the form $a\pi + b\sqrt{3}$, where *a* and *b* are simplified fractions. (7)

6.

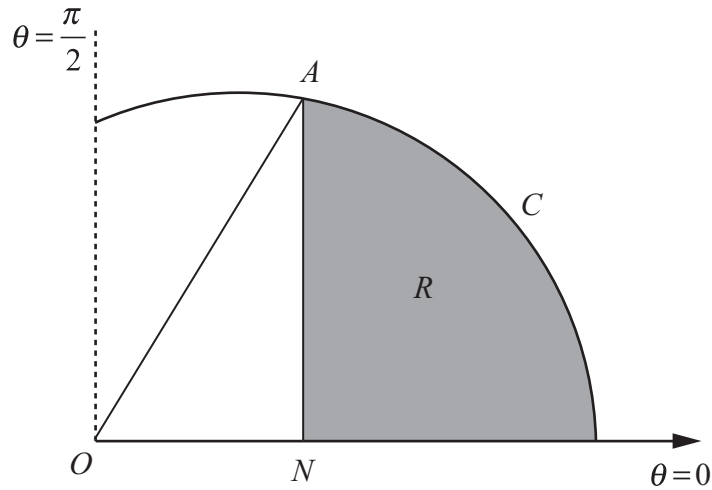


Figure 1

The curve C shown in Figure 1 has polar equation

$$r = 2 + \cos \theta, \quad 0 \leq \theta \leq \frac{\pi}{2}$$

At the point A on C , the value of r is $\frac{5}{2}$.

The point N lies on the initial line and AN is perpendicular to the initial line.

The finite region R , shown shaded in Figure 1, is bounded by the curve C , the initial line and the line AN .

Find the exact area of the shaded region R .

(9)

8.

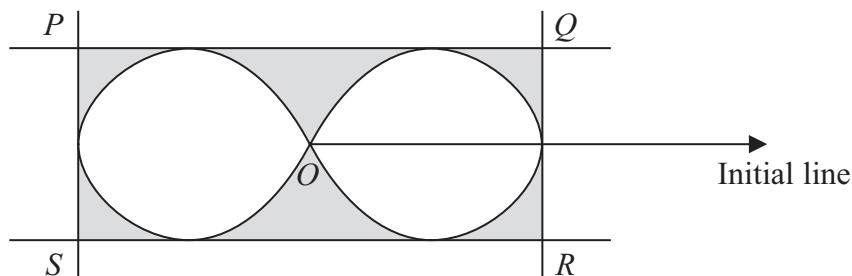


Figure 1

Figure 1 shows a closed curve C with equation

$$r = 3(\cos 2\theta)^{\frac{1}{2}}, \quad \text{where } -\frac{\pi}{4} < \theta \leq \frac{\pi}{4}, \quad \frac{3\pi}{4} < \theta \leq \frac{5\pi}{4}$$

The lines PQ , SR , PS and QR are tangents to C , where PQ and SR are parallel to the initial line and PS and QR are perpendicular to the initial line. The point O is the pole.

- (a) Find the total area enclosed by the curve C , shown unshaded inside the rectangle in Figure 1. (4)

- (b) Find the total area of the region bounded by the curve C and the four tangents, shown shaded in Figure 1. (9)



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8.

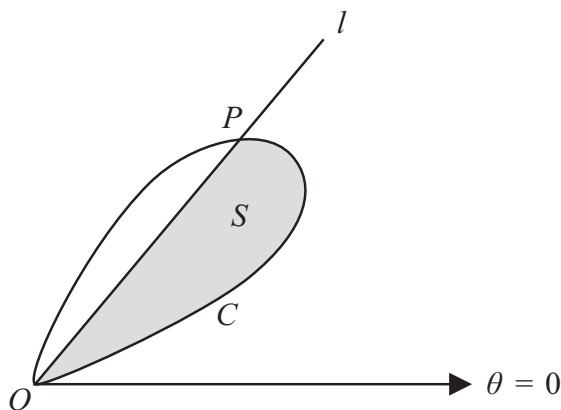


Figure 1

Figure 1 shows a curve C with polar equation $r = a \sin 2\theta$, $0 \leq \theta \leq \frac{\pi}{2}$, and a half-line l .

The half-line l meets C at the pole O and at the point P . The tangent to C at P is parallel to the initial line. The polar coordinates of P are (R, ϕ) .

(a) Show that $\cos \phi = \frac{1}{\sqrt{3}}$ (6)

(b) Find the exact value of R . (2)

The region S , shown shaded in Figure 1, is bounded by C and l .

(c) Use calculus to show that the exact area of S is

$$\frac{1}{36} a^2 \left(9 \arccos \left(\frac{1}{\sqrt{3}} \right) + \sqrt{2} \right)$$
(7)



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Question 8 continued

Q8

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END



Further Pure Mathematics FP2

Candidates sitting FP2 may also require those formulae listed under Further Pure Mathematics FP1 and Core Mathematics C1–C4.

Area of a sector

$$A = \frac{1}{2} \int r^2 \, d\theta \quad (\text{polar coordinates})$$

Complex numbers

$$e^{i\theta} = \cos \theta + i \sin \theta$$

$$\{r(\cos \theta + i \sin \theta)\}^n = r^n (\cos n\theta + i \sin n\theta)$$

The roots of $z^n = 1$ are given by $z = e^{\frac{2\pi k i}{n}}$, for $k = 0, 1, 2, \dots, n-1$

Maclaurin's and Taylor's Series

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \dots + \frac{x^r}{r!} f^{(r)}(0) + \dots$$

$$f(x) = f(a) + (x-a) f'(a) + \frac{(x-a)^2}{2!} f''(a) + \dots + \frac{(x-a)^r}{r!} f^{(r)}(a) + \dots$$

$$f(a+x) = f(a) + x f'(a) + \frac{x^2}{2!} f''(a) + \dots + \frac{x^r}{r!} f^{(r)}(a) + \dots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^r}{r!} + \dots \quad \text{for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{r+1} \frac{x^r}{r} + \dots \quad (-1 < x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \dots \quad \text{for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^r \frac{x^{2r}}{(2r)!} + \dots \quad \text{for all } x$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots + (-1)^r \frac{x^{2r+1}}{2r+1} + \dots \quad (-1 \leq x \leq 1)$$

Further Pure Mathematics FP1

Candidates sitting FP1 may also require those formulae listed under Core Mathematics C1 and C2.

Summations

$$\sum_{r=1}^n r^2 = \frac{1}{6} n(n+1)(2n+1)$$

$$\sum_{r=1}^n r^3 = \frac{1}{4} n^2(n+1)^2$$

Numerical solution of equations

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Conics

	Parabola	Rectangular Hyperbola
Standard Form	$y^2 = 4ax$	$xy = c^2$
Parametric Form	$(at^2, 2at)$	$\left(ct, \frac{c}{t} \right)$
Foci	$(a, 0)$	Not required
Directrices	$x = -a$	Not required

Matrix transformations

Anticlockwise rotation through θ about O : $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

Reflection in the line $y = (\tan \theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

In FP1, θ will be a multiple of 45° .

Core Mathematics C4

Candidates sitting C4 may also require those formulae listed under Core Mathematics C1, C2 and C3.

Integration (+ constant)

$f(x)$	$\int f(x) \, dx$
$\sec^2 kx$	$\frac{1}{k} \tan kx$
$\tan x$	$\ln \sec x $
$\cot x$	$\ln \sin x $
$\operatorname{cosec} x$	$-\ln \operatorname{cosec} x + \cot x , \quad \ln\left \tan\left(\frac{1}{2}x\right)\right $
$\sec x$	$\ln \sec x + \tan x , \quad \ln\left \tan\left(\frac{1}{2}x + \frac{1}{4}\pi\right)\right $

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Core Mathematics C3

Candidates sitting C3 may also require those formulae listed under Core Mathematics C1 and C2.

Logarithms and exponentials

$$e^{x \ln a} = a^x$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi)$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Differentiation

f(x)	f'(x)
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \dots + \binom{n}{r} a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^n C_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2} x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{1 \times 2 \times \dots \times r} x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

The trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2} h \{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

Core Mathematics C1

Mensuration

$$\text{Surface area of sphere} = 4\pi r^2$$

$$\text{Area of curved surface of cone} = \pi r \times \text{slant height}$$

Arithmetic series

$$u_n = a + (n - 1)d$$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$$