FP2 Arc length and area of surface of revolution

5 (a) Use the definition $\cosh x = \frac{1}{2}(e^x + e^{-x})$ to show that $\cosh 2x = 2\cosh^2 x - 1$.

(2 marks)

(b) (i) The arc of the curve $y = \cosh x$ between $x = 0$ and $x = \ln a$ is rotated through 2π radians about the x-axis. Show that S, the surface area generated, is given by

$$S = 2\pi \int_0^{\ln a} \cosh^2 x \, dx$$

(3 marks)

(ii) Hence show that

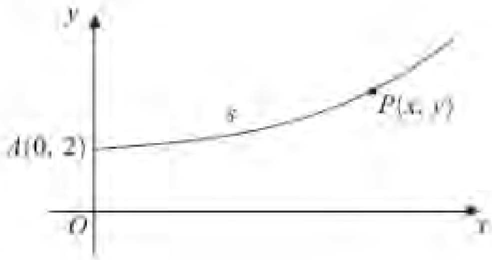
$$S = \pi\left(\ln a + \frac{a^4 - 1}{4a^2}\right)$$

(5 marks)

7 The diagram shows a curve which starts from the point A with coordinates $(0, 2)$. The curve is such that, at every point P on the curve,

$$\frac{dy}{dx} = \frac{1}{2}s$$

where s is the length of the arc AP.



(a) (i) Show that

$$\frac{ds}{dx} = \frac{1}{2}\sqrt{4 + s^2}$$

(3 marks)

(ii) Hence show that

$$s = 2\sinh\frac{x}{2}$$

(4 marks)

(iii) Hence find the cartesian equation of the curve.

(3 marks)

(b) Show that

$$y^2 = 4 + s^2$$

(2 marks)

5 (a) Using the identities

$$\cosh^2 t - \sinh^2 t = 1, \quad \tanh t = \frac{\sinh t}{\cosh t} \quad \text{and} \quad \operatorname{sech} t = \frac{1}{\cosh t}$$

show that:

(i) $\tanh^2 t + \operatorname{sech}^2 t = 1$; *(2 marks)*

(ii) $\frac{d}{dt}(\tanh t) = \operatorname{sech}^2 t$; *(3 marks)*

(iii) $\frac{d}{dt}(\operatorname{sech} t) = -\operatorname{sech} t \tanh t$. *(3 marks)*

(b) A curve C is given parametrically by

$$x = \operatorname{sech} t, \quad y = 4 - \tanh t$$

(i) Show that the arc length, s, of C between the points where $t = 0$ and $t = \frac{1}{2}\ln 3$ is given by

$$s = \int_0^{\frac{1}{2}\ln 3} \operatorname{sech} t \, dt$$ *(4 marks)*

(ii) Using the substitution $u = e^t$, find the exact value of s. *(6 marks)*

6 (a) Given that

$$x = \ln(\sec t + \tan t) - \sin t$$

show that

$$\frac{dx}{dt} = \sin t \tan t$$ *(4 marks)*

(b) A curve is given parametrically by the equations

$$x = \ln(\sec t + \tan t) - \sin t, \quad y = \cos t$$

The length of the arc of the curve between the points where $t = 0$ and $t = \frac{\pi}{3}$ is denoted by s.

Show that $s = \ln p$, where p is an integer. *(6 marks)*

5 (a) The arc of the curve $y^2 = x^2 + 8$ between the points where $x = 0$ and $x = 6$ is rotated through 2π radians about the x-axis. Show that the area S of the curved surface formed is given by

$$S = 2\sqrt{2}\pi \int_0^6 \sqrt{x^2 + 4}\,\mathrm{d}x$$ *(5 marks)*

(b) By means of the substitution $x = 2\sinh\theta$, show that

$$S = \pi(24\sqrt{5} + 4\sqrt{2}\sinh^{-1} 3)$$ *(8 marks)*

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6 (a) Show that

$$\frac{1}{4}(\cosh 4x + 2\cosh 2x + 1) = \cosh^2 x \cosh 2x$$ *(3 marks)*

(b) Show that, if $y = \cosh^2 x$, then

$$1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = \cosh^2 2x$$ *(3 marks)*

(c) The arc of the curve $y = \cosh^2 x$ between the points where $x = 0$ and $x = \ln 2$ is rotated through 2π radians about the x-axis. Show that the area S of the curved surface formed is given by

$$S = \frac{\pi}{256}(a\ln 2 + b)$$

where a and b are integers. *(7 marks)*

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6 A curve is defined parametrically by

$$x = t^3 + 5, \qquad y = 6t^2 - 1$$

The arc length between the points where $t = 0$ and $t = 3$ on the curve is s.

(a) Show that $s = \int_0^3 3t\sqrt{t^2 + A}\,\mathrm{d}t$, stating the value of the constant A. *(4 marks)*

(b) Hence show that $s = 61$. *(4 marks)*

7 (a) (i) Show that

$$\frac{\mathrm{d}}{\mathrm{d}u}\left(2u\sqrt{1+4u^2}+\sinh^{-1}2u\right)=k\sqrt{1+4u^2}$$

where k is an integer. *(4 marks)*

(ii) Hence show that

$$\int_0^1 \sqrt{1+4u^2}\,\mathrm{d}u = p\sqrt{5}+q\sinh^{-1}2$$

where p and q are rational numbers. *(2 marks)*

(b) The arc of the curve with equation $y=\frac{1}{2}\cos 4x$ between the points where $x=0$ and $x=\frac{\pi}{8}$ is rotated through 2π radians about the x-axis.

(i) Show that the area S of the curved surface formed is given by

$$S=\pi\int_0^{\frac{\pi}{8}}\cos 4x\sqrt{1+4\sin^2 4x}\,\mathrm{d}x$$ *(2 marks)*

(ii) Use the substitution $u=\sin 4x$ to find the exact value of S. *(4 marks)*

3 A curve C is defined parametrically by

$$x = \frac{t^2+1}{t}, \quad y = 2\ln t$$

(a) Show that $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \left(1+\frac{1}{t^2}\right)^2$.

[4 marks]

(b) The arc of C from $t=1$ to $t=2$ is rotated through 2π radians about the x-axis. Find the area of the surface generated, giving your answer in the form $\pi(m\ln 2 + n)$, where m and n are integers.

[5 marks]

8 A curve has equation $y = 2\sqrt{x-1}$, where $x > 1$. The length of the arc of the curve between the points on the curve where $x = 2$ and $x = 9$ is denoted by s.

(a) Show that $s = \int_2^9 \sqrt{\frac{x}{x-1}}\, dx$.

[3 marks]

(b) (i) Show that $\cosh^{-1} 3 = 2\ln(1+\sqrt{2})$.

[2 marks]

(ii) Use the substitution $x = \cosh^2\theta$ to show that

$$s = m\sqrt{2} + \ln(1+\sqrt{2})$$

where m is an integer.

[6 marks]

3 The arc of the curve with equation $y = 4 - \ln(1-x^2)$ from $x = 0$ to $x = \frac{3}{4}$ has length s.

(a) Show that $s = \int_0^{\frac{3}{4}} \left(\frac{1+x^2}{1-x^2}\right) dx$.

[4 marks]

(b) Find the value of s, giving your answer in the form $p + \ln N$, where p is a rational number and N is an integer.

[6 marks]

Q	Solution	Marks	Total	Comments
5(a)	$\left(e^x + e^{-x}\right)^2$ expanded correctly	B1		$e^{2x} + 2e^0 + e^{-2x}$ is acceptable
	Result	B1	2	AG
(b)(i)	$\frac{dy}{dx} = \sinh x$	B1		
	$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\sinh^2 x}$			
	$= \cosh x$	M1		use of $\cosh^2 x - \sinh^2 x = 1$
	$S = 2\pi\int_0^{\ln a} \cosh^2 x \, dx$	A1	3	AG (clearly derived)
(ii)	Use of $\cosh^2 x = \frac{1}{2}(1+\cosh 2x)$	M1		allow one slip in formula M0 if $\int \cosh^2 x \, dx$ is given as $\sinh^2 x$
	$S = \pi\left[x + \frac{1}{2}\sinh 2x\right]_0^{\ln a}$	A1		
	$= \pi\left[\ln a + \frac{1}{2}\left(\frac{e^{2\ln a} - e^{-2\ln a}}{2}\right)\right]$	M1		
	$= \pi\left[\ln a + \frac{1}{4}\left(a^2 - a^{-2}\right)\right]$	A1F		
	$= \pi\left[\ln a + \frac{1}{4a^2}\left(a^4 - 1\right)\right]$	A1	5	AG
	Total		**10**	

7(a)(i)	$\frac{ds}{dx} = \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\left(\frac{s}{2}\right)^2}$	M1A1		Allow M1 for $s = \int\sqrt{1+\left(\frac{s}{2}\right)^2}\,dx$ then A1 for $\frac{dy}{dx}$
	$= \frac{1}{2}\sqrt{4+s^2}$	A1	3	AG
(ii)	$\int \frac{ds}{\sqrt{4+s^2}} = \int \frac{1}{2}\, dx$	M1		For separation of variables; allow without integral sign
	$\sinh^{-1}\frac{s}{2} = \frac{1}{2}x + C$	A1		Allow if C is missing
	$C = 0$	A1		
	$s = 2\sinh\frac{1}{2}x$	A1	4	AG if C not mentioned allow $\frac{3}{4}$ **SC** incomplete proof of (a)(ii), differentiating $s = 2\sinh\frac{x}{2}$ to arrive at $\frac{ds}{dx} = \frac{1}{2}\sqrt{4+s^2}$ allow M1A1 only $\left(\frac{2}{4}\right)$
(iii)	$\frac{dy}{dx} = \sinh\frac{1}{2}x$	M1		
	$y = 2\cosh\frac{1}{2}x + C$	A1		Allow if C is missing
	$C = 0$	A1	3	Must be shown to be zero and CAO
(b)	$y^2 = 4\left(1+\sinh^2\frac{x}{2}\right)$	M1		Use of $\cosh^2 = 1 + \sinh^2$
	$= 4 + s^2$	A1	2	AG
	Total		**12**	

5(a)(i)	Divide $\cosh^2 t - \sinh^2 t = 1$ by $\cosh^2 t$	M1		Or $\frac{\sinh^2 t}{\cosh^2 t} + \frac{1}{\cosh^2 t}$
	Rearrange	A1	2	AG If solved back to front with no conclusion ending $\cosh^2 t - \sinh^2 t = 1$ B1 only
(ii)	$\frac{d}{dt}\left(\frac{\sinh t}{\cosh t}\right) = \frac{\cosh^2 t - \sinh^2 t}{\cosh^2 t}$	M1A1		
	$= \text{sech}^2 t$	A1	3	AG
(iii)	$\frac{d}{dt}(\text{sech}\, t) = -(\cosh t)^{-2} \sinh t$	M1A1		Allow A1 if negative sign missing
	$= -\text{sech}\, t \tanh t$	A1	3	AG
(b)(i)	$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \text{sech}^4 t + \text{sech}^2 t \tanh^2 t$	M1		Allow slips of sign before squaring for this M1
	Use of $\tanh^2 t + \text{sech}^2 t = 1$	m1		Correct formula only for m1
	$= \text{sech}^2 t$	A1		
	$\therefore s = \int_0^{\frac{1}{2}\ln 3} \text{sech}\, t \, dt$	A1	4	AG (including limits)
(ii)	$u = e^t \quad du = e^t dt$	B1		
	$\int \text{sech}\, t \, dt = \int \frac{2}{u^2+1} du$	M1A1		CAO M1 for putting integrand in terms of u (<u>no</u> sech (ln$u$))
	$\left[2\tan^{-1} u\right]$	A1		Or $2\tan^{-1} e^t$
	Change limits correctly or change back to t	m1		At some stage
	$= \frac{2\pi}{3} - \frac{2\pi}{4} = \frac{\pi}{6}$	A1	6	CAO
	Total		**18**	

6(a)	$\frac{dx}{dt} = \sec t - \cos t$	B1,B1		use of FB for $\sec t$; if done from first principles, allow B1 when $\sec t$ is arrived at
	Use of $1 - \cos^2 t = \sin^2 t$	M1		
	$\frac{dx}{dt} = \sin t \tan t$	A1	4	AG
(b)	$\dot{x}^2 + \dot{y}^2 = \sin^2 t \tan^2 t + \sin^2 t$	M1A1		sign error in $\frac{dy}{dt}$ A0
	Use of $1 + \tan^2 t = \sec^2 t$	m1		
	$\sqrt{\dot{x}^2 + \dot{y}^2} = \tan t$	A1F		ft sign error in $\frac{dy}{dt}$
	$\int_0^{\frac{\pi}{3}} \tan t \, dt = \left[\ln \sec t\right]_0^{\frac{\pi}{3}}$	A1F		ft sign error in $\frac{dy}{dt}$
	$= \ln 2$	A1	6	CAO
	Total		**10**	

Q	Solution	Marks	Total	Comments
5(a)	$2y\frac{dy}{dx}=2x$	B1		Or $\frac{dy}{dx}=x\left(x^2+8\right)^{-\frac{1}{2}}$
	$S=2\pi\int_0^6 y\sqrt{1+\frac{x^2}{y^2}}\,dx$	M1 A1F		M1 for use of formula provided $\frac{dy}{dx}$ is a function of x A1 for substitution for $\frac{dy}{dx}$ (one slip)
	Eliminating all y	m1		
	$=2\sqrt{2}\pi\int_0^6\sqrt{x^2+4}\,dx$	A1	5	AG
(b)	$dx=2\cosh\theta\,d\theta$ or $\frac{dx}{d\theta}=2\cosh\theta$	B1		
	$S=2\sqrt{2}\pi\int\sqrt{4\sinh^2 x+4}\,.\,2\cosh\theta\,d\theta$	M1		For eliminating x completely and use of $d\theta$, ie $d\theta$ attempted
	$S=\left(2\sqrt{2}\right)\pi\int 2\cosh\theta.\,2\cosh\theta\,d\theta$	m1		Use of $\cosh^2\theta-\sinh^2\theta=1$ (ignore limits)
	$=4\sqrt{2}\pi\int(\cosh 2\theta+1)\,d\theta$	m1		Use of formula for $\cosh 2\theta$; must be correct
	$=4\sqrt{2}\pi\left[\frac{\sinh 2\theta}{2}+\theta\right]$	B1F		Correct integration of $a\cosh 2\theta+b$
	$=4\sqrt{2}\pi[\sinh\theta\cosh\theta+\theta]$	m1		Use of $\sinh 2\theta=2\sinh\theta\cosh\theta$ Must be seen
	$=4\sqrt{2}\pi\left[\frac{x}{2}\sqrt{\frac{x^2}{4}+1}+\sinh^{-1}\frac{x}{2}\right]_0^6$	M1		Or change limits
	$=\pi\left[24\sqrt{5}+4\sqrt{2}\sinh^{-1}3\right]$	A1	8	AG
	Total		**13**	

Q	Solution	Marks	Total	Comments
6(a)	$7+4x-2x^2=9-2(x-1)^2$	M1A1	2	
(b)	Put $u=\sqrt{2}(x-1)$	M1		allow $u=k(x-1)$ any k
	$du=\sqrt{2}\,dx$	A1F		
	$I=\frac{1}{\sqrt{2}}\int\frac{du}{\sqrt{9-u^2}}$	A1F		ft error in (a); must have u^2 only, ie $\frac{1}{\sqrt{2}}$ outside integrand
	$=\frac{1}{\sqrt{2}}\sin^{-1}\frac{u}{3}$	A1		for $\sin^{-1}\frac{u}{p}$
	Change limits or replace u	m1		provided $\sin^{-1}$
	$=\frac{\pi}{4\sqrt{2}}$ or $\frac{\pi\sqrt{2}}{8}$	A1	6	CAO
	Alternative – if integration is attempted without substitution:			
	$\sin^{-1}$	(M1)		
	$\frac{1}{\sqrt{2}}$	(A1F)		
	$(x-1)$	(A1)		
	$\frac{\sqrt{2}}{3}$	(A1F)		
	Substitution of limits	(m1)		
	$\frac{\pi}{4\sqrt{2}}$	(A1)	(6)	CAO
	Total		**8**	

6(a)	Use of $\cosh 2x = 2\cosh^2 x - 1$	M1		or $\cosh 4x = 2\cosh^2 2x - 1$
	$\text{RHS} = \frac{1}{2}\cosh 2x + \frac{1}{2}\cosh^2 2x$	A1		
	$= \frac{1}{4}(1 + 2\cosh 2x + \cosh 4x)$	A1	3	
	If substituted for both $\cosh 4x$ and $\cosh 2x$ in LHS M1 only, until corrected If RHS is put in terms of e^x M1 for correct substitution A1 for correct expansion A1 for correct result			
(b)	$\frac{dy}{dx} = 2\cosh x \sinh x = \sinh 2x$	M1A1		allow A1 for $1 + \left(\frac{dy}{dx}\right)^2 = 1 - 4\cosh^2 x + 4\cosh^4 x$ Incorrect form for $\cosh^2 x$ in terms of $\cosh 2x$ M1 only
	Or $y = \left(\frac{e^x + e^{-x}}{2}\right)^2 = \frac{e^{2x} + 2 + e^{-2x}}{4}$			
	$\frac{dy}{dx} = \frac{2e^{2x} - 2e^x}{4}$	(M1)		
	$= \sinh 2x$	(A1)		
	$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \sinh^2 2x = \cosh^2 2x$	A1	3	AG
(c)	$S = 2\pi \int_{(0)}^{(\ln 2)} \cosh^2 x \cosh 2x \, dx$	M1A1		allow even if limits missing
	$= 2\pi \int_0^{\ln 2} \frac{1}{4}(1 + 2\cosh 2x + \cosh 4x)\,dx$	m1		
	$= \frac{2\pi}{4}\left[x + \frac{2\sinh 2x}{2} + \frac{\sinh 4x}{4}\right]$	A1		Integrated correctly
	Correct use of limits	m1		
	$a = 128$, $b = 495$	A1,A1	7	accept correct answers written down with no working. Only one A1 if 2π not used
	Total		**13**	

Q	Solution	Marks	Total	Comments
6(a)	$\frac{dx}{dt} = 3t^2 \quad \frac{dy}{dt} = 12t$	B1		both correct
	$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 9t^4 + 144t^2$	M1		'their' $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2$
	$s = \int \sqrt{9t^4 + 144t^2}\,(dt)$	A1		OE
	$s = \int_0^3 3t\sqrt{t^2 + 16}\,dt$	A1cso	4	$A = 16$
(b)	$k(t^2 + A)^{\frac{3}{2}}$	M1		where k is a constant; ft their A
	$(t^2 + 16)^{\frac{3}{2}}$	A1		
	$25^{\frac{3}{2}} - 16^{\frac{3}{2}}$	m1		$F(3) - F(0)$
	$= 61$	A1 cso	4	**AG**
	Total		**8**	

7(a)(i)	$\frac{d}{du}\left(2u\sqrt{1+4u^2}\right)=\frac{8u^2}{\sqrt{1+4u^2}}+2\sqrt{1+4u^2}$	M1		M1 for clear use of product rule (condone one error in one term)
		A1		correct unsimplified
	$\frac{d}{du}\left(\sinh^{-1}2u\right)=\frac{2}{\sqrt{1+4u^2}}$	B1		
	$\frac{8u^2+2}{\sqrt{1+4u^2}}=\frac{2(1+4u^2)}{\sqrt{1+4u^2}}=2\sqrt{1+4u^2}$			be convinced – must see this line OE
	$\frac{d}{du}\left(2u\sqrt{1+4u^2}+4\sinh^{-1}2u\right)=4\sqrt{1+4u^2}$	A1cso	4	all working must be correct (not enough to just say $k=4$)
(ii)	$\frac{1}{\text{"their"}k}\left[2u\sqrt{1+4u^2}+\sinh^{-1}2u\right]_0^1$	M1		anti differentiation
	$=\frac{\sqrt{5}}{2}+\frac{1}{4}\sinh^{-1}2$	A1√	2	FT "their" k or even use of k
(b)(i)	$y=\frac{1}{2}\cos 4x$ and $\frac{dy}{dx}=A\sin 4x$			$\frac{dy}{dx}=-2\sin 4x$
	substituted into $\int K\,y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,(dx)$	M1		clear attempt to use formula for CSA
	$(S=)\ \int_0^{\frac{\pi}{8}}2\pi\times\frac{1}{2}\cos 4x\sqrt{1+4\sin^2 4x}\,dx$ = printed answer (combining $2\times\frac{1}{2}$)	A1cso	2	**AG** $\frac{dy}{dx}=-2\sin 4x$ **and** $2\times\frac{1}{2}$ **and** dx must be seen to award A1cso
(ii)	$u=\sin 4x\Rightarrow du=4\cos 4x\,dx$	M1		condone $du=B\cos 4x\,dx$ for M1
	$(S=)\ \frac{\pi}{4}\int_0^1\sqrt{1+4u^2}\,(du)$	A1		condone limits seen later
		m1		use of their result from (a)(ii) **correctly** FT "their" B
	$(S=)\ \frac{\pi\sqrt{5}}{8}+\frac{\pi}{16}\sinh^{-1}2$	A1cso	4	OE
	Total		**12**	

8(a)	$y = 2(x-1)^{\frac{1}{2}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = (x-1)^{-\frac{1}{2}}$	B1		
	$1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = 1+\frac{1}{x-1}$	M1		ft their $\frac{\mathrm{d}y}{\mathrm{d}x}$
	$(s=)\int_{(2)}^{(9)} \sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,(\mathrm{d}x) \quad (=)$			$s = \int_2^9 \sqrt{1+\frac{1}{x-1}}\,\mathrm{d}x$
	$\int_2^9 \sqrt{\frac{x}{x-1}}\,\mathrm{d}x$	A1	3	(be convinced) **AG** (must have limits & dx on final line)
(b)(i)	$\cosh^{-1} 3 = \ln\left(3+\sqrt{8}\right)$	M1		
	$\left(1+\sqrt{2}\right)^2 = 3+2\sqrt{2} = 3+\sqrt{8}$			need to see this line OE
	$\cosh^{-1} 3 = \ln\left(1+\sqrt{2}\right)^2 = 2\ln\left(1+\sqrt{2}\right)$	A1	2	**AG** (be convinced)
(ii)	$x = \cosh^2\theta \Rightarrow \mathrm{d}x = 2\cosh\theta\sinh\theta\,\mathrm{d}\theta$	M1		$\frac{\mathrm{d}x}{\mathrm{d}\theta} = k\cosh\theta\sinh\theta$ **OE**
	$(s=)\int \frac{\cosh\theta}{\sinh\theta} 2\cosh\theta\sinh\theta\,\mathrm{d}\theta$	A1		including $\mathrm{d}\theta$ on this or later line
	$2\cosh^2\theta = 1+\cosh 2\theta$ **OE**	B1		double angle formula or $\frac{1}{2}\left(\mathrm{e}^{2\theta}+2+\mathrm{e}^{-2\theta}\right)$
	$(s=)\ \theta+\frac{1}{2}\sinh 2\theta$	A1		or $\left(\frac{1}{4}\mathrm{e}^{2\theta}+\theta-\frac{1}{4}\mathrm{e}^{-2\theta}\right)$
	$\cosh^{-1}3+\frac{1}{2}\sinh(2\cosh^{-1}3)$ $-\cosh^{-1}\sqrt{2}-\frac{1}{2}\sinh\left(2\cosh^{-1}\sqrt{2}\right)$	m1		correct use of correct limits
	$(s=2\ln\left(1+\sqrt{2}\right)-\ln\left(1+\sqrt{2}\right)+6\sqrt{2}-\sqrt{2}$			must see this line OE
	$= 5\sqrt{2}+\ln\left(1+\sqrt{2}\right)$	A1	6	**partial AG** (be convinced)

Q3	Solution	Mark	Total	Comment
(a)	$\frac{dx}{dt}=1-\frac{1}{t^2}$	B1		OE eg $\frac{t(2t)-(t^2+1)}{t^2}$ ACF
	$\frac{dy}{dt}=\frac{2}{t}$	B1		
	$\left(\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2=\right)\quad 1-\frac{2}{t^2}+\frac{1}{t^4}+\frac{4}{t^2}$	M1		squaring and adding their expressions and attempting to multiply out
	$1+\frac{2}{t^2}+\frac{1}{t^4}\quad=\left(1+\frac{1}{t^2}\right)^2$	A1	4	AG be convinced
(b)	$2\pi\int_1^2(2\ln t)\left(1+\frac{1}{t^2}\right)dt$	B1		must have 2π , limits and dt
		M1		integration by parts - clear attempt to integrate $1+\frac{1}{t^2}$ and differentiate $2\ln t$
	$(2\pi)\left\{(2\ln t)\left(t-\frac{1}{t}\right)-\int\frac{2}{t}\left(t-\frac{1}{t}\right)(dt)\right\}$	A1		correct (may omit limits, 2π and dt)
	$2\pi\left[(2\ln t)\left(t-\frac{1}{t}\right)-\left(2t+\frac{2}{t}\right)\right]$	A1		correct including 2π (no limits required)
	$=2\pi(3\ln 2-5+4)$ $=\pi(6\ln 2-2)$	A1	5	
	Total		9	