FP2 Inverse Trig

6 By using the substitution $u = x - 2$, or otherwise, find the exact value of

$$\int_{-1}^{5} \frac{dx}{\sqrt{32 + 4x - x^2}}$$ *(5 marks)*

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5 (a) Given that $u = \sqrt{1 - x^2}$, find $\frac{du}{dx}$. *(2 marks)*

(b) Use integration by parts to show that

$$\int_0^{\frac{\sqrt{3}}{2}} \sin^{-1} x \, dx = a\sqrt{3}\pi + b$$

where a and b are rational numbers. *(6 marks)*

6 (a) Express $7 + 4x - 2x^2$ in the form $a - b(x - c)^2$, where a, b and c are integers. *(2 marks)*

(b) By means of a suitable substitution, or otherwise, find the exact value of

$$\int_1^{\frac{5}{2}} \frac{dx}{\sqrt{7 + 4x - 2x^2}}$$ *(6 marks)*

5 The function f, where $f(x) = \sec x$, has domain $0 \leqslant x < \frac{\pi}{2}$ and has inverse function f^{-1}, where $f^{-1}(x) = \sec^{-1} x$.

(a) Show that

$$\sec^{-1} x = \cos^{-1} \frac{1}{x}$$ *(2 marks)*

(b) Hence show that

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{\sqrt{x^4 - x^2}}$$ *(4 marks)*

7 (a) Given that $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$ and $x \neq 1$, show that $\frac{dy}{dx} = \frac{1}{1+x^2}$.

[4 marks]

(b) Hence, given that $x < 1$, show that $\tan^{-1}\left(\frac{1+x}{1-x}\right) - \tan^{-1} x = \frac{\pi}{4}$.

[3 marks]

6	$u = x - 2$			
	$du = dx$ or $\frac{du}{dx} = 1$	B1		clearly seen
	$32 + 4x - x^2 = 36 - u^2$	B1		if $32 + 4x - x^2$ is written as $36 - (x-2)^2$, give B2
	$\int \frac{du}{\sqrt{36 - u^2}} = \sin^{-1}\frac{u}{6}$	M1		allow if dx is used instead of du
	limits -3 and 3 or substitute back to give $\sin^{-1}\frac{x-2}{6}$	A1		
	$I = \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$	A1	5	

5(a)	$\frac{du}{dx} = \frac{1}{2}(1 - x^2)^{-\frac{1}{2}}$	B1		
	$\times(-2x)$	B1	2	
(b)	$\int \sin^{-1} x \, dx = x\sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} dx$	M1 A1A1		A1 for each part of the integration by parts
	$\int -\frac{x}{\sqrt{1-x^2}} dx = \sqrt{1-x^2}$ used	A1F		ft sign error in $\frac{du}{dx}$
	$\frac{\sqrt{3}}{2}\frac{\pi}{3} + \sqrt{1 - \frac{3}{4}} - 1$	m1		substitution of limits
	$\frac{1}{6}\sqrt{3}\pi - \frac{1}{2}$	A1	6	CAO

6(a)	$\frac{dx}{dt} = \sec t - \cos t$	B1,B1		use of FB for $\sec t$; if done from first principles, allow B1 when $\sec t$ is arrived at
	Use of $1 - \cos^2 t = \sin^2 t$	M1		
	$\frac{dx}{dt} = \sin t \tan t$	A1	4	AG
(b)	$\dot{x}^2 + \dot{y}^2 = \sin^2 t \tan^2 t + \sin^2 t$	M1A1		sign error in $\frac{dy}{dt}$ A0
	Use of $1 + \tan^2 t = \sec^2 t$	m1		
	$\sqrt{\dot{x}^2 + \dot{y}^2} = \tan t$	A1F		ft sign error in $\frac{dy}{dt}$
	$\int_0^{\frac{\pi}{3}} \tan t \, dt = [\ln \sec t]_0^{\frac{\pi}{3}}$	A1F		ft sign error in $\frac{dy}{dt}$
	$= \ln 2$	A1	6	CAO
	Total		**10**	

5(a)	$\frac{1}{x}=\cos y$ or $\frac{1}{y}=\cos x$	M1		
	$y=\cos^{-1}\frac{1}{x}$ ie result	A1	2	CSO
(b)	$\frac{d}{dx}\left(\sec^{-1}x\right)=\frac{d}{dx}\left(\cos^{-1}\frac{1}{x}\right)$	M1		
	$=-\frac{1}{\sqrt{1-\frac{1}{x^2}}}$ if in terms of u A0	A1		
	$\times\left(-\frac{1}{x^2}\right)$	A1		
	$=\frac{1}{\sqrt{x^4-x^2}}$	A1	4	clearly shown (AG)
	Alternative			
	$\cos y=\frac{1}{x}$			Use of $\sec y=x$ M0
	$-\sin y\frac{dy}{dx}=\frac{-1}{x^2}$	(M1) (A1)		
	Substitute for $\sin y$	(A1)		
	Result	(A1)		
	Total		**6**	

7 (a)	$\frac{d}{dx}\left(\frac{1+x}{1-x}\right)=\frac{1-x+1+x}{(1-x)^2}=\frac{2}{(1-x)^2}$	**B1**		**ACF**
	$\frac{dy}{dx}=\frac{1}{1+u^2}$	**M1**		where $u=\frac{1+x}{1-x}$
	$\times\frac{2}{(1-x)^2}$	**A1**		correct unsimplified
	$=\frac{2}{(1-x)^2+(1+x)^2}=\frac{1}{1+x^2}$	**A1**	**4**	**AG** be convinced
(b)	either $\frac{d}{dx}(\tan^{-1}x)=\frac{1}{1+x^2}$ or $\int\frac{1}{1+x^2}dx=\tan^{-1}x\quad(+c)$	**B1**		
	$\Rightarrow\tan^{-1}\left(\frac{1+x}{1-x}\right)=\tan^{-1}x+C$	**M1**		
	Putting $x=0$ gives $C=\tan^{-1}1=\frac{\pi}{4}$			
	$\Rightarrow\tan^{-1}\left(\frac{1+x}{1-x}\right)-\tan^{-1}x=\frac{\pi}{4}$	**A1**	**3**	**AG**
	Total		**7**	