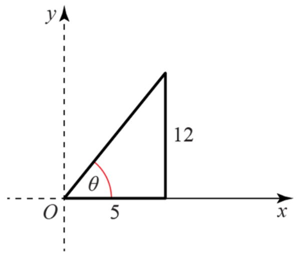
Exercise 8A

1 a

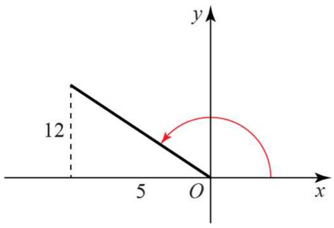


$\arctan\left(\frac{12}{5}\right)=1.176$

$r=\sqrt{5^2+12^2}=13$

$\therefore$ point is (13, 1.176)

b

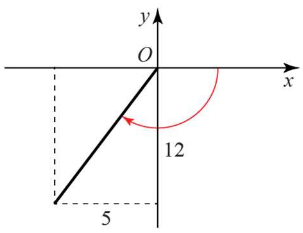


$r=\sqrt{(-5)^2+12^2}=13$

$\theta=\pi-\arctan\left(\frac{12}{5}\right)=1.966$

$\therefore$ point is (13, 1.966)

c



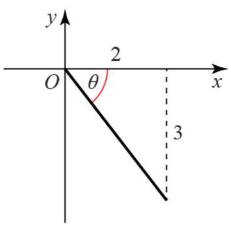
$\theta=-\left(\pi-\arctan\frac{12}{5}\right)$

$=-1.966$

$r=\sqrt{(-5)^2+(-12)^2}=13$

$\therefore$ point is (13, −1.966)

d

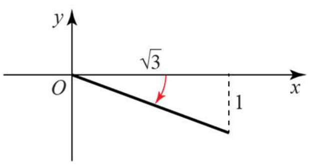


$\theta=-\arctan\frac{3}{2}=-0.983$

$r=\sqrt{2^2+(-3)^2}=\sqrt{13}$

$\therefore$ point is $(\sqrt{13},-0.983)$

e



$\theta=-\arctan\frac{1}{\sqrt{3}}=-\frac{\pi}{6}$

$r=\sqrt{\sqrt{3}^2+(-1)^2}=\sqrt{4}=2$

$\therefore$ point is $(2,-\frac{\pi}{6})$

2 a $x=6\cos\left(\frac{\pi}{6}\right)=\frac{6\sqrt{3}}{2}=3\sqrt{3}$

$y=6\sin\left(\frac{\pi}{6}\right)=3$ $\therefore$ point is $(3\sqrt{3}, 3)$

b $x=6\cos\left(-\frac{\pi}{6}\right)=\frac{6\sqrt{3}}{2}=3\sqrt{3}$

$y=6\sin\left(-\frac{\pi}{6}\right)=-3$ $\therefore$ point is $(3\sqrt{3}, -3)$

2 c $x = 6\cos\left(\frac{3\pi}{4}\right) = -\frac{6}{\sqrt{2}}$ or $-3\sqrt{2}$

$y = 6\sin\left(\frac{3\pi}{4}\right) = \frac{6}{\sqrt{2}} = 3\sqrt{2}$ $\therefore$ point is $(-3\sqrt{2}, 3\sqrt{2})$

d $x = 10\cos\left(\frac{5\pi}{4}\right) = -\frac{10}{\sqrt{2}} = -5\sqrt{2}$

$y = 10\sin\left(\frac{5\pi}{4}\right) = -\frac{10}{\sqrt{2}} = -5\sqrt{2}$ $\therefore$ point is $(-5\sqrt{2}, -5\sqrt{2})$

e $x = 2\cos(\pi) = -2$

$y = 2\sin(\pi) = 0$ $\therefore$ point is $(-2, 0)$

3 a $r = 2$ is $x^2 + y^2 = 4$

b $r = 3\sec\theta$

$\Rightarrow r\cos\theta = 3$ i.e. $x = 3$

c $r = 5\operatorname{cosec}\theta$

$\Rightarrow r\sin\theta = 5$ i.e. $y = 5$

d $r = 4a\tan\theta\sec\theta$

$r = \frac{4a\sin\theta}{\cos^2\theta}$

$r\cos^2\theta = 4a\sin\theta$ ← Multiply by *r*.

$r^2\cos^2\theta = 4ar\sin\theta$

$\therefore x^2 = 4ay$ or $y = \frac{x^2}{4a}$

e $r = 2a\cos\theta$

$r^2 = 2ar\cos\theta$

$\therefore x^2 + y^2 = 2ax$ or $(x-a)^2 + y^2 = a^2$

f $r = 3a\sin\theta$ ← Multiply by *r*.

$r^2 = 3ar\sin\theta$

$x^2 + y^2 = 3ay$ or $x^2 + \left(y - \frac{3a}{2}\right)^2 = \frac{9a^2}{4}$

g $r = 4(1-\cos 2\theta)$ ← Use $\cos 2\theta = 1 - 2\sin^2\theta$ $\therefore 2\sin^2\theta = 1 - \cos 2\theta$

$r = 4 \times 2\sin^2\theta$

$r^3 = 8r^2\sin^2\theta$ ← $\times r^2$

$\therefore \left(x^2 + y^2\right)^{\frac{3}{2}} = 8y^2$

3 h $r = 2\cos^2\theta$

$r^3 = 2r^2\cos^2\theta$ ← $\times\, r^2$

$(x^2+y^2)^{\frac{3}{2}} = 2x^2$

i $r^2 = 1+\tan^2\theta$

$\therefore \quad r^2 = \sec^2\theta$ ← Use $\sec^2\theta = 1+\tan^2\theta$.

$\therefore r^2\cos^2\theta = 1$

i.e. $x^2 = 1$ or $x = \pm 1$

4 a $x^2+y^2 = 16$

$\Rightarrow \quad r^2 = 16$ or $r = 4$

b $xy = 4$

$\Rightarrow r\cos\theta\; r\sin\theta = 4$

$$r^2 = \frac{4}{\cos\theta\sin\theta} = \frac{8}{2\cos\theta\sin\theta}$$

i.e. $r^2 = 8 \text{ cosec } 2\theta$

c $\left(x^2+y^2\right)^2 = 2xy$

$\Rightarrow \quad \left(r^2\right)^2 = 2r\cos\theta\; r\sin\theta$

$r^4 = 2r^2\cos\theta\sin\theta$

$r^2 = \sin 2\theta$

d $x^2+y^2-2x = 0$

$\Rightarrow \quad r^2 - 2r\cos\theta = 0$

$r^2 = 2r\cos\theta$

$r = 2\cos\theta$

e $(x+y)^2 = 4$

$\Rightarrow \quad x^2+y^2+2xy = 4$

$\Rightarrow \quad r^2 + 2r\cos\theta\; r\sin\theta = 4$

$\Rightarrow \quad r^2(1+\sin 2\theta) = 4$

$$r^2 = \frac{4}{1+\sin 2\theta}$$

4 f

$$x - y = 3$$
$$r\cos\theta - r\sin\theta = 3$$
$$r(\cos\theta - \sin\theta) = 3$$
$$r\left(\frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta\right) = \frac{3}{\sqrt{2}}$$
$$r\cos\left(\theta + \frac{\pi}{4}\right) = \frac{3}{\sqrt{2}}$$
$$\therefore \quad r = \frac{3}{\sqrt{2}}\sec\left(\theta + \frac{\pi}{4}\right)$$

g

$$y = 2x$$
$$\Rightarrow \quad r\sin\theta = 2r\cos\theta$$
$$\tan\theta = 2 \quad \text{or} \quad \theta = \arctan 2$$

h

$$y = -\sqrt{3}x + a$$
$$r\sin\theta = -\sqrt{3}\,r\cos\theta + a$$
$$r(\sin\theta + \sqrt{3}\cos\theta) = a$$
$$r\left(\frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta\right) = \frac{a}{2}$$
$$r\sin\left(\theta + \frac{\pi}{3}\right) = \frac{a}{2}$$
$$\therefore \quad r = \frac{a}{2}\operatorname{cosec}\left(\theta + \frac{\pi}{3}\right)$$

i

$$y = x(x - a)$$
$$r\sin\theta = r\cos\theta\,(r\cos\theta - a)$$
$$\tan\theta = r\cos\theta - a$$
$$r\cos\theta = \tan\theta + a$$
$$r = \tan\theta\sec\theta + a\sec\theta$$

Challenge

First we convert
(r_1, θ_1) and (r_2, θ_2) to their Cartesian coordinate representation by using the relations
$r\cos\theta = x$ and $r\sin\theta = y$
This gives the Cartesian points
$(x_1, y_1) = (r_1\cos\theta_1, r_1\sin\theta_1)$ and $(x_2, y_2) = (r_2\cos\theta_2, r_2\sin\theta_2)$
In Cartesian coordinates, the distance between two points,
(x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Substituting $(x_1, y_1) = (r_1\cos\theta_1, r_1\sin\theta_1)$ and $(x_2, y_2) = (r_2\cos\theta_2, r_2\sin\theta_2)$ into this expression for d, we obtain

$$d = \sqrt{(r_2\cos\theta_2 - r_1\cos\theta_1)^2 + (r_2\sin\theta_2 - r_1\sin\theta_1)^2}$$
$$= \sqrt{\begin{array}{l}(r_2^2\cos^2\theta_2 - 2r_2r_1\cos\theta_2\cos\theta_1 + r_1^2\cos^2\theta_1) \\ +(r_2^2\sin^2\theta_2 - 2r_2r_1\sin\theta_2\sin\theta_1 + r_1^2\sin^2\theta_1)\end{array}}$$
$$= \sqrt{r_2^2 + r_1^2 - 2r_2r_1(\cos\theta_2\cos\theta_1 + \sin\theta_2\sin\theta_1)}$$
$$= \sqrt{r_1^2 + r_2^2 - 2r_2r_1\cos(\theta_1 - \theta_2)}$$