Solutionbank FP2
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

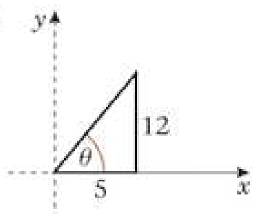
Find the polar coordinates of the following points

a $(5, 12)$ **b** $(-5, 12)$ **c** $(-5, -12)$

d $(2, -3)$ **e** $(\sqrt{3}, -1)$

Solution:

a

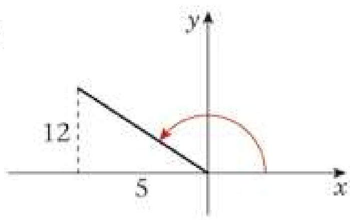


$\arctan\left(\frac{12}{5}\right) = 67.4°$

$r = \sqrt{5^2 + 12^2} = 13$

$\therefore$ point is $(13, 67.4°)$

b

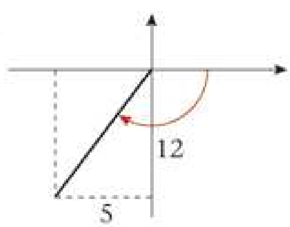


$r = \sqrt{(-5)^2 + 12^2} = 13$

$\theta = 180 - \arctan\left(\frac{12}{5}\right) = 112.6°$

$\therefore$ point is $(13, 112.6°)$

c



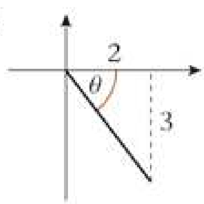
$\theta = -\left(180 - \arctan\frac{12}{5}\right)$

$= -112.6°$

$r = \sqrt{(-5)^2 + (-12)^2} = 13$

$\therefore$ point is $(13, -112.6°)$

d

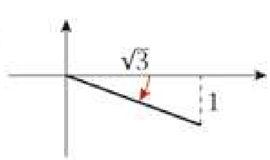


$\theta = -\arctan\frac{3}{2} = -56.3°$

$r = \sqrt{2^2 + (-3)^2} = \sqrt{13}$

$\therefore$ point is $(\sqrt{13}, -56.3°)$

e



$\theta = -\arctan\frac{1}{\sqrt{3}} = -30°$

$r = \sqrt{\sqrt{3}^2 + (-1)^2} = \sqrt{4} = 2$

$\therefore$ point is $(2, -30°)$

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Exercise A, Question 2

Question:

Find Cartesian coordinates of the following points. Angles are measured in radians.

a $\left(6, \frac{\pi}{6}\right)$ **b** $\left(6, -\frac{\pi}{6}\right)$ **c** $\left(6, \frac{3\pi}{4}\right)$

d $\left(10, \frac{5\pi}{4}\right)$ **e** $(2, \pi)$

Solution:

a $x = 6\cos\left(\frac{\pi}{6}\right) \quad = \frac{6\sqrt{3}}{2} = 3\sqrt{3}$

$y = 6\sin\frac{\pi}{6} \quad = 3$ $\quad\therefore$ point is $(3\sqrt{3}, 3)$

b $x = 6\cos\left(-\frac{\pi}{6}\right) \quad = \frac{6\sqrt{3}}{2} = 3\sqrt{3}$

$y = 6\sin\left(-\frac{\pi}{6}\right) \quad = -3$ $\quad\therefore$ point is $(3\sqrt{3}, -3)$

c $x = 6\cos\left(\frac{3\pi}{4}\right) \quad = -\frac{6}{\sqrt{2}}$ or $-3\sqrt{2}$

$y = 6\sin\left(\frac{3\pi}{4}\right) \quad = \frac{6}{\sqrt{2}} = 3\sqrt{2}$ $\quad\therefore$ point is $(-3\sqrt{2}, 3\sqrt{2})$

d $x = 10\cos\left(\frac{5\pi}{4}\right) = -\frac{10}{\sqrt{2}} = -5\sqrt{2}$

$y = 10\sin\left(\frac{5\pi}{4}\right) = \frac{-10}{\sqrt{2}} = -5\sqrt{2}$ $\quad\therefore$ point is $(-5\sqrt{2}, -5\sqrt{2})$

e $x = 2\cos(\pi) \quad = -2$

$y = 2\sin(\pi) \quad = 0$ $\quad\therefore$ point is $(-2, 0)$

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Exercise B, Question 1

Question:

Find Cartesian equations for the following curves where a is a positive constant.

a $r = 2$ **b** $r = 3 \sec \theta$ **c** $r = 5 \operatorname{cosec} \theta$

Solution:

a $r = 2$ is $x^2 + y^2 = 4$

b $r = 3 \sec \theta$

$\Rightarrow \quad r \cos \theta = 3$ i.e. $x = 3$

c $r = 5 \operatorname{cosec} \theta$

$\Rightarrow \quad r \sin \theta = 5$ i.e. $y = 5$

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Exercise B, Question 2

Question:

Find Cartesian equations for the following curves where a is a positive constant.

a $r = 4a \tan\theta \sec\theta$ **b** $r = 2a\cos\theta$ **c** $r = 3a\sin\theta$

Solution:

a

$$r = 4a\tan\theta\sec\theta$$

$$r = \frac{4a\sin\theta}{\cos^2\theta}$$

$$r\cos^2\theta = 4a\sin\theta$$ (Multiply by r.)

$$r^2\cos^2\theta = 4ar\sin\theta$$

$$\therefore \quad x^2 = 4ay \quad \text{or} \quad y = \frac{x^2}{4a}$$

b

$$r = 2a\cos\theta$$

$$r^2 = 2ar\cos\theta$$

$$\therefore \quad x^2 + y^2 = 2ax \quad \text{or} \quad (x-a)^2 + y^2 = a^2$$

c

$$r = 3a\sin\theta$$ (Multiply by r.)

$$r^2 = 3ar\sin\theta$$

$$x^2 + y^2 = 3ay \quad \text{or} \quad x^2 + \left(y - \frac{3a}{2}\right)^2 = \frac{9a^2}{4}$$

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Exercise B, Question 3

Question:

Find Cartesian equations for the following curves where a is a positive constant.

a $r = 4(1 - \cos 2\theta)$ **b** $r = 2\cos^2\theta$ **c** $r^2 = 1 + \tan^2\theta$

Solution:

a
$$r = 4(1 - \cos 2\theta)$$
Use $\cos 2\theta = 1 - 2\sin^2\theta$ $\therefore$ $2\sin^2\theta = 1 - \cos 2\theta$
$$r = 4 \times 2\sin^2\theta$$
$$r^3 = 8r^2\sin^2\theta$$
$\times r^2$
$$\therefore \quad (x^2 + y^2)^{\frac{3}{2}} = 8y^2$$

b
$$r = 2\cos^2\theta$$
$$r^3 = 2r^2\cos^2\theta$$
$\times r^2$
$$(x^2 + y^2)^{\frac{3}{2}} = 2x^2$$

c
$$r^2 = 1 + \tan^2\theta$$
$$\therefore \quad r^2 = \sec^2\theta$$
Use $\sec^2\theta = 1 + \tan^2\theta.$
$$\therefore \quad r^2\cos^2\theta = 1$$
i.e. $x^2 = 1$ or $x = \pm 1$

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Exercise B, Question 4

Question:

Find polar equations for the following curves:

a $x^2 + y^2 = 16$ **b** $xy = 4$ **c** $(x^2 + y^2)^2 = 2xy$

Solution:

a $x^2 + y^2 = 16$

$\Rightarrow \quad r^2 = 16 \quad \text{or} \quad r = 4$

b
$$xy = 4$$
$$\Rightarrow \quad r\cos\theta\, r\sin\theta = 4$$
$$r^2 = \frac{4}{\cos\theta\sin\theta} = \frac{8}{2\cos\theta\sin\theta}$$
i.e.
$$r^2 = 8\,\text{cosec}\,2\theta$$

c $(x^2 + y^2)^2 = 2xy$
$$\Rightarrow \quad (r^2)^2 = 2r\cos\theta\, r\sin\theta$$
$$r^4 = 2r^2\cos\theta\sin\theta$$
$$r^2 = \sin 2\theta$$

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Exercise B, Question 5

Question:

Find polar equations for the following curves:

a $x^2 + y^2 - 2x = 0$ **b** $(x + y)^2 = 4$ **c** $x - y = 3$

Solution:

a
$$x^2 + y^2 - 2x = 0$$
$$\Rightarrow r^2 - 2r\cos\theta = 0$$
$$r^2 = 2r\cos\theta$$
$$r = 2\cos\theta$$

b
$$(x + y)^2 = 4$$
$$\Rightarrow x^2 + y^2 + 2xy = 4$$
$$\Rightarrow r^2 + 2r\cos\theta\, r\sin\theta = 4$$
$$\Rightarrow r^2(1 + \sin 2\theta) = 4$$
$$r^2 = \frac{4}{1 + \sin 2\theta}$$

c
$$x - y = 3$$
$$r\cos\theta - r\sin\theta = 3$$
$$r(\cos\theta - \sin\theta) = 3$$
$$r\left(\frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta\right) = \frac{3}{\sqrt{2}}$$
$$r\cos\left(\theta + \frac{\pi}{4}\right) = \frac{3}{\sqrt{2}}$$
$$\therefore \quad r = \frac{3}{\sqrt{2}}\sec\left(\theta + \frac{\pi}{4}\right)$$

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Exercise B, Question 6

Question:

Find polar equations for the following curves:

a $y = 2x$ **b** $y = -\sqrt{3}x + a$ **c** $y = x(x - a)$

Solution:

a
$$y = 2x$$
$$\Rightarrow \quad r\sin\theta = 2r\cos\theta$$
$$\tan\theta = 2 \quad \text{or} \quad \theta = \arctan 2$$

b
$$y = -\sqrt{3}x + a$$
$$r\sin\theta = -\sqrt{3}r\cos\theta + a$$
$$r(\sin\theta + \sqrt{3}\cos\theta) = a$$
$$r\left(\tfrac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta\right) = \frac{a}{2}$$
$$r\sin\left(\theta + \frac{\pi}{3}\right) = \frac{a}{2}$$
$$\therefore \quad r = \frac{a}{2}\operatorname{cosec}\left(\theta + \frac{\pi}{3}\right)$$

c
$$y = x(x - a)$$
$$r\sin\theta = r\cos\theta\,(r\cos\theta - a)$$
$$\tan\theta = r\cos\theta - a$$
$$r\cos\theta = \tan\theta + a$$
$$r = \tan\theta\sec\theta + a\sec\theta$$

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Exercise C, Question 1

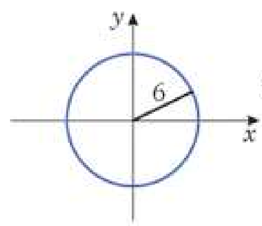
Question:

Sketch the following curves.

a $r = 6$ **b** $\theta = \frac{5\pi}{4}$ **c** $\theta = -\frac{\pi}{4}$

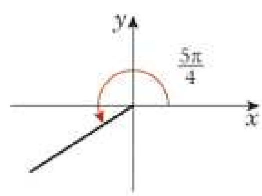
Solution:

a $r = 6$



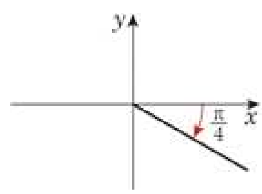
Circle radius 6

b $\theta = \frac{5\pi}{4}$



Half-line
(half of $y = x$)

c $\theta = -\frac{\pi}{4}$



Half-line
(half of $y = -x$)

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Exercise C, Question 2

Question:

Sketch the following curves.

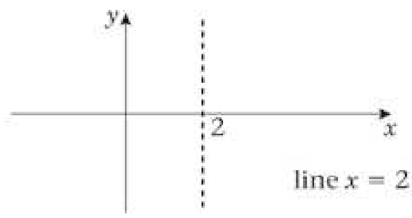
a $r = 2\sec\theta$ **b** $r = 3\operatorname{cosec}\theta$ **c** $r = 2\sec\left(\theta - \frac{\pi}{3}\right)$

Solution:

a $r = 2\sec\theta$

$\Rightarrow \quad r\cos\theta = 2$

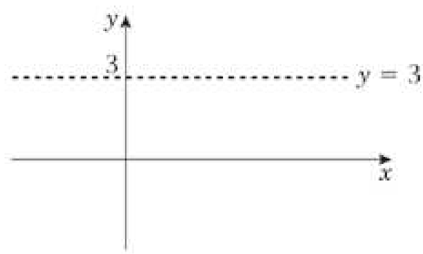
i.e. $\quad x = 2$



b $r = 3\operatorname{cosec}\theta$

$\Rightarrow \quad r\sin\theta = 3$

i.e. $\quad y = 3$



c

$$r = 2\sec\left(\theta - \frac{\pi}{3}\right)$$

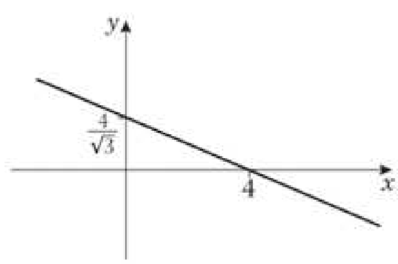
$$r\cos\left(\theta - \frac{\pi}{3}\right) = 2$$

$$\Rightarrow \quad r\cos\theta\cos\frac{\pi}{3} + r\sin\theta\sin\frac{\pi}{3} = 2$$

$$\Rightarrow \quad \frac{x}{2} + y\frac{\sqrt{3}}{2} = 2$$

$$x + y\sqrt{3} = 4$$

or $$y = \frac{4}{\sqrt{3}} - \frac{1}{\sqrt{3}}x$$



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Exercise C, Question 3

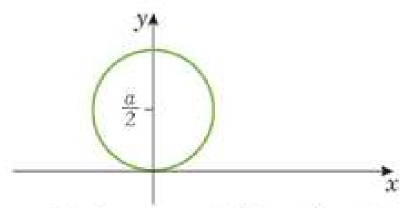
Question:

Sketch the following curves.

a $r = a\sin\theta$ **b** $r = a(1 - \cos\theta)$ **c** $r = a\cos 3\theta$

Solution:

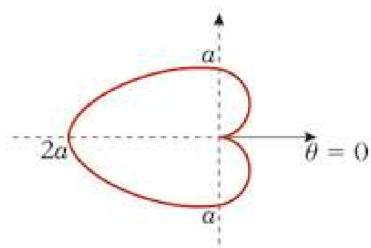
a
$$r = a\sin\theta$$
$$\Rightarrow \quad r^2 = ar\sin\theta$$
$$x^2 + y^2 = ay$$
$$x^2 + \left(y - \frac{a}{2}\right)^2 = \frac{a^2}{4}$$



Circle centre $(0, \frac{a}{2})$ radius $\frac{a}{2}$

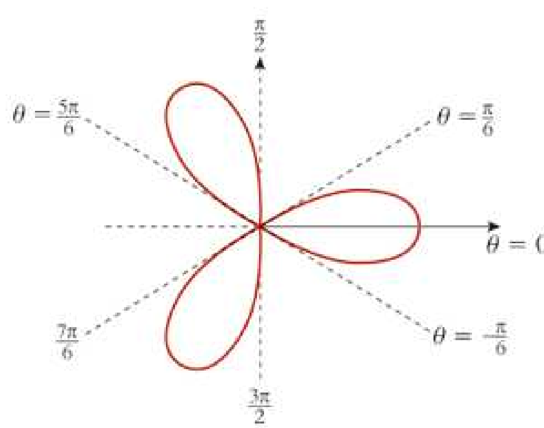
b $r = a(1 - \cos\theta)$

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
r	0	a	$2a$	a	0



c $r = a\cos 3\theta$

θ	0	$\frac{\pi}{6}$	$-\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$
r	a	0	0	0	a	0	0	a	0



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Exercise C, Question 4

Question:

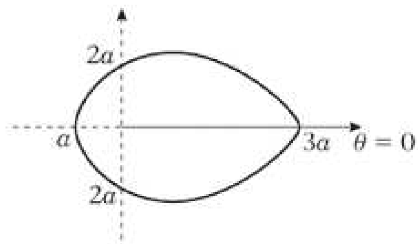
Sketch the following curves.

a $r = a(2 + \cos\theta)$ **b** $r = a(6 + \cos\theta)$ **c** $r = a(4 + 3\cos\theta)$

Solution:

a $r = a(2 + \cos\theta)$

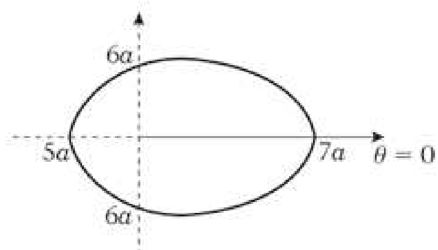
θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
r	$3a$	$2a$	a	$2a$	$3a$



$2 = 2 \times 1 \quad \therefore$ no dimple.

b $r = a(6 + \cos\theta)$

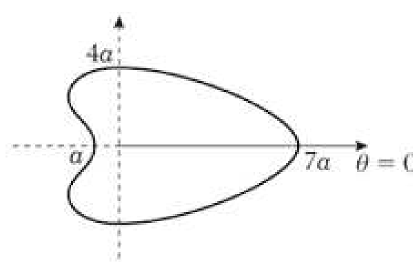
θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
r	$7a$	$6a$	$5a$	$6a$	$7a$



$6 > 2 \times 1 \quad \therefore$ no dimple.

c $r = a(4 + 3\cos\theta)$

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
r	$7a$	$4a$	a	$4a$	$7a$



$4 < 2 \times 3 \quad \therefore$ a dimple at $\theta = \pi$.

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Exercise C, Question 5

Question:

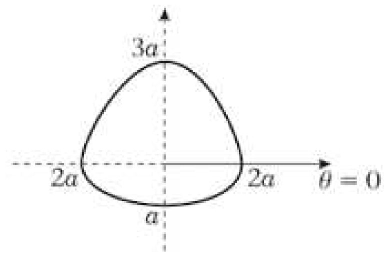
Sketch the following curves.

a $r = a(2 + \sin\theta)$ **b** $r = a(6 + \sin\theta)$ **c** $r = a(4 + 3\sin\theta)$

Solution:

a $r = a(2 + \sin\theta)$

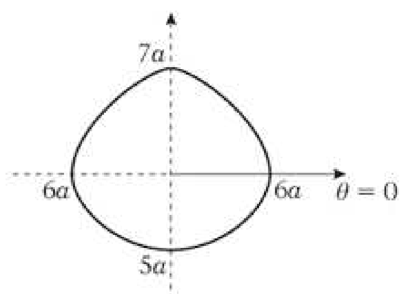
θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
r	$2a$	$3a$	$2a$	a	$2a$



$2 = 2 \times 1$ so no dimple

b $r = a(6 + \sin\theta)$

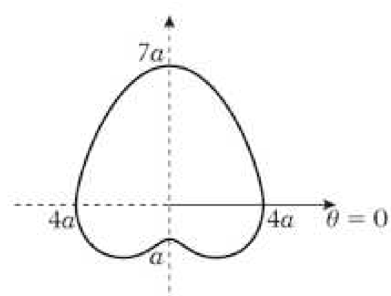
θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
r	$6a$	$7a$	$6a$	$5a$	$6a$



$6 > 2 \times 1$ so no dimple

c $r = a(4 + 3\sin\theta)$

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
r	$4a$	$7a$	$4a$	a	$4a$



$4 < 2 \times 3 \therefore$ there is a dimple at $\theta = \frac{3\pi}{2}$

The graphs in question 5 are simply rotations of the graphs in question 4.

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Exercise C, Question 6

Question:

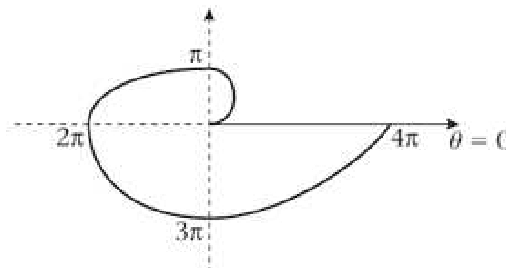
Sketch the following curves.

a $r = 2\theta$ **b** $r^2 = a^2 \sin \theta$ **c** $r^2 = a^2 \sin 2\theta$

Solution:

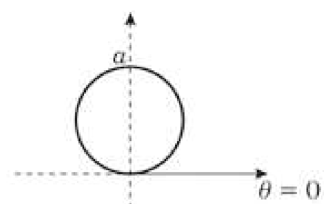
a $r = 2\theta$

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
r	0	π	2π	3π	4π



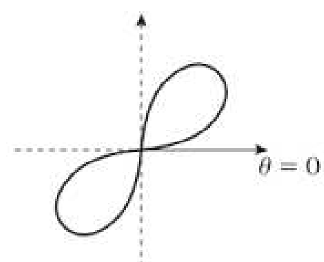
b $r^2 = a^2 \sin \theta$

θ	0	$\frac{\pi}{2}$	π
r	0	a	0



c $r^2 = a^2 \sin 2\theta$

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$
r	0	a	0	0	a	0



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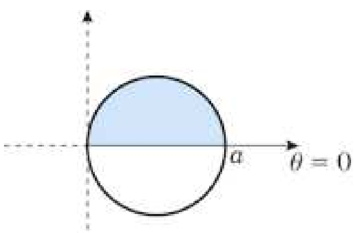
Exercise D, Question 1

Question:

Find the area of the finite region bounded by the curve with the given polar equation and the half lines $\theta = \alpha$ and $\theta = \beta$.

$r = a\cos\theta$, $\alpha = 0, \beta = \frac{\pi}{2}$

Solution:



$r = a\cos\theta$

$$\text{Area} = \frac{1}{2}a^2\int_0^{\frac{\pi}{2}}\cos^2\theta\,d\theta$$

$\cos 2\theta = 2\cos^2\theta - 1$

$$= \frac{a^2}{4}\int_0^{\frac{\pi}{2}}(\cos 2\theta + 1)$$

$$= \frac{a^2}{4}\left[\frac{1}{2}\sin 2\theta + \theta\right]_0^{\frac{\pi}{2}}$$

$$= \frac{a^2}{4}\left[\left(0 + \frac{\pi}{2}\right) - (0)\right]$$

$$= \frac{\pi a^2}{8}$$

$r = a\cos\theta$ is a circle centre $\left(\frac{a}{2}, 0\right)$ and radius $\frac{a}{2}$.
The area of the semicircle is $\therefore\ \frac{1}{2}\pi\frac{a^2}{4} = \frac{a^2\pi}{8}$.

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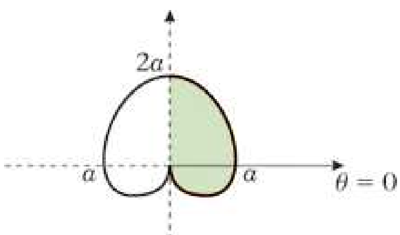
Exercise D, Question 2

Question:

Find the area of the finite region bounded by the curve with the given polar equation and the half lines $\theta = \alpha$ and $\theta = \beta$.

$r = a(1 + \sin\theta)$, $\alpha = -\frac{\pi}{2}, \beta = \frac{\pi}{2}$

Solution:



$r = a(1 + \sin\theta)$

$$\text{Area} = \frac{1}{2}a^2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(1 + 2\sin\theta + \sin^2\theta)\,d\theta$$

$$= \frac{1}{2}a^2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(1 + 2\sin\theta + \frac{1}{2} - \frac{1}{2}\cos 2\theta\right)d\theta$$

Use $\cos 2\theta = 1 - 2\sin^2\theta$.

$$= \frac{1}{2}a^2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{3}{2} + 2\sin\theta - \frac{1}{2}\cos 2\theta\right)d\theta$$

$$= \frac{1}{2}a^2\left[\frac{3}{2}\theta - 2\cos\theta - \frac{1}{4}\sin 2\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{1}{2}a^2\left[\left(\frac{3\pi}{4} - 0 - 0\right) - \left(-\frac{3\pi}{4} - 0 - 0\right)\right]$$

$$= \frac{3\pi a^2}{4}$$

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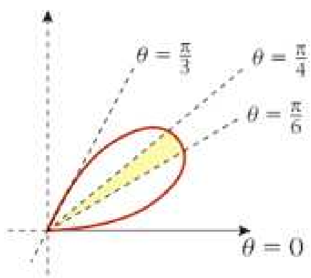
Exercise D, Question 3

Question:

Find the area of the finite region bounded by the curve with the given polar equation and the half lines $\theta = \alpha$ and $\theta = \beta$.

$r = a \sin 3\theta$, $\alpha = \frac{\pi}{6}, \beta = \frac{\pi}{4}$

Solution:



$r = a \sin 3\theta$

$$\text{Area} = \frac{1}{2} a^2 \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sin^2 3\theta \, d\theta$$

$$= \frac{a^2}{4} \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (1 - \cos 6\theta) \, d\theta$$

Use $\cos 6\theta = 1 - 2\sin^2 3\theta$.

$$= \frac{a^2}{4} \left[\theta - \frac{1}{6} \sin 6\theta \right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$$

$$= \frac{a^2}{4} \left[\left(\frac{\pi}{4} - \frac{1}{6} \sin \frac{3\pi}{2} \right) - \left(\frac{\pi}{6} - \frac{1}{6} \sin \pi \right) \right]$$

$$= \frac{a^2}{4} \left(\frac{\pi}{4} + \frac{1}{6} - \frac{\pi}{6} \right)$$

$$= \frac{a^2}{4} \left(\frac{\pi}{12} + \frac{2}{12} \right)$$

$$= \frac{(\pi + 2) a^2}{48}$$

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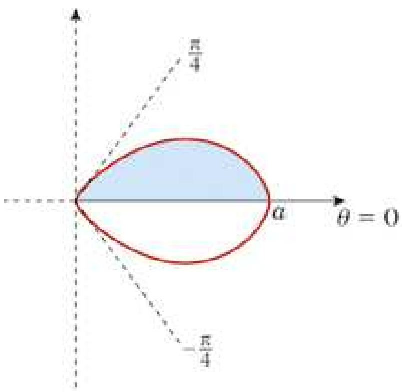
Exercise D, Question 4

Question:

Find the area of the finite region bounded by the curve with the given polar equation and the half lines $\theta = \alpha$ and $\theta = \beta$.

$r^2 = a^2 \cos 2\theta,$ $\alpha = 0, \beta = \frac{\pi}{4}$

Solution:



$r = a^2 \cos 2\theta$

$$\text{Area} = \frac{1}{2} a^2 \int_0^{\frac{\pi}{4}} \cos 2\theta \, d\theta$$

$$= \left[\frac{a^2}{4} \sin 2\theta\right]_0^{\frac{\pi}{4}}$$

$$= \left(\frac{a^2}{4} \sin \frac{\pi}{2}\right) - (0)$$

$$= \frac{a^2}{4}$$

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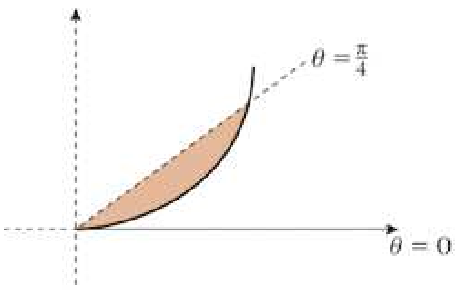
Exercise D, Question 5

Question:

Find the area of the finite region bounded by the curve with the given polar equation and the half lines $\theta = \alpha$ and $\theta = \beta$.

$r^2 = a^2 \tan \theta$, $\alpha = 0, \beta = \frac{\pi}{4}$

Solution:



$r^2 = a^2 \tan \theta$

$$\text{Area} = \frac{1}{2} a^2 \int_0^{\frac{\pi}{4}} \tan \theta \, d\theta$$

$$= \left[\frac{1}{2} a^2 \ln \sec \theta\right]_0^{\frac{\pi}{4}}$$

$$= \left(\frac{1}{2} a^2 \ln \sqrt{2}\right) - (0)$$

$$= \frac{a^2 \ln \sqrt{2}}{2} \quad \text{or} \quad \frac{a^2 \ln 2}{4}$$

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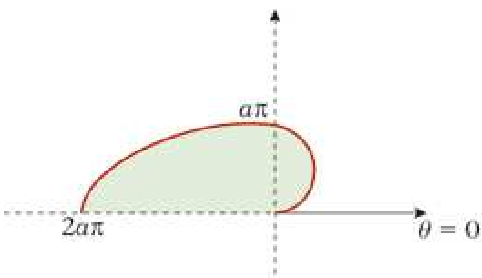
Exercise D, Question 6

Question:

Find the area of the finite region bounded by the curve with the given polar equation and the half lines $\theta = \alpha$ and $\theta = \beta$.

$r = 2a\theta$, $\alpha = 0, \beta = \pi$

Solution:



$$r = 2a\theta$$

$$\text{Area} = \frac{1}{2}\int_0^{\pi} 4a^2\theta^2\,d\theta$$

$$= 2a^2\left[\frac{\theta^3}{3}\right]_0^{\pi}$$

$$= 2a^2\left[\left(\frac{\pi^3}{3}\right) - (0)\right]$$

$$= \frac{2a^2\pi^3}{3}$$

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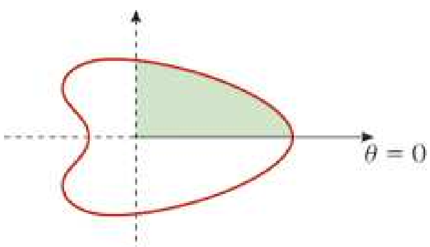
Exercise D, Question 7

Question:

Find the area of the finite region bounded by the curve with the given polar equation and the half lines $\theta = \alpha$ and $\theta = \beta$.

$r = a(3 + 2\cos\theta)$, $\alpha = 0, \beta = \frac{\pi}{2}$

Solution:



$r = a(3 + 2\cos\theta)$

$$\text{Area} = \frac{a^2}{2}\int_0^{\frac{\pi}{2}} (9 + 12\cos\theta + 4\cos^2\theta)\,d\theta$$

$$= \frac{a^2}{2}\int_0^{\frac{\pi}{2}} (11 + 12\cos\theta + 2\cos 2\theta)\,d\theta$$

Use $\cos 2\theta = 2\cos^2\theta - 1$.

$$= \frac{a^2}{2}\left[11\theta + 12\sin\theta + \sin 2\theta\right]_0^{\frac{\pi}{2}}$$

$$= \frac{a^2}{2}\left[\left(\frac{11\pi}{2} + 12 + 0\right) - (0)\right]$$

$$= \frac{a^2}{4}(11\pi + 24)$$

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Exercise D, Question 8

Question:

Show that the area enclosed by the curve with polar equation
$r = a(p + q\cos\theta)$ is $\frac{2p^2 + q^2}{2}\pi a^2$.

Solution:

$$\text{Area} = \frac{1}{2}a^2\int_0^{2\pi}(p^2 + 2pq\cos\theta + q^2\cos^2\theta)\,\mathrm{d}\theta$$

$$= \frac{1}{2}a^2\int_0^{2\pi}\left(p^2 + 2pq\cos\theta + \frac{q^2}{2}\cos 2\theta + \frac{q^2}{2}\right)\mathrm{d}\theta$$

Use $\cos 2\theta = 2\cos^2\theta - 1$.

$$= \frac{1}{2}a^2\int_0^{2\pi}\left(\left[\frac{2p^2 + q^2}{2}\right] + 2pq\cos\theta + \frac{q^2}{2}\cos 2\theta\right)\mathrm{d}\theta$$

$$= \frac{1}{2}a^2\left[\left[\frac{2p^2 + q^2}{2}\right]\theta + 2pq\sin\theta + \frac{q^2}{4}\sin 2\theta\right]_0^{2\pi}$$

$$= \frac{1}{2}a^2\left[\left(\left[\frac{2p^2 + q^2}{\not 2}\right]\pi \times \not 2 + 0 + 0\right) - (0)\right]$$

$$= \frac{a^2(2p^2 + q^2)\pi}{2}$$

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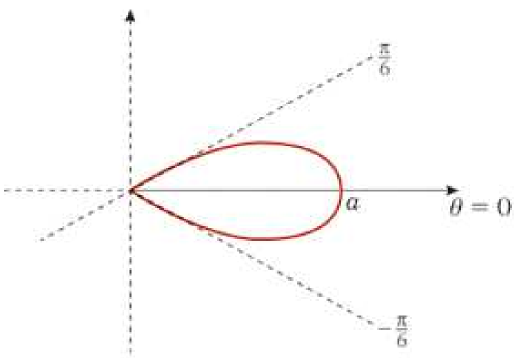
Exercise D, Question 9

Question:

Find the area of a single loop of the curve with equation $r = a\cos 3\theta$.

Solution:



$$\text{Area} = \frac{1}{2}a^2\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \cos^2 3\theta \, d\theta = 2 \times \frac{1}{2}a^2 \int_0^{\frac{\pi}{6}} \cos^2 3\theta \, d\theta$$

$$= \frac{a^2}{2}\int_0^{\frac{\pi}{6}} (1 + \cos 6\theta)\, d\theta$$

Use $\cos 6\theta = 2\cos^2 3\theta - 1$.

$$= \frac{a^2}{2}\left[\theta + \frac{1}{6}\sin 6\theta\right]_0^{\frac{\pi}{6}}$$

$$= \frac{a^2}{2}\left[\left(\frac{\pi}{6} + 0\right) - (0)\right]$$

$$= \frac{\pi a^2}{12}$$

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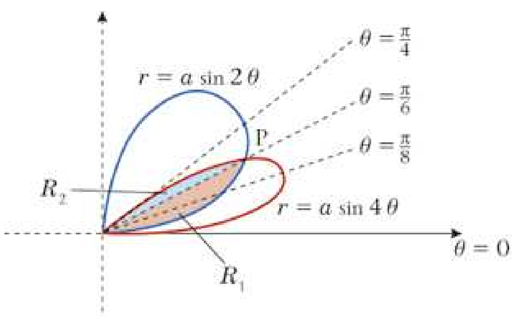
Exercise D, Question 10

Question:

Find the finite area enclosed between $r = a \sin 4\theta$ and $r = a \sin 2\theta$ for $0 \leqslant \theta \leqslant \frac{\pi}{2}$.

Solution:



Find P

$$a \sin 2\theta = a \sin 4\theta$$
$$\Rightarrow \quad \sin 2\theta = 2 \sin 2\theta \cos 2\theta$$
$$\Rightarrow \quad 0 = \sin 2\theta (2 \cos 2\theta - 1)$$
$$\Rightarrow \quad \sin 2\theta = 0, \ \theta = 0, \frac{\pi}{2}$$
$$\cos 2\theta = \tfrac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$$

$$R_1 = \tfrac{1}{2} a^2 \int_0^{\frac{\pi}{6}} \sin^2 2\theta \, d\theta$$

$\cos 2A = 1 - 2\sin^2 A.$

$$= \frac{a^2}{4} \int_0^{\frac{\pi}{6}} (1 - \cos 4\theta) \, d\theta$$
$$= \frac{a^2}{4} \left[\theta - \tfrac{1}{4} \sin 4\theta\right]_0^{\frac{\pi}{6}} = \frac{a^2}{4} \left[\left(\frac{\pi}{6} - \tfrac{1}{4} \sin \frac{2\pi}{3}\right) - (0)\right]$$
$$= \frac{a^2}{4} \left[\frac{\pi}{6} - \frac{\sqrt{3}}{8}\right]$$

$$R_2 = \tfrac{1}{2} a^2 \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sin^2 4\theta \, d\theta = \frac{a^2}{4} \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (1 - \cos 8\theta) \, d\theta$$
$$= \frac{a^2}{4} \left[\theta - \tfrac{1}{8} \sin 8\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}} = \frac{a^2}{4} \left[\left(\frac{\pi}{4} - \tfrac{1}{8} \sin 2\pi\right) - \left(\frac{\pi}{6} - \tfrac{1}{8} \sin \frac{4\pi}{3}\right)\right]$$
$$= \frac{a^2}{4} \left[\frac{\pi}{12} - \frac{\sqrt{3}}{16}\right]$$

$$\therefore \quad \text{enclosed area} = R_1 + R_2 = \frac{a^2}{4} \left[\frac{\pi}{4} - \frac{3\sqrt{3}}{16}\right]$$

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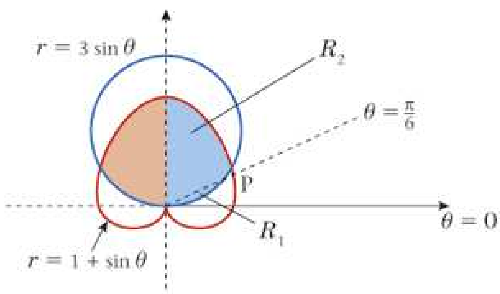
Exercise D, Question 11

Question:

Find the area of the finite region R enclosed by the curve with equation $r = (1 + \sin\theta)$ that lies entirely within the curve with equation $r = 3\sin\theta$.

Solution:



First find P:

$$1 + \sin\theta = 3\sin\theta$$

$$\Rightarrow \quad 1 = 2\sin\theta$$

$$\Rightarrow \quad \sin\theta = \tfrac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$$

Just finding RHS of the required area, so total $= 2(R_1 + R_2)$

$$R_1 = \tfrac{1}{2}\int_0^{\frac{\pi}{6}} (3\sin\theta)^2\,d\theta = \tfrac{9}{4}\int_0^{\frac{\pi}{6}} (1 - \cos 2\theta)\,d\theta$$

$$R_2 = \tfrac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1 + \sin\theta)^2\,d\theta$$

So $$R_2 = \tfrac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1 + 2\sin\theta + \sin^2\theta)\,d\theta = \tfrac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(\tfrac{3}{2} + 2\sin\theta - \tfrac{1}{2}\cos 2\theta\right)d\theta$$

$$R_1 = \tfrac{9}{4}\left[\theta - \tfrac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{6}} = \tfrac{9}{4}\left[\left(\frac{\pi}{6} - \tfrac{1}{2}\sin\frac{\pi}{3}\right) - (0)\right]$$

$$R_1 = \frac{3\pi}{8} - \frac{9\sqrt{3}}{16}$$

$$R_2 = \tfrac{1}{2}\left[\tfrac{3}{2}\theta - 2\cos\theta - \tfrac{1}{4}\sin 2\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \tfrac{1}{2}\left[\left(\frac{3\pi}{4} - 0\right) - \left(\frac{\pi}{4} - \sqrt{3} - \frac{\sqrt{3}}{8}\right)\right]$$

$$R_2 = \frac{\pi}{4} + \frac{9\sqrt{3}}{16}$$

$$\therefore \quad R_1 + R_2 = \frac{5\pi}{8}$$

$\therefore$ Area required is $\dfrac{5\pi}{4}$

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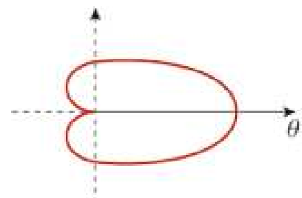
Exercise E, Question 1

Question:

Find the points on the cardioid $r = a(1 + \cos\theta)$ where the tangents are perpendicular to the initial line.

Solution:



$r = a(1 + \cos\theta)$

Require $\frac{\mathrm{d}}{\mathrm{d}\theta}(r\cos\theta) = 0$

i.e. $\frac{\mathrm{d}}{\mathrm{d}\theta}(a\cos\theta + a\cos^2\theta) = a[-\sin\theta - 2\cos\theta\sin\theta]$

So $\quad 0 = -a\sin\theta\,[1 + 2\cos\theta]$

$\sin\theta = 0 \Rightarrow \quad \theta = 0, \pi \quad$ (from sketch π is not allowed)

$\cos\theta = -\frac{1}{2} \Rightarrow \theta = \pm\frac{2\pi}{3} \Rightarrow r = a\left(1 - \frac{1}{2}\right) = \frac{a}{2}$

$\therefore$ points are $(2a, 0)$ and $\left(\frac{a}{2}, \frac{2\pi}{3}\right)\left(\frac{a}{2}, \frac{-2\pi}{3}\right)$

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Exercise E, Question 2

Question:

Find the points on the spiral $r = e^{2\theta}$, $0 \leqslant \theta \leqslant \pi$, where the tangents are

a perpendicular, **b** parallel

to the initial line. Give your answers to 3 s.f.

Solution:

$$r = e^{2\theta}$$

a $x = r\cos\theta = e^{2\theta}\cos\theta$

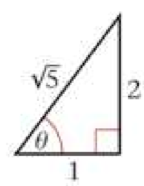
$$\frac{dx}{d\theta} = 0 \Rightarrow 0 = 2e^{2\theta}\cos\theta - \sin\theta\, e^{2\theta}$$

$$0 = e^{2\theta}(2\cos\theta - \sin\theta)$$

$\Rightarrow \quad \tan\theta = 2$

$\therefore \quad \theta = 1.107$ (rads)

$$r = e^{2 \times 1.107} = 9.1549\ldots$$



So at (9.15, 1.11) the tangent is perpendicular to initial line.

b $y = r\sin\theta = e^{2\theta}\sin\theta$

$$\frac{dy}{d\theta} = 0 \Rightarrow 0 = 2e^{2\theta}\sin\theta + \cos\theta\, e^{2\theta}$$

$$0 = e^{2\theta}(2\sin\theta + \cos\theta)$$

$\Rightarrow \quad \tan\theta = -\frac{1}{2}$

$\therefore \quad \theta = (-0.463\ldots,)\ 2.6779\ldots$

$$r = e^{2 \times 2.6779\ldots} = 211.852\ldots$$

So at (212, 2.68) the tangent is parallel to initial line.

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Exercise E, Question 3

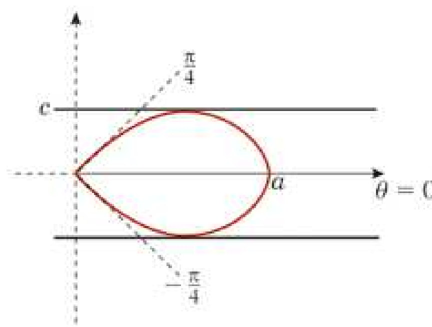
Question:

a Find the points on the curve $r = a\cos 2\theta$, $-\frac{\pi}{4} \leqslant \theta \leqslant \frac{\pi}{4}$, where the tangents are parallel to the initial line, giving your answers to 3 s.f. where appropriate.

b Find the equation of these tangents.

Solution:

$$r = a\cos 2\theta$$



a $y = r\sin\theta = a\sin\theta\cos 2\theta$

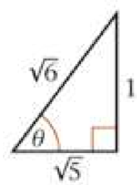
$$\frac{dy}{d\theta} = 0 \Rightarrow 0 = a[\cos\theta\cos 2\theta - 2\sin 2\theta\sin\theta]$$

$$0 = a\cos\theta\,[\cos 2\theta - 4\sin^2\theta]$$

$$0 = a\cos\theta\,[\cos^2\theta - 5\sin^2\theta]$$

$\cos\theta = \Rightarrow \theta = \frac{\pi}{2}$ (outside range)

$\therefore \quad \tan^2\theta = \frac{1}{5} \Rightarrow \tan\theta = \pm\frac{1}{\sqrt{5}}$



$$\theta = \pm 0.42053\ldots$$

$$r = a[\cos^2\theta - \sin^2\theta] = a\left[\frac{5}{6} - \frac{1}{6}\right] = \frac{2a}{3}$$

$\therefore$ points are $\left(\frac{2a}{3}, \pm 0.421\right)$

b The lines are $y = \pm c$ where $c = r\sin(0.42053\ldots)$

$$= \frac{2a}{3} \times \frac{1}{\sqrt{6}} = \frac{a\sqrt{6}}{9}$$

The line $y = c$ is $r\sin\theta = \frac{a\sqrt{6}}{9}$

$\therefore$ Tangents have equations $r = \pm\frac{a\sqrt{6}}{9}\operatorname{cosec}\theta$

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Exercise E, Question 4

Question:

Find the points on the curve with equation $r = a(7 + 2\cos\theta)$ where the tangents are parallel to the initial line.

Solution:

$$r = a(7 + 2\cos\theta)$$

$$y = r\sin\theta = a(7\sin\theta + 2\cos\theta\sin\theta)$$

$$\Downarrow$$

$$\sin 2\theta$$

$$\frac{\mathrm{d}y}{\mathrm{d}\theta} = 0 \Rightarrow 0 = a(7\cos\theta + 2\cos 2\theta)$$

$$\Rightarrow \quad 0 = 4\cos^2\theta + 7\cos\theta - 2$$

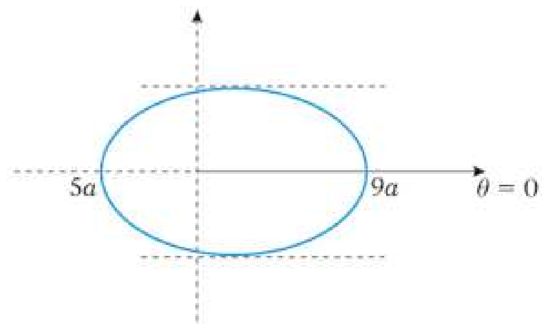
$$0 = (4\cos\theta - 1)(\cos\theta + 2)$$

$$\cos\theta = \tfrac{1}{4} \text{ (or } -2)$$

$$\Rightarrow \quad \theta = \pm 1.318\ldots$$

$$r = a\left(7 + \tfrac{2}{4}\right) = 7\tfrac{1}{2}a$$

$\therefore$ tangents are parallel at $\left(7\tfrac{1}{2}a, \pm 1.32\right)$



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Exercise E, Question 5

Question:

Find the equation of the tangents to $r = 2 + \cos\theta$ that are perpendicular to the initial line.

Solution:

$$r = 2 + \cos\theta$$

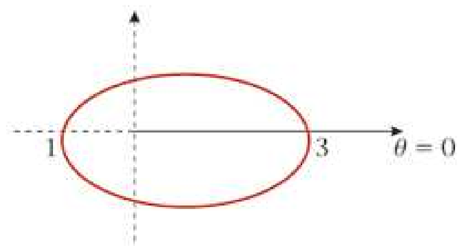
$$x = r\cos\theta = 2\cos\theta + \cos^2\theta$$

$$\frac{dx}{d\theta} = 0 \Rightarrow 0 = -2\sin\theta - 2\cos\theta\sin\theta$$

$$0 = -2\sin\theta(1 + \cos\theta)$$

$$\sin\theta = 0 \quad \Rightarrow \theta = 0, \pi$$

$$\cos\theta = -1 \Rightarrow \theta = \pi$$



$\therefore$ tangents are perpendicular to the initial line at:

$(3, 0)$ and $(1, \pi)$

The equations are

$r\cos\theta = 3 \qquad r\cos\theta = -1$

$r = 3\sec\theta \qquad r = -\sec\theta$

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Exercise E, Question 6

Question:

Find the point on the curve with equation $r = a(1 + \tan\theta)$, $0 \leqslant \theta < \frac{\pi}{2}$, where the tangent is perpendicular to the initial line.

Solution:

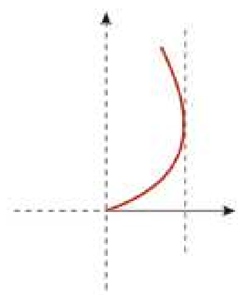
$$r = a(1 + \tan\theta)$$

$$x = r\cos\theta = a(\cos\theta + \sin\theta)$$

$$\frac{dx}{d\theta} = 0 \Rightarrow \quad 0 = a[-\sin\theta + \cos\theta]$$

$$\Rightarrow \quad \tan\theta = 1$$

$$\Rightarrow \quad \theta = \frac{\pi}{4}$$



$\therefore$ point is $\left(2a, \frac{\pi}{4}\right)$

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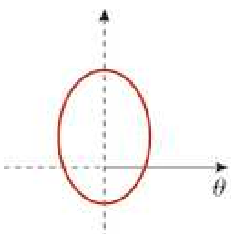
Exercise F, Question 1

Question:

Determine the area enclosed by the curve with equation
$r = a(1 + \frac{1}{2}\sin\theta)$, $a > 0$, $0 \leqslant \theta < 2\pi$,
giving your answer in terms of a and π.

Solution:



$r = a\left(1 + \frac{1}{2}\sin\theta\right)$

$$\text{Area} = \frac{1}{2}a^2\int_0^{2\pi}\left(1 + \frac{1}{2}\sin\theta\right)^2 d\theta$$

$$= \frac{a^2}{2}\int_0^{2\pi}\left(1 + \sin\theta + \frac{1}{4}\sin^2\theta\right)d\theta$$

$$= \frac{a^2}{2}\int_0^{2\pi}\left(\frac{9}{8} + \sin\theta - \frac{\cos 2\theta}{8}\right)d\theta$$

Use $\cos 2\theta = 1 - 2\sin^2\theta$.

$$= \frac{a^2}{2}\left[\frac{9}{8}\theta - \cos\theta - \frac{\sin 2\theta}{16}\right]_0^{2\pi}$$

$$= \frac{a^2}{2}\left[\left(\frac{9\pi}{4} - 1 - 0\right) - (0 - 1 - 0)\right]$$

$$= \frac{9\pi a^2}{8}$$

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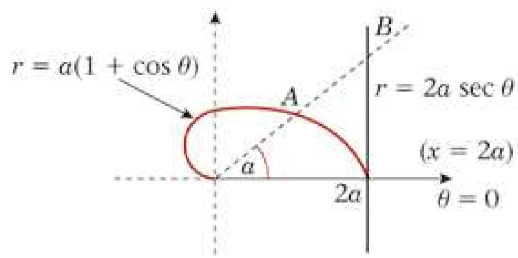
Exercise F, Question 2

Question:

Sketch the curve with equation $r = a(1 + \cos\theta)$ for $0 \leqslant \theta \leqslant \pi$, where $a > 0$.
Sketch also the line with equation $r = 2a\sec\theta$ for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, on the same diagram.
The half-line with equation $\theta = \alpha$, $0 < \alpha < \frac{\pi}{2}$, meets the curve at A and the line with equation $r = 2a\sec\theta$ at B. If O is the pole, find the value of $\cos\alpha$ for which $OB = 2OA$.

Solution:



$OB = 2a\sec\alpha$

$OA = a(1 + \cos\alpha)$

$2OA = OB \Rightarrow 1 + \cos\alpha = \sec\alpha$

$\cos^2\alpha + \cos\alpha - 1 = 0$

$\cos\alpha = \dfrac{-1 \pm \sqrt{1+4}}{2}$

$\therefore$ α is acute.

$\cos\alpha = \dfrac{\sqrt{5} - 1}{2}$

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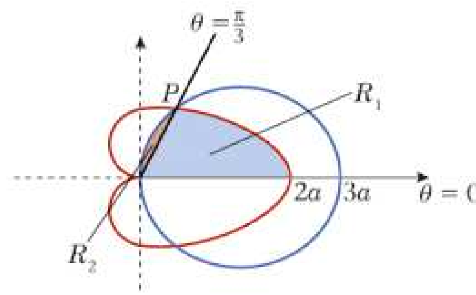
Exercise F, Question 3

Question:

Sketch, in the same diagram, the curves with equations $r = 3\cos\theta$ and $r = 1 + \cos\theta$ and find the area of the region lying inside both curves.

Solution:



First find P:

$$1 + \cos\theta = 3\cos\theta$$
$$1 = 2\cos\theta$$
$$\Rightarrow \quad \theta = \arccos\tfrac{1}{2} = \frac{\pi}{3}$$

By symmetry the required area $= 2(R_1 + R_2)$

$$R_1 = \tfrac{1}{2}\int_0^{\frac{\pi}{3}} (1 + \cos\theta)^2 \, d\theta = \tfrac{1}{2}\int_0^{\frac{\pi}{3}} (1 + 2\cos\theta + \cos^2\theta) \, d\theta$$

$$R_1 = \tfrac{1}{2}\int_0^{\frac{\pi}{3}} \left(\tfrac{3}{2} + 2\cos\theta + \frac{\cos 2\theta}{2}\right) d\theta$$

Use $\cos 2\theta = 2\cos^2\theta - 1$.

$$= \tfrac{1}{2}\left[\tfrac{3}{2}\theta + 2\sin\theta + \tfrac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{3}}$$

$$= \tfrac{1}{2}\left[\left(\frac{\pi}{2} + 2\sin\frac{\pi}{3} + \tfrac{1}{4}\sin\frac{2\pi}{3}\right) - (0)\right]$$

$$= \tfrac{1}{2}\left[\frac{\pi}{2} + \sqrt{3} + \frac{\sqrt{3}}{8}\right] = \frac{\pi}{4} + \frac{9\sqrt{3}}{16}$$

$$R_2 = \tfrac{9}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos^2\theta \, d\theta = \tfrac{9}{4}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (1 + \cos 2\theta) \, d\theta$$

$$= \tfrac{9}{4}\left[\theta + \tfrac{1}{2}\sin 2\theta\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} = \tfrac{9}{4}\left[\left(\frac{\pi}{2} + 0\right) - \left(\frac{\pi}{3} + \frac{\sqrt{3}}{4}\right)\right]$$

$$= \frac{3\pi}{8} - \frac{9\sqrt{3}}{16}$$

$$\therefore \quad \text{Area required} = 2\left(\frac{3\pi}{8} + \frac{\pi}{4}\right) = \frac{5\pi}{4}$$

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Exercise F, Question 4

Question:

Find the polar coordinates of the points on $r^2 = a^2 \sin 2\theta$ where the tangent is perpendicular to the initial line.

Solution:

$$r^2 = a^2 \sin 2\theta \qquad \left(\text{must have } 0 \leqslant \theta \leqslant \frac{\pi}{2}\right)$$

$$r = a\sqrt{\sin 2\theta}$$

$$x = r\cos\theta = a\cos\theta\sqrt{\sin 2\theta}$$

$$\frac{dx}{d\theta} = 0 \Rightarrow 0 = -\sin\theta\sqrt{\sin 2\theta} + \tfrac{1}{2}\cos\theta\frac{1}{\sqrt{\sin 2\theta}}2\cos 2\theta$$

i.e. $0 = -\sin\theta \times \sin 2\theta + \cos\theta\cos 2\theta$

i.e. $0 = \cos 3\theta$

$\therefore$ $3\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\therefore$ $\theta = \frac{\pi}{6}, \frac{\pi}{2}$

So $\left(a\sqrt{\frac{\sqrt{3}}{2}}, \frac{\pi}{6}\right)$ and $\left(0, \frac{\pi}{2}\right)$

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Exercise F, Question 5

Question:

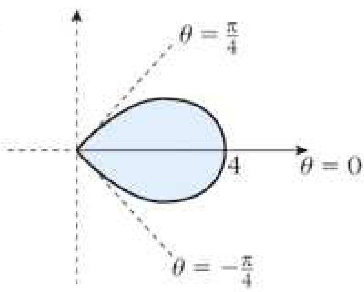
a Shade the region C for which the polar coordinates r, θ satisfy

$r \leqslant 4\cos 2\theta$ for $-\frac{\pi}{4} \leqslant \theta \leqslant \frac{\pi}{4}$

b Find the area of C.

Solution:

a



b

$$\text{Area} = 2 \times \frac{1}{2} \int_0^{\frac{\pi}{4}} 16\cos^2 2\theta \, d\theta$$

$$= \int_0^{\frac{\pi}{4}} (8 + 8\cos 4\theta) \, d\theta$$

$2\cos^2\theta = 1 + \cos 2\theta.$

$$= \left[8\theta + 2\sin 4\theta\right]_0^{\frac{\pi}{4}}$$

$$= 2\pi + 0 - 0$$

$$= 2\pi$$

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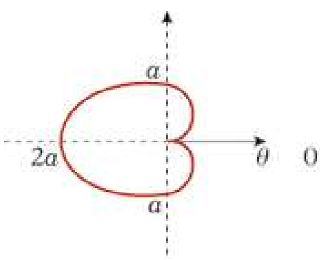
Exercise F, Question 6

Question:

Sketch the curve with polar equation $r = a(1 - \cos\theta)$, where $a > 0$, stating the polar coordinates of the point on the curve at which r has its maximum value.

Solution:



Max r is $2a$ at point $(2a, \pi)$

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Exercise F, Question 7

Question:

a On the same diagram, sketch the curve C_1 with polar equation

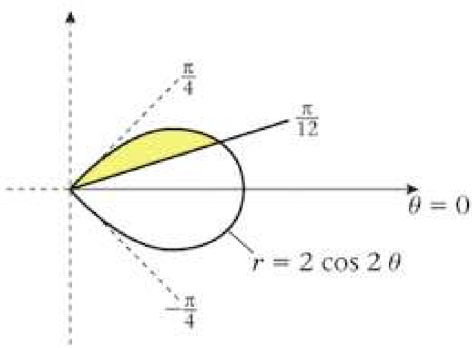
$r = 2\cos 2\theta, \quad -\frac{\pi}{4} < \theta \leqslant \frac{\pi}{4}$

and the curve C_2 with polar equation $\theta = \frac{\pi}{12}$.

b Find the area of the smaller region bounded by C_1 and C_2.

Solution:

a



b Area $= \frac{1}{2}\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} 4\cos^2 2\theta$

$= \int_{\frac{\pi}{12}}^{\frac{\pi}{4}} (1 + \cos 4\theta)\,d\theta$ $\quad$ $\boxed{\cos 4\theta = 2\cos^2 2\theta - 1}$

$= \left[\theta + \frac{1}{4}\sin 4\theta\right]_{\frac{\pi}{12}}^{\frac{\pi}{4}}$

$= \left(\frac{\pi}{4} + 0\right) - \left(\frac{\pi}{12} + \frac{1}{4}\sin\frac{\pi}{3}\right)$

$= \frac{\pi}{6} - \frac{1}{4} \times \frac{\sqrt{3}}{2}$

$= \frac{\pi}{6} - \frac{\sqrt{3}}{8}$

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Exercise F, Question 8

Question:

a Sketch on the same diagram the circle with polar equation $r = 4\cos\theta$ and the line with polar equation $r = 2\sec\theta$.

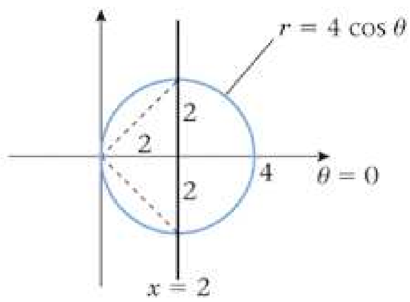
b State polar coordinates for their points of intersection.

Solution:

a $r = 2\sec\theta$

$r\cos\theta = 2$

$x = 2$



b $x = 2$ is a diameter

$r = \sqrt{2^2 + 2^2} = 2\sqrt{2}$

So polar coordinates are

$\left(2\sqrt{2}, \frac{\pi}{4}\right)$ $\left(2\sqrt{2}, -\frac{\pi}{4}\right)$

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Exercise F, Question 9

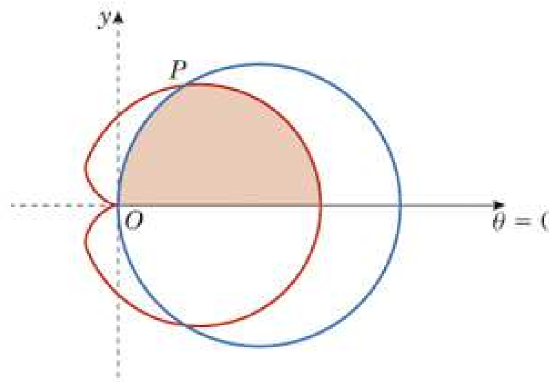
Question:

The diagram shows a sketch of the curves with polar equations

$r = a(1 + \cos\theta)$ and $r = 3a\cos\theta$, $a > 0$

a Find the polar coordinates of the point of intersection P of the two curves.

b Find the area, shaded in the figure, bounded by the two curves and by the initial line $\theta = 0$, giving your answer in terms of a and π.



Solution:

a $a(1+\cos\theta) = 3a\cos\theta$

$$1 = 2\cos\theta$$

$$\cos\theta = \tfrac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

So P is $\left(\frac{3}{2}a, \frac{\pi}{3}\right)$

b Area $= \dfrac{a^2}{2}\displaystyle\int_0^{\frac{\pi}{3}}\left(\tfrac{3}{2} + 2\cos\theta + \frac{\cos 2\theta}{2}\right)d\theta + \tfrac{9}{2}a^2\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos^2\theta\, d\theta$

$$= \frac{a^2}{2}\left[\tfrac{3}{2}\theta + 2\sin\theta + \tfrac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{3}} + \tfrac{9}{4}a^2\left[\theta + \tfrac{1}{2}\sin 2\theta\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$= \frac{a^2}{2}\left[\frac{\pi}{2} + \sqrt{3} + \frac{\sqrt{3}}{8}\right] + \tfrac{9}{4}a^2\left[\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right]$$

$$= \frac{5\pi}{8}a^2$$

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Exercise F, Question 10

Question:

Obtain a Cartesian equation for the curve with polar equation

a $r^2 = \sec 2\theta$, **b** $r^2 = \text{cosec } 2\theta$.

Solution:

a

$$r^2 = \sec 2\theta$$
$$r^2 \cos 2\theta = 1$$
$$r^2(2\cos^2\theta - 1) = 1$$
$$2r^2\cos^2\theta = 1 + r^2$$
$$2x^2 = 1 + x^2 + y^2$$
$$\therefore \quad y^2 = x^2 - 1$$

b

$$r^2 = \text{cosec } 2\theta$$
$$\Rightarrow \quad r^2 \sin 2\theta = 1$$
$$\Rightarrow \quad 2r\sin\theta \, r\cos\theta = 1$$
$$\Rightarrow \quad 2xy = 1$$
$$y = \frac{1}{2x}$$