Edexcel Maths FP2

Topic Questions from Papers

Maclaurin & Taylor Series

5.

$$y = \sec^2 x$$

(a) Show that $\frac{d^2y}{dx^2} = 6\sec^4 x - 4\sec^2 x.$

(4)

(b) Find a Taylor series expansion of $\sec^2 x$ in ascending powers of $\left(x - \frac{\pi}{4}\right)$, up to and including the term in $\left(x - \frac{\pi}{4}\right)^3$.

(6)

Question 5 continued	blank



2. The displacement x metres of a particle at time t seconds is given by the differential equation

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + x + \cos x = 0$$

When t = 0, x = 0 and $\frac{dx}{dt} = \frac{1}{2}$.

Find a Taylor series solution for x in ascending powers of t, up to and including the term in t^3 .

(5)

2.

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \mathrm{e}^x \left(2y \frac{\mathrm{d}y}{\mathrm{d}x} + y^2 + 1 \right)$$

(a) Show that

$$\frac{d^{3}y}{dx^{3}} = e^{x} \left[2y \frac{d^{2}y}{dx^{2}} + 2\left(\frac{dy}{dx}\right)^{2} + ky \frac{dy}{dx} + y^{2} + 1 \right],$$

where k is a constant to be found.

(3)

Given that, at x = 0, y = 1 and $\frac{dy}{dx} = 2$,

(b) find a series solution for y in ascending powers of x, up to and including the term in x^3 .

(4)

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 $x\frac{\mathrm{d}y}{\mathrm{d}x} = 3x + y^2$

(a) Show that

$$x\frac{d^{2}y}{dx^{2}} + (1 - 2y)\frac{dy}{dx} = 3$$
(2)

Given that y = 1 at x = 1,

(b) find a series solution for y in ascending powers of (x-1), up to and including the term in $(x-1)^3$.

(8)

estion 5 continued		



4. Given that

$$y\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + 5y = 0$$

(a) find $\frac{d^3y}{dx^3}$ in terms of $\frac{d^2y}{dx^2}$, $\frac{dy}{dx}$ and y.

(4)

Given that y = 2 and $\frac{dy}{dx} = 2$ at x = 0

(b) find a series solution for y in ascending powers of x, up to and including the term in x^3 .

(5)

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estion 4 continued	

3.	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 4y - \sin x = 0$
	$\mathrm{d}x^2$

Given that $y = \frac{1}{2}$ and $\frac{dy}{dx} = \frac{1}{8}$ at x = 0,

find a series expansion for y in terms of x, up to and including the term in x^3 .

(5)

Further Pure Mathematics FP2

Candidates sitting FP2 may also require those formulae listed under Further Pure Mathematics FP1 and Core Mathematics C1–C4.

Area of a sector

$$A = \frac{1}{2} \int r^2 d\theta$$
 (polar coordinates)

Complex numbers

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\{r(\cos\theta + i\sin\theta)\}^n = r^n(\cos n\theta + i\sin n\theta)$$
The roots of $z^n = 1$ are given by $z = e^{\frac{2\pi k i}{n}}$, for $k = 0, 1, 2, ..., n-1$

Maclaurin's and Taylor's Series

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \dots + \frac{x^r}{r!} f^{(r)}(0) + \dots$$

$$f(x) = f(a) + (x - a) f'(a) + \frac{(x - a)^2}{2!} f''(a) + \dots + \frac{(x - a)^r}{r!} f^{(r)}(a) + \dots$$

$$f(a + x) = f(a) + x f'(a) + \frac{x^2}{2!} f''(a) + \dots + \frac{x^r}{r!} f^{(r)}(a) + \dots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^r}{r!} + \dots \quad \text{for all } x$$

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{r+1} \frac{x^r}{r} + \dots \quad (-1 < x \le 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \dots \quad \text{for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^r \frac{x^{2r}}{(2r)!} + \dots \quad \text{for all } x$$

$$\arcsin x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \dots \quad \text{for all } x$$

$$\arcsin x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \dots \quad \text{for all } x$$

Further Pure Mathematics FP1

Candidates sitting FP1 may also require those formulae listed under Core Mathematics C1 and C2.

Summations

$$\sum_{r=1}^{n} r^2 = \frac{1}{6} n(n+1)(2n+1)$$

$$\sum_{n=1}^{n} r^3 = \frac{1}{4} n^2 (n+1)^2$$

Numerical solution of equations

The Newton-Raphson iteration for solving f(x) = 0: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Conics

	Parabola	Rectangular Hyperbola
Standard Form	$y^2 = 4ax$	$xy = c^2$
Parametric Form	(at ² , 2at)	$\left(ct, \frac{c}{t}\right)$
Foci	(a, 0)	Not required
Directrices	x = -a	Not required

Matrix transformations

Anticlockwise rotation through θ about O: $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

Reflection in the line $y = (\tan \theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

In FP1, θ will be a multiple of 45°.

Candidates sitting C4 may also require those formulae listed under Core Mathematics C1, C2 and C3.

Integration (+ constant)

$$\int \mathbf{f}(x) \, dx$$

$$\sec^2 kx \qquad \frac{1}{k} \tan kx$$

$$\tan x \qquad \ln|\sec x|$$

$$\cot x \qquad \ln|\sin x|$$

$$\csc x \qquad -\ln|\csc x + \cot x|, \quad \ln|\tan(\frac{1}{2}x)|$$

$$\sec x \qquad \ln|\sec x + \tan x|, \quad \ln|\tan(\frac{1}{2}x + \frac{1}{4}\pi)|$$

Candidates sitting C3 may also require those formulae listed under Core Mathematics C1 and C2.

Logarithms and exponentials

$$e^{x \ln a} = a^x$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \frac{1}{2})\pi)$$

$$\sin A + \sin B = 2 \sin \frac{A + B}{2} \cos \frac{A - B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A + B}{2} \sin \frac{A - B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A + B}{2} \cos \frac{A - B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A + B}{2} \sin \frac{A - B}{2}$$

Differentiation

f(x) f'(x)

$$\tan kx$$
 $k \sec^2 kx$
 $\sec x$ $\sec x \tan x$
 $\cot x$ $-\csc^2 x$
 $\csc x$ $-\csc x \cot x$

$$\frac{f(x)}{g(x)}$$

$$\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Binomial series

$$(a+b)^{n} = a^{n} + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^{2} + \dots + \binom{n}{r} a^{n-r}b^{r} + \dots + b^{n} \quad (n \in \mathbb{N})$$
where $\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$

$$(1+x)^{n} = 1 + nx + \frac{n(n-1)}{1 \times 2}x^{2} + \dots + \frac{n(n-1)\dots(n-r+1)}{1 \times 2 \times \dots \times r}x^{r} + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_{\infty} = \frac{a}{1-r}$$
 for $|r| < 1$

Numerical integration

The trapezium rule:
$$\int_{a}^{b} y \, dx \approx \frac{1}{2} h\{(y_0 + y_n) + 2(y_1 + y_2 + ... + y_{n-1})\}$$
, where $h = \frac{b - a}{n}$

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times \text{slant height}$

Arithmetic series

$$u_n = a + (n-1)d$$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n[2a+(n-1)d]$$