FP2 Summation of finite series

2 (a) Given that

$$\frac{1}{r(r+1)(r+2)} = \frac{A}{r(r+1)} + \frac{B}{(r+1)(r+2)}$$

show that $A = \frac{1}{2}$ and find the value of B. *(3 marks)*

(b) Use the method of differences to find

$$\sum_{r=10}^{98} \frac{1}{r(r+1)(r+2)}$$

giving your answer as a rational number. *(4 marks)*

2 (a) Given that

$$\frac{1}{4r^2-1} = \frac{A}{2r-1} + \frac{B}{2r+1}$$

find the values of A and B. *(2 marks)*

(b) Use the method of differences to show that

$$\sum_{r=1}^{n} \frac{1}{4r^2-1} = \frac{n}{2n+1}$$ *(3 marks)*

(c) Find the least value of n for which $\sum_{r=1}^{n} \frac{1}{4r^2-1}$ differs from 0.5 by less than 0.001 .

(3 marks)

2 (a) Express $\frac{1}{r(r+2)}$ in partial fractions. *(3 marks)*

(b) Use the method of differences to find

$$\sum_{r=1}^{48} \frac{1}{r(r+2)}$$

giving your answer as a rational number. *(5 marks)*

6 (a) Show that $\dfrac{1}{(k+2)!} - \dfrac{k+1}{(k+3)!} = \dfrac{2}{(k+3)!}$. *(2 marks)*

(b) Prove by induction that, for all positive integers n,

$$\sum_{r=1}^{n} \frac{r \times 2^r}{(r+2)!} = 1 - \frac{2^{n+1}}{(n+2)!}$$ *(6 marks)*

2 (a) Given that

$$u_r = \tfrac{1}{6}r(r+1)(4r+11)$$

show that

$$u_r - u_{r-1} = r(2r+3)$$ *(3 marks)*

(b) Hence find the sum of the first hundred terms of the series

$$1 \times 5 + 2 \times 7 + 3 \times 9 + \ldots + r(2r+3) + \ldots$$ *(3 marks)*

7 (a) Given that

$$\mathrm{f}(k) = 12^k + 2 \times 5^{k-1}$$

show that

$$\mathrm{f}(k+1) - 5\mathrm{f}(k) = a \times 12^k$$

where a is an integer. *(3 marks)*

(b) Prove by induction that $12^n + 2 \times 5^{n-1}$ is divisible by 7 for all integers $n \geqslant 1$. *(4 marks)*

3 (a) Show that

$$(r+1)! - (r-1)! = (r^2 + r - 1)(r-1)!$$ *(2 marks)*

(b) Hence show that

$$\sum_{r=1}^{n} (r^2 + r - 1)(r-1)! = (n+2)n! - 2$$ *(4 marks)*

6 (a) Show that

$$(k+1)(4(k+1)^2-1) = 4k^3+12k^2+11k+3 \qquad \textit{(2 marks)}$$

(b) Prove by induction that, for all integers $n \geqslant 1$,

$$1^2+3^2+5^2+\ldots+(2n-1)^2 = \tfrac{1}{3}n(4n^2-1) \qquad \textit{(6 marks)}$$

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4 The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = \frac{3}{4} \qquad u_{n+1} = \frac{3}{4-u_n}$$

Prove by induction that, for all $n \geqslant 1$,

$$u_n = \frac{3^{n+1}-3}{3^{n+1}-1} \qquad \textit{(6 marks)}$$

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3 (a) Show that

$$\frac{2^{r+1}}{r+2} - \frac{2^r}{r+1} = \frac{r2^r}{(r+1)(r+2)} \qquad \textit{(3 marks)}$$

(b) Hence find

$$\sum_{r=1}^{30} \frac{r2^r}{(r+1)(r+2)}$$

giving your answer in the form $2^n - 1$, where n is an integer. *(3 marks)*

QUESTION | Answer space for question 3

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7 (a) Prove by induction that, for all integers $n \geqslant 1$,

$$\frac{3}{1^2\times 2^2} + \frac{5}{2^2\times 3^2} + \frac{7}{3^2\times 4^2} + \ldots + \frac{2n+1}{n^2(n+1)^2} = 1 - \frac{1}{(n+1)^2} \qquad \textit{(7 marks)}$$

(b) Find the smallest integer n for which the sum of the series differs from 1 by less than 10^{-5}. *(2 marks)*

3 (a) Show that $\dfrac{1}{5r-2} - \dfrac{1}{5r+3} = \dfrac{A}{(5r-2)(5r+3)}$, stating the value of the constant A. *(2 marks)*

(b) Hence use the method of differences to show that

$$\sum_{r=1}^{n} \frac{1}{(5r-2)(5r+3)} = \frac{n}{3(5n+3)}$$ *(4 marks)*

(c) Find the value of

$$\sum_{r=1}^{\infty} \frac{1}{(5r-2)(5r+3)}$$ *(1 mark)*

7 The polynomial $\mathrm{p}(n)$ is given by $\mathrm{p}(n) = (n-1)^3 + n^3 + (n+1)^3$.

(a) (i) Show that $\mathrm{p}(k+1) - \mathrm{p}(k)$, where k is a positive integer, is a multiple of 9. *(3 marks)*

(ii) Prove by induction that $\mathrm{p}(n)$ is a multiple of 9 for all integers $n \geqslant 1$. *(4 marks)*

(b) Using the result from part **(a)(ii)**, show that $n(n^2+2)$ is a multiple of 3 for any positive integer n. *(2 marks)*

3 The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 2, \qquad u_{n+1} = \frac{5u_n - 3}{3u_n - 1}$$

Prove by induction that, for all integers $n \geqslant 1$,

$$u_n = \frac{3n+1}{3n-1}$$ *(6 marks)*

4 (a) Given that $\mathrm{f}(r) = r^2(2r^2 - 1)$, show that

$$\mathrm{f}(r) - \mathrm{f}(r-1) = (2r-1)^3$$ *(3 marks)*

(b) Use the method of differences to show that

$$\sum_{r=n+1}^{2n} (2r-1)^3 = 3n^2(10n^2 - 1)$$ *(4 marks)*

3 (a) Express $(k+1)^2+5(k+1)+8$ in the form k^2+ak+b, where a and b are constants.

[1 mark]

(b) Prove by induction that, for all integers $n \geqslant 1$,

$$\sum_{r=1}^{n} r(r+1)\left(\frac{1}{2}\right)^{r-1} = 16-(n^2+5n+8)\left(\frac{1}{2}\right)^{n-1}$$

[6 marks]

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1 (a) Express $\frac{1}{(r+2)r!}$ in the form $\frac{A}{(r+1)!}+\frac{B}{(r+2)!}$, where A and B are integers.

[3 marks]

(b) Hence find $\sum_{r=1}^{n}\frac{1}{(r+2)r!}$.

[2 marks]

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4 The expression $f(n)$ is given by $f(n)=2^{4n+3}+3^{3n+1}$.

(a) Show that $f(k+1)-16f(k)$ can be expressed in the form $A\times 3^{3k}$, where A is an integer.

[3 marks]

(b) Prove by induction that $f(n)$ is a multiple of 11 for all integers $n \geqslant 1$.

[4 marks]

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1 (a) Given that $f(r)=\frac{1}{4r-1}$, show that

$$f(r)-f(r+1)=\frac{A}{(4r-1)(4r+3)}$$

where A is an integer.

[2 marks]

(b) Use the method of differences to find the value of $\sum_{r=1}^{50}\frac{1}{(4r-1)(4r+3)}$, giving your answer as a fraction in its simplest form.

[4 marks]

7 Given that $p \geqslant -1$, prove by induction that, for all integers $n \geqslant 1$,

$$(1+p)^n \geqslant 1+np$$

[6 marks]

2(a)	$1 = A(r+2)+Br$	M1		
	$2A = 1, \quad A = \frac{1}{2}$	A1		
	$A + B = 0, \quad B = -\frac{1}{2}$	A1	3	
(b)	$r = 10 \quad \frac{1}{2}\left(\frac{1}{10.11} - \frac{1}{11.12}\right)$			if (a) is incorrect but $A = \frac{1}{2}$ and $B = -\frac{1}{2}$ used, allow full marks for (b)
	$r = 11 \quad \frac{1}{2}\left(\frac{1}{11.12} - \frac{1}{12.13}\right)$			
	....			
	$r = 98 \quad \frac{1}{2}\left(\frac{1}{98.99} - \frac{1}{99.100}\right)$	M1A1		3 relevant rows seen
	$S = \frac{1}{2}\left(\frac{1}{10.11} - \frac{1}{99.100}\right)$	m1		if split into $\frac{1}{2r} - \frac{1}{r+1} + \frac{1}{2(r+2)}$, follow mark scheme, in which case $\frac{1}{2.10} - \frac{1}{2.11} + \frac{1}{2.100} - \frac{1}{2.99}$ scores m1
	$= \frac{89}{19800}$	A1	4	
	Total		7	

2(a)	$A=\frac{1}{2},\ B=-\frac{1}{2}$	B1, B1F	2	For either A or B For the other
(b)	Method of differences clearly shown	M1		
	$\text{Sum}=\frac{1}{2}\left(1-\frac{1}{2n+1}\right)$	A1		
	$=\frac{n}{2n+1}$	A1	3	AG
(c)	$\frac{1}{2(2n+1)}<0.001$ or $\frac{n}{2n+1}>0.499$	M1		Condone use of equals sign
	$1<0.004n+0.002$ or $n>0.998n+0.499$			
	$n>\frac{0.998}{0.004}$ or $0.004n>0.998$	A1		OE
	$n=250$	A1F	3	ft if say 0.4999 used If method of trial and improvement used, award full marks for a completely correct solution showing working
	Total		**8**	

2(a)	$\frac{1}{r(r+2)}=\frac{A}{r}+\frac{B}{r+2}$	M1		
	$A=\frac{1}{2},\ B=-\frac{1}{2}$	A1, A1F	3	ft incorrect A
(b)	$r=1\quad \frac{1}{1.3}=\frac{1}{2}\left(\frac{1}{1}-\frac{1}{3}\right)$			
	$r=2\quad \frac{1}{2.4}=\frac{1}{2}\left(\frac{1}{2}-\frac{1}{4}\right)$			
	$r=3\quad \frac{1}{3.5}=\frac{1}{2}\left(\frac{1}{3}-\frac{1}{5}\right)$	M1		3 rows (PI) numerical values only
	$r=48\quad \frac{1}{48.50}=\frac{1}{2}\left(\frac{1}{48}-\frac{1}{50}\right)$	A1F		Last row – could be implied
	Cancelling appropriate pairs	M1		
	$\text{Sum}=\frac{1}{2}\left(\frac{1}{1}+\frac{1}{2}-\frac{1}{49}-\frac{1}{50}\right)$	A1F		Allow if the $\frac{1}{2}$ is missing only
	$=\frac{894}{1225}$	A1	5	CAO (or equivalent fraction)
	Total		**8**	

Q	Solution	Marks	Total	Comments
6(a)	$\frac{1}{(k+2)!}=\frac{k+3}{(k+3)!}$	M1		
	Result	A1	2	
(b)	Assume true for $n=k$			
	For $n=k+1$			
	$\sum_{r=1}^{k+1}\frac{r\times 2^r}{(r+2)!}=1-\frac{2^{k+1}}{(k+2)!}+\frac{(k+1)2^{k+1}}{(k+3)!}$	M1A1		If no LHS of equation, M1A0
	$=1-2^{k+1}\left(\frac{1}{(k+2)!}-\frac{k+1}{(k+3)!}\right)$	m1		m1 for a suitable combination clearly shown
	$=1-\frac{2^{k+2}}{(k+3)!}$	A1		clearly shown or stated true for $n=k+1$
	True for $n=1$	B1		Shown
	Method of induction set out properly	E1	6	Provided previous 5 marks all earned
	Total		**8**	

2(a)	$u_r-u_{r-1}=$			
	$\frac{1}{6}r(r+1)(4r+11)-\frac{1}{6}(r-1)r(4r+7)$	M1		
	Correct expansion in any form, eg			
	$\frac{1}{6}r\left(4r^2+15r+11-4r^2-3r+7\right)$	A1		
	$=r(2r+3)$	A1	3	AG
(b)	Attempt to use method of differences	M1		
	$S_{100}=u_{100}-u_0$	A1		
	$=691850$	A1	3	CAO
	Total		**6**	

7(a)	$f(k+1)-5f(k)$			
	$=12^{k+1}+2\times5^k-5\left(12^k+2\times5^{k-1}\right)$	M1		
	$=12^{k+1}+2\times5^k-5\times12^k-2\times5^k$	A1		for expansion of bracket $5\times5^{k-1}=5^k$ used
	$=12\times12^k-5\times12^k=7\times12^k$	A1	3	clearly shown
(b)	Assume $f(k)=M(7)$			
	Then $f(k+1)=5f(k)+M(7)$	M1		Not merely a repetition of part (a)
	$=M(7)$	A1		clearly shown
	$f(1)=12+2=14=M(7)$	B1		
	Correct inductive process	E1	4	(award only if all 3 previous marks earned)
	Total		**7**	

Q	Solution	Marks	Total	Comments
3(a)	$(r+1)!=(r+1)r(r-1)!$	M1		
	Result	A1	2	AG
(b)	Attempt to use method of differences	M1		
	$\sum_{r=1}^{n}\left(r^2+r-1\right)(r-1)!=(n+1)!+n!-1!-0!$	A1		
	$(n+1)!=(n+1)n!$	m1		Must be seen
	$(n+2)n!-2$	A1	4	AG
	Total		**6**	

6(a)	Expansion of $(k+1)\left(4(k+1)^2-1\right)$	M1		Any valid method – first step correct
	$= 4k^3 + 12k^2 + 11k + 3$	A1	2	AG
(b)	Assume true for $n = k$			
	For $n = k+1$:			
	$\sum_{r=1}^{k+1}(2r-1)^2 = \frac{1}{3}k\left(4k^2-1\right)+(2k+1)^2$	M1A1		No LHS M1A0
	$= \frac{1}{3}\left(4k^3+12k^2+11k+3\right)$	A1F		ft error in $(2k + 1)$
	$= \frac{1}{3}(k+1)\left(4(k+1)^2-1\right)$	A1		Using part (a)
	True for $n = 1$ shown	B1		
	Proof by induction set out properly (if factorised by 3 linear factors, allow A1 at this particular point)	E1	6	Dependent on all marks correct
	Total		**8**	

4	Assume result true for $n = k$			
	Then $u_{k+1} = \dfrac{3}{4-\left(\dfrac{3^{k+1}-3}{3^{k+1}-1}\right)}$	M1		
	$= \dfrac{3(3^{k+1}-1)}{4(3^{k+1}-1)-(3^{k+1}-3)}$	A1		
	$4\times 3^{k+1} - 3^{k+1} = 3^{k+2}$	A1		clearly shown
	$u_{k+1} = \dfrac{3^{k+2}-3}{3^{k+2}-1}$	A1		
	$n=1$ $\quad \dfrac{3^2-3}{3^2-1} = \dfrac{3}{4} = u_1$	B1		
	Induction proof set out properly	E1	6	must have earned previous 5 marks
	Total		**6**	

3(a)	Attempt to put LHS over common denominator	M1		
	$\dfrac{2^{r+1}(r+1)-2^r(r+2)}{(r+1)(r+2)}$	A1		any form
	$= \dfrac{r(2^{r+1}-2^r)}{(r+1)(r+2)}$			
	$= \dfrac{r2^r}{(r+1)(r+2)}$ must see $r2^{r+1} = 2r2^r$	A1	3	clearly shown as AG
(b)	$\dfrac{2^2}{3} - \dfrac{2}{2}$ $\dfrac{2^3}{4} - \dfrac{2^2}{3}$ ………. $\dfrac{2^{31}}{32} - \dfrac{2^{30}}{31}$	M1		3 rows indicated (PI)
	$S_{30} = \dfrac{2^{31}}{32} - 1$ or $S_n = \dfrac{2^{n+1}}{n+2} - 1$	A1		
	$= 2^{26} - 1$	A1	3	CAO
	Total		**6**	

7(a)	Assume true for $n = k$			
	Then $\sum_{r=1}^{k+1} \frac{2r+1}{r^2(r+1)^2}$			
	$= 1 - \frac{1}{(k+1)^2} + \frac{2k+3}{(k+1)^2(k+2)^2}$	M1A1		M1A0 if no LHS
	$= 1 - \frac{1}{(k+1)^2}\left(1 - \frac{2k+3}{(k+2)^2}\right)$	m1		attempt to factorise or put over a common denominator
	$= 1 - \frac{1}{(k+1)^2}\left(\frac{k^2+2k+1}{(k+2)^2}\right)$	A1		any correct combination starting 1–
	$= 1 - \frac{1}{(k+2)^2}$	A1		
	True for $n = 1$ LHS = RHS = $\frac{3}{4}$	B1		
	Method of induction set out properly	E1	7	must score all 6 previous marks for this mark
(b)	$(n+1)^2 > 10^5$ or $\frac{1}{(n+1)^2} > 10^{-5}$	M1		Condone equals
	$n+1 > 316.2$ $n > 315.2$ $n = 316$	A1	2	
	Total		**9**	

3(a)	$\frac{1}{5r-2} - \frac{1}{5r+3} = \frac{5r+3-(5r-2)}{(5r-2)(5r+3)}$	M1		condone omission of brackets for M1
	$= \frac{5}{(5r-2)(5r+3)}$	A1cso	2	$A = 5$
(b)	Attempt to use method of differences	M1		at least 2 terms of correct form seen
	$k\left\{\frac{1}{3} - \frac{1}{5n+3}\right\}$	A1		correct cancellation leaving correct two fractions
	$k\left\{\frac{(5n+3)-3}{3(5n+3)}\right\}$	m1		attempt to write with common denominator
	$S_n = \frac{1}{5}\left\{\frac{(5n+3)-3}{3(5n+3)}\right\} = \frac{n}{3(5n+3)}$	A1cso	4	**AG** $k = \frac{1}{5}$ used correctly throughout
(c)	$S_\infty = \frac{1}{15}$	B1	1	
	Total		**7**	

7(a)(i)	$p(k+1) - p(k) = k^3 + (k+1)^3 + (k+2)^3$ $- (k-1)^3 - k^3 - (k+1)^3$	M1		
	$= (k+2)^3 - (k-1)^3$			
	$= k^3 + 6k^2 + 12k + 8 - (k^3 - 3k^2 + 3k - 1)$	A1		multiplied out & correct unsimplified
	$= 9k^2 + 9k + 9 \quad = 9(k^2 + k + 1)$ which is a multiple of 9 (since $k^2 + k + 1$ is an integer)	A1cso	3	correct algebra plus statement
(ii)	$p(1) = 1 + 8 = 9$ $\Rightarrow p(1)$ is a multiple of 9	B1		result true for $n = 1$
	$p(k+1) = p(k) + 9(k^2 + k + 1)$ *or* $p(k+1) = p(k) + 9N$	M1		$p(k+1) = \ldots$ **and** result from part (i) considered **and** mention of divisible by 9
	Assume $p(k)$ is a multiple of 9 so $p(k) = 9M$, where M is integer $\Rightarrow p(k+1) = 9M + 9N = 9(M + N)$ $\Rightarrow p(k+1)$ is a multiple of 9	A1		must have word such as "assume" for A1; convincingly shown
	Result true for $n = 1$ therefore true for $n = 2$, $n = 3$ etc by induction. (*or* $p(n)$ is a multiple of 9 for all integers $n \geqslant 1$)	E1	4	must earn previous 3 marks before E1 is scored
(b)	$p(n) = (n-1)^3 + n^3 + (n+1)^3$ $= 3n^3 + 6n$	B1		need to see this OE as evidence *or* $3n(n^2 + 2)$
	$p(n) = 3n(n^2 + 2)$ & $p(n)$ is a multiple of 9. Therefore $n(n^2 + 2)$ is a multiple of 3 (for any positive integer n.)	E1	2	both of these required plus concluding statement

Q	Solution	Marks	Total	Comments
3	$n=1$, $\frac{3+1}{3-1}=\frac{4}{2}=2$			be convinced they have used $u_n=\frac{3n+1}{3n-1}$
	($u_1=2$ so formula is) true when $n=1$	B1		
	Assume formula is true for $n=k$ (*)			
	$(u_{k+1}=)\ \dfrac{5\frac{3k+1}{3k-1}-3}{3\frac{3k+1}{3k-1}-1}$	M1		clear attempt at RHS of this formula
		m1		clear attempt to remove "double fraction"
	$(u_{k+1}=)\ \dfrac{5(3k+1)-3(3k-1)}{3(3k+1)-(3k-1)}$	A1		$\frac{6k+8}{6k+4}$
	$u_{k+1}=\frac{3k+4}{3k+2}$ *or* $u_{k+1}=\frac{3(k+1)+1}{3(k+1)-1}$ Hence formula is true for $n = k+1$ (**)	A1cso		must have "$u_{k+1}=$" on at least this line
	must have lines (*) & (**) **and** "Result true for $n=1$ therefore true for $n=2$, $n=3$ etc by induction."	E1	6	must also have earned previous 5 marks before E1 is scored
	Total		**6**	

4(a)	$f(r)-f(r-1)=$			
	$r^2(2r^2-1)-(r-1)^2(2(r-1)^2-1)$			condone one slip here
	$=2r^4-r^2-(r^2-2r+1)(2r^2-4r+1)$	M1		attempt to multiply out "their" $f(r-1)$
	$=2r^4-r^2-(2r^4-8r^3+11r^2-6r+1)$	A1		$f(r)$ & $f(r-1)$ expanded correctly condone correct unsimplified
	$=8r^3-12r^2+6r-1$ $=(2r-1)^3$	A1cso	3	**AG**
(b)	Attempt to use method of differences	M1		
	$f(2n)-f(n)$	m1		$(2n)^2\{2(2n^2)-1\}-n^2(2n^2-1)$
	$f(2n)-f(n)=4n^2(8n^2-1)-n^2(2n^2-1)$	A1		
	$=30n^4-3n^2$ $=3n^2(10n^2-1)$	A1cso	4	**AG** be convinced
	Total		**7**	

3 (a)	$k^2 + 7k + 14$	**B1**	**1**	
(b)	When $n = 1$ LHS = $1\times2\times1 = 2$ RHS = $16 - 14 = 2$ Therefore true for $n = 1$	**B1**		
	Assume formula is true for $n=k$ (*) Add $(k+1)$th term (to both sides) $\sum_{r=1}^{k+1} r(r+1)\left(\frac{1}{2}\right)^{r-1}$	**M1**		$(k+1)$th term must be correct
	$= 16 - \left(k^2+5k+8\right)\left(\frac{1}{2}\right)^{k-1}$ $+(k+1)(k+2)\left(\frac{1}{2}\right)^{k}$	**A1**		**A0** if only considering RHS
	$=16-\left(\frac{1}{2}\right)^k\left(2k^2+10k+16-k^2-3k-2\right)$ $=16-\left(\frac{1}{2}\right)^k\left(k^2+7k+14\right)$	**A1**		
	$=16-\left(\left(k+1\right)^2+5\left(k+1\right)+8\right)\left(\frac{1}{2}\right)^k$	**A1**		from part (a)
	Hence formula is true when $n = k+1$ (**) but true for $n = 1$ so true for $n = 2, 3, \ldots$ by induction (***)	**E1**	**6**	must have (*), (**) and (***) and must have earned previous 5 marks
	Total		**7**	

Q1	Solution	Mark	Total	Comment
(a)	$r+1 = A(r+2)+B$ **or** $1 = \frac{A(r+2)}{r+1} + \frac{B}{r+1}$	**M1**		**OE** with factorials removed
	either $A=1$ or $B=-1$	**A1**		**correctly** obtained
	$\frac{1}{(r+2)r!} = \frac{1}{(r+1)!} - \frac{1}{(r+2)!}$	**A1**	**3**	allow if seen in part **(b)**
(b)	$\frac{1}{2!}-\frac{1}{3!}+\frac{1}{3!}-\frac{1}{4!}+\ldots$ $\frac{1}{(n+1)!} - \frac{1}{(n+2)!}$	**M1**		use of their result from part **(a)** at least twice
	Sum $= \frac{1}{2} - \frac{1}{(n+2)!}$	**A1**	**2**	must simplify 2! and must have scored at least **M1 A1** in part **(a)**
	Total		**5**	

Q4	Solution	Mark	Total	Comment
(a)	$f(k+1) = 2^{4k+7} + 3^{3k+4}$	M1		
	convincingly showing $2^{4k+7} = 16 \times 2^{4k+3}$ $f(k+1) - 16f(k)$ $= (81 - 16 \times 3) \times 3^{3k}$	E1		must see $16 = 2^4$ OE
	$= 33 \times 3^{3k}$	A1	3	
(b)	$f(1) = 209$ therefore $f(1)$ is a multiple of 11	B1		$f(1) = 209 = 11 \times 19$ *or* $209 \div 11 = 19$ etc therefore true when n=1
	Assume $f(k)$ is a multiple of 11 (*) $f(k+1) = 16f(k) + 33 \times 3^{3k}$	M1		attempt at $f(k+1) = \ldots$ using their result from part **(a)**
	$= 11M + 11N = 11(M+N)$ Therefore $f(k+1)$ is a multiple of 11	A1		where M and N are integers
	Since f(1) is multiple of 11 then f(2), f(3),… are multiples of 11 by induction (or is a multiple of 11 for all integers $n \geq 1$)	E1	4	must earn previous 3 marks and have (*) before **E1** can be awarded
	Total		7	

Q1	Solution	Mark	Total	Comment
(a)	$f(r) - f(r+1) = \frac{1}{4r-1} - \frac{1}{4(r+1)-1}$	M1		or $\frac{1}{4r-1} - \frac{1}{4r+3}$
	$= \frac{4}{(4r-1)(4r+3)}$	A1	2	
(b)	$\frac{1}{3} - \frac{1}{7} + \frac{1}{7} - \frac{1}{11} + \ldots$ **OE** **or** $f(1) - f(2) + f(2) - f(3) + \ldots$	M1		Clear attempt to use **method of differences** possibly with one error **PI** by first **A1**
	$\sum_{r=1}^{50} [f(r) - f(r+1)] = f(1) - f(51)$			
	$= \frac{1}{3} - \frac{1}{203}$	A1		
	$\sum_{r=1}^{50} \frac{1}{(4r-1)(4r+3)} = \frac{1}{4}\left(\frac{1}{3} - \frac{1}{203}\right)$	m1		"*their*" $\frac{1}{4} \times$ "*their*" $\left(\frac{1}{3} - \frac{1}{203}\right)$
	$= \frac{50}{609}$	A1	4	
	Total		6	

Q7	Solution	Mark	Total	Comment
	$n=1$; LHS $=1+p$; RHS $=1+p$ Therefore result is true when $n=1$	**B1**		
	Assume inequality is true for $n = k$ (*)			
	Multiply both sides by $1+p$ $(1+p)^{k+1} \ldots (1+kp)(1+p)$ Inequality only valid since multiplication by positive number because $1+p \ldots 0$	**E1**		and stating $1+p \ldots 0$ before multiplying both sides by $1+p$ or justifying why inequality remains …
	Considering $(1+kp)(1+p)$	**M1**		and attempt to multiply out
	RHS $= 1+kp+p+kp^2$	**A1**		
	RHS $\ldots 1+kp+p$ $\Rightarrow (1+p)^{k+1} \ldots 1+(k+1)p$	**A1**		must have … correct algebra and inequalities throughout
	Hence inequality is true when $n = k+1$ (**) but true for $n = 1$ so true for $n = 2, 3, \ldots$ by induction (***) (or true for all integers $n \ldots 1$ (***))	**E1**	**6**	must have (*), (**) and (***) and must have earned previous **B1, M1, A1, A1** marks
	Total		**6**	