Solutionbank FP2

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

a Show that $r = \frac{1}{2}(r(r+1) - r(r-1))$.

b Hence show that $\sum_{r=1}^{n} r = \frac{n}{2}(n+1)$ using the method of differences.

Solution:

a $\frac{1}{2}(r(r+1) - r(r-1))$ — Consider RHS.

$= \frac{1}{2}(r^2 + r - r^2 + r)$ — Expand and simplify.

$= \frac{1}{2}(2r)$

$= r$

$=$ LHS

b $\sum_{r=1}^{n} r = \frac{1}{2}\sum_{r=1}^{n} r(r+1) - \frac{1}{2}\sum_{r=1}^{n} r(r-1)$ — Use above.

Use method of differences.

$r = 1$	$\frac{1}{2} \times 1 \times 2$	$-\frac{1}{2} \times 1 \times 0$
$r = 2$	$\frac{1}{2} \times 2 \times 3$	$-\frac{1}{2} \times 2 \times 1$
$r = 3$	$\frac{1}{2} \times 3 \times 4$	$-\frac{1}{2} \times 3 \times 2$
…	…	
$r = n-1$	$\frac{1}{2}(n-1)(n)$	$-\frac{1}{2}(n-1)(n-2)$
$r = n$	$\frac{1}{2}n(n+1)$	$-\frac{1}{2}n(n-1)$

When you add, all terms cancel except $\frac{1}{2}n(n+1)$.

Hence $\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$

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Exercise A, Question 2

Question:

Given $\frac{1}{r(r+1)(r+2)} \equiv \frac{1}{2r(r+1)} - \frac{1}{2(r+1)(r+2)}$

find $\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)}$ using the method of differences.

Solution:

$$\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)} = \sum_{r=1}^{n} \frac{1}{2r(r+1)} - \sum_{r=1}^{n} \frac{1}{2(r+1)(r+2)}$$

Use the information given and equate the summations.

Put $r = 1$ $\quad \frac{1}{2\times1\times2} - \frac{1}{2\times2\times3}$

Use method of differences.

$r = 2$ $\quad \frac{1}{2\times2\times3} - \frac{1}{2\times3\times4}$

All terms cancel except first and last.

$r = 3$ $\quad \frac{1}{2\times3\times4} - \frac{1}{2\times4\times5}$

⋮

$r = n$ $\quad \frac{1}{2n(n+1)} - \frac{1}{2(n+1)(n+2)}$

Add $\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)} = \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$

First and last from above.

$$= \frac{(n+1)(n+2) - 2}{4(n+1)(n+2)}$$

Simplify.

$$= \frac{n^2 + 3n + 2 - 2}{4(n+1)(n+2)}$$

$$= \frac{n(n+3)}{4(n+1)(n+2)}$$

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Exercise A, Question 3

Question:

a Express $\frac{1}{r(r+2)}$ in partial fractions.

b Hence find the sum of the series $\sum_{r=1}^{n} \frac{1}{r(r+2)}$ using the method of differences.

Solution:

a $\frac{1}{r(r+2)} \equiv \frac{A}{r} + \frac{B}{r+2}$ — Set $\frac{1}{r(r+2)}$ identical to $\frac{A}{r} + \frac{B}{r+2}$.

$\equiv \frac{A(r+2)+Br}{r(r+2)}$ — Add the two fractions.

$1 \equiv A(r+2) + Br$

Put $r = 0$

$1 = 2A$

$A = \frac{1}{2}$

~~$\frac{1}{2} = A$~~

Put $r = 1$

$1 = \frac{1}{2}(3) + B$

$B = -\frac{1}{2}$

$\therefore \frac{1}{r(r+2)} = \frac{1}{2r} - \frac{1}{2(r+2)}$

b $\sum_{r=1}^{n} \frac{1}{r(r+2)} = \sum_{r=1}^{n} \frac{1}{2r} - \sum_{r=1}^{n} \frac{1}{2(r+2)}$ — Use method of differences.

$r = 1 \quad \frac{1}{2\times 1} - \cancel{\frac{1}{2\times 3}}$

$r = 2 \quad \frac{1}{2\times 2} - \cancel{\frac{1}{2\times 4}}$

$r = 3 \quad \cancel{\frac{1}{2\times 3}} - \cancel{\frac{1}{2\times 5}}$

$\vdots$

$r = n-1 \quad \cancel{\frac{1}{2(n-1)}} - \frac{1}{2(n+1)}$

$r = n \quad \cancel{\frac{1}{2n}} - \frac{1}{2(n+2)}$

All terms cancel except $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{2(n+1)}$ and $\frac{1}{2(n+2)}$

Add

$$\sum_{r=1}^{n} \frac{1}{r(r+2)} = \frac{1}{2} + \frac{1}{4} - \frac{1}{2(n+1)} - \frac{1}{2(n+2)}$$

$$= \frac{2(n+1)(n+2) + (n+1)(n+2) - 2(n+2) - 2(n+1)}{4(n+1)(n+2)}$$

$$= \frac{2n^2 + 6n + 4 + n^2 + 3n + 2 - 2n - 4 - 2n - 2}{4(n+1)(n+2)}$$

$$= \frac{3n^2 + 5n}{4(n+1)(n+2)}$$

$$= \frac{n(3n+5)}{4(n+1)(n+2)}$$

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Exercise A, Question 4

Question:

a Express $\frac{1}{(r+2)(r+3)}$ in partial fractions.

b Hence find the sum of the series $\sum_{r=1}^{n} \frac{1}{(r+2)(r+3)}$ using the method of differences.

Solution:

a $\frac{1}{(r+2)(r+3)} \equiv \frac{A}{r+2} + \frac{B}{r+3}$

Set $\frac{1}{(r+2)(r+3)}$ identical to $\frac{A}{r+2} + \frac{B}{r+3}$.

$\equiv \frac{A(r+3) + B(r+2)}{(r+2)(r+3)}$

Add the two fractions.

$1 \equiv A(r+3) + B(r+2)$

Compare numerators as they are equivalent.

$r = -3 \Rightarrow B = -1$

$r = -2 \Rightarrow A = 1$

Solve for A and B.

$\therefore \frac{1}{(r+2)(r+3)} = \frac{1}{r+2} - \frac{1}{r+3}$

b $\sum_{r=1}^{n} \frac{1}{(r+2)(r+3)} \equiv \sum_{r=1}^{n} \frac{1}{(r+2)} - \sum_{r=1}^{n} \frac{1}{(r+3)}$

Use the method of differences.

$r = 1 \quad \frac{1}{3} - \frac{1}{4}$

$r = 2 \quad \frac{1}{4} - \frac{1}{5}$

$r = 3 \quad \frac{1}{5} - \frac{1}{6}$

$\vdots$

$r = n \quad \frac{1}{n+2} - \frac{1}{n+3}$

All cancel except first and last.

Add $\sum_{r=1}^{n} \frac{1}{(r+2)(r+3)} = \frac{1}{3} - \frac{1}{n+3}$

$= \frac{n+3-3}{3(n+3)}$

$= \frac{n}{3(n+3)}$

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Exercise A, Question 5

Question:

a Express $\dfrac{5r+4}{r(r+1)(r+2)}$ in partial fractions.

b Hence or otherwise, show that $\displaystyle\sum_{r=1}^{n} \frac{5r+4}{r(r+1)(r+2)} = \frac{7n^2+11n}{2(n+1)(n+2)}$

Solution:

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Exercise A, Question 6

Question:

Given that $\frac{r}{(r+1)!} \equiv \frac{1}{r!} - \frac{1}{(r+1)!}$

find $\sum_{r=1}^{n} \frac{r}{(r+1)!}$

Solution:

$$\sum_{r=1}^{n} \frac{r}{(r+1)!} \equiv \sum_{r=1}^{n} \frac{1}{r!} - \sum_{r=1}^{n} \frac{1}{(r+1)!}$$

Use given.

$r = 1 \quad \frac{1}{1!} - \frac{1}{2!}$

$r = 2 \quad \frac{1}{2!} - \frac{1}{3!}$

$r = 3 \quad \frac{1}{3!} - \frac{1}{4!}$

$\vdots$

$r = n \quad \frac{1}{n!} - \frac{1}{(n+1)!}$

Use method of differences.

All cancel except first and last term.

$\therefore$

Add $\quad \sum_{r=1}^{n} \frac{r}{(r+1)!} = 1 - \frac{1}{(n+1)!}$

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Exercise A, Question 7

Question:

Given that $\frac{2r+1}{r^2(r+1)^2} \equiv \frac{1}{r^2} - \frac{1}{(r+1)^2}$

find $\sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2}$.

Solution:

$$\sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2} = \sum_{r=1}^{n} \frac{1}{r^2} - \sum_{r=1}^{n} \frac{1}{(r+1)^2}$$

Use given.

Use method of differences.

$r = 1$: $\frac{1}{1} - \cancel{\frac{1}{2^2}}$

$r = 2$: $\cancel{\frac{1}{2^2}} - \cancel{\frac{1}{3^2}}$

$r = 3$: $\cancel{\frac{1}{3^2}} - \cancel{\frac{1}{4^2}}$

$\vdots$

$r = n$: $\cancel{\frac{1}{n^2}} - \frac{1}{(n+1)^2}$

All terms cancel except first and last.

So adding $\sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2} = 1 - \frac{1}{(n+1)^2}$

$$= \frac{(n+1)^2 - 1}{(n+1)^2}$$

Simplify.

$$= \frac{n^2 + 2n}{(n+1)^2}$$

$$= \frac{n(n+2)}{(n+1)^2}$$