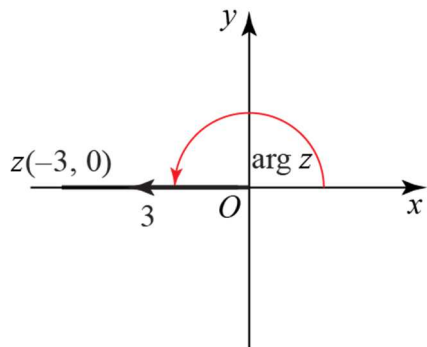


Exercise 3A

1 a -3

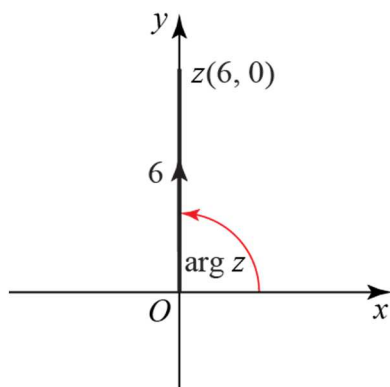


$$r = 3$$

$$\theta = \arg z = \pi$$

$$\therefore -3 = 3e^{\pi i}$$

b 6i

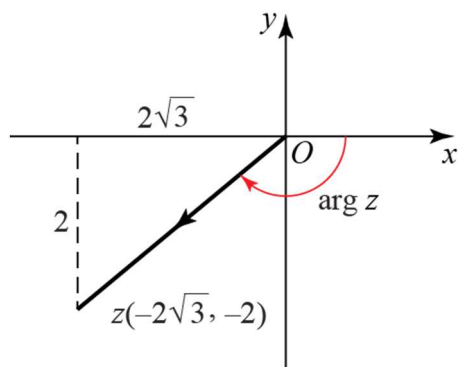


$$r = 6$$

$$\theta = \arg z = \frac{\pi}{2}$$

$$\therefore 6i = 6e^{\frac{\pi i}{2}}$$

1 c $-2\sqrt{3} - 2i$

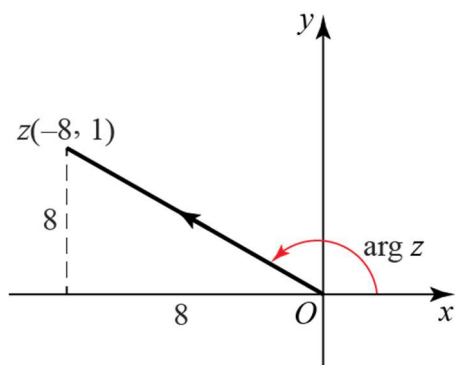


$$r = \sqrt{(-2\sqrt{3})^2 + (-2)^2} = \sqrt{12 + 4} = \sqrt{16} = 4$$

$$\theta = \arg z = -\pi + \tan^{-1}\left(\frac{2}{2\sqrt{3}}\right) = -\pi + \frac{\pi}{6} = -\frac{5\pi}{6}$$

$$\therefore -2\sqrt{3} - 2i = 4e^{-\frac{5\pi}{6}}$$

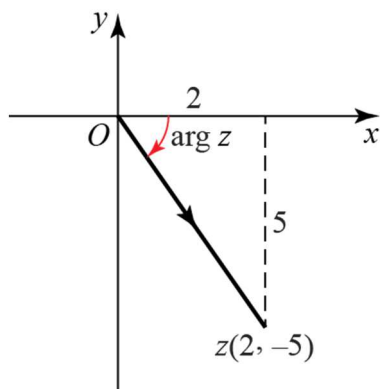
d $-8 + i$



$$r = \sqrt{(-8)^2 + 1^2} = \sqrt{65}$$

$$\theta = \pi - \tan^{-1}\left(\frac{1}{8}\right) = 3.02 \text{ rad (2 d.p.)}$$

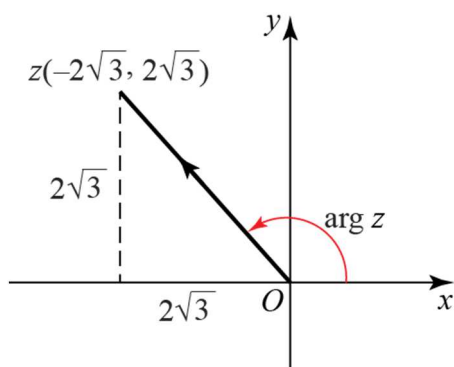
$$\therefore -8 + i = \sqrt{65} e^{3.02i}$$

1 e $2 - 5i$ 

$$r = \sqrt{2^2 + (-5)^2} = \sqrt{29}$$

$$\theta = -\tan^{-1}\left(\frac{5}{2}\right) = -1.19 \text{ rad (2 d.p.)}$$

$$\therefore 2 - 5i = \sqrt{29} e^{-1.19i}$$

f $-2\sqrt{3} + 2\sqrt{3}i$ 

$$r = \sqrt{(-2\sqrt{3})^2 + (2\sqrt{3})^2} = \sqrt{12 + 12} = \sqrt{24}$$

$$= \sqrt{4} \sqrt{6} = 2\sqrt{6}$$

$$\theta = \pi - \tan^{-1}\left(\frac{2\sqrt{3}}{2\sqrt{3}}\right) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

$$\therefore -2\sqrt{3} + 2\sqrt{3}i = 2\sqrt{6} e^{\frac{3\pi}{4}}$$

Further Pure Maths 2

Solution Bank

$$\begin{aligned} 1 \text{ g } \sqrt{8} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right) \\ = 2\sqrt{2} e^{\frac{\pi i}{4}} \end{aligned}$$

$$\begin{aligned} r &= \sqrt{8} = \sqrt{4} \sqrt{2} = 2\sqrt{2} \\ \theta &= \frac{\pi}{4} \end{aligned}$$

$$\begin{aligned} \text{h } 8 \left(\cos \frac{\pi}{6} - i \sin \frac{\pi}{6} \right) \\ = 8 \left(\cos \left(-\frac{\pi}{6} \right) + i \sin \left(-\frac{\pi}{6} \right) \right) \\ = 8e^{-\frac{\pi i}{6}} \end{aligned}$$

$$r = 8, \theta = -\frac{\pi}{6}$$

$$\begin{aligned} \text{i } 2 \left(\cos \frac{\pi}{5} - i \sin \frac{\pi}{5} \right) \\ = 2 \left(\cos \left(-\frac{\pi}{5} \right) + i \sin \left(-\frac{\pi}{5} \right) \right) \\ = 2e^{-\frac{\pi i}{5}} \end{aligned}$$

$$r = 2, \theta = -\frac{\pi}{5}$$

$$\begin{aligned} 2 \text{ a } e^{\frac{\pi i}{3}} &= \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \\ &= \frac{1}{2} + \frac{\sqrt{3}}{2} i \end{aligned}$$

$$\begin{aligned} \text{b } 4e^{\pi i} &= 4(\cos \pi + i \sin \pi) \\ &= 4(-1 + i(0)) \\ &= -4 \end{aligned}$$

$$\begin{aligned} \text{c } 3\sqrt{2} e^{\frac{\pi i}{4}} &= 3\sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right) \\ &= 3\sqrt{2} \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i \right) \\ &= 3 + 3i \end{aligned}$$

$$\begin{aligned} \text{d } 8e^{\frac{\pi i}{6}} &= 8 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) \\ &= 8 \left(\frac{\sqrt{3}}{2} + \frac{1}{2} i \right) \\ &= 4\sqrt{3} + 4i \end{aligned}$$

$$\begin{aligned} \text{e } 3e^{-\frac{\pi i}{2}} &= 3 \left(\cos \left(-\frac{\pi}{2} \right) + i \sin \left(-\frac{\pi}{2} \right) \right) \\ &= 3(0 - i) \\ &= -3i \end{aligned}$$

$$\begin{aligned}
 2 \text{ f } e^{\frac{5\pi i}{6}} &= \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \\
 &= -\frac{\sqrt{3}}{2} + \frac{1}{2}i
 \end{aligned}$$

$$\begin{aligned}
 \text{g } e^{-\pi i} &= \cos(-\pi) + i \sin(-\pi) \\
 &= -1 + i(0) \\
 &= -1
 \end{aligned}$$

$$\begin{aligned}
 \text{h } 3\sqrt{2}e^{\frac{3\pi i}{4}} &= 3\sqrt{2} \left(\cos\left(-\frac{3\pi}{4}\right) + i \sin\left(-\frac{3\pi}{4}\right) \right) \\
 &= 3\sqrt{2} \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i \right) \\
 &= -3 - 3i
 \end{aligned}$$

$$\begin{aligned}
 \text{i } 8e^{\frac{4\pi i}{3}} &= 8 \left(\cos\left(-\frac{4\pi}{3}\right) + i \sin\left(-\frac{4\pi}{3}\right) \right) \\
 &= 8 \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) \\
 &= -4 + 4\sqrt{3}i
 \end{aligned}$$

$$\begin{aligned}
 3 \text{ a } e^{\frac{16\pi i}{13}} &= \cos\left(\frac{16\pi}{13}\right) + i \sin\left(\frac{16\pi}{13}\right) \\
 &= \cos\left(-\frac{10\pi}{13}\right) + i \sin\left(-\frac{10\pi}{13}\right)
 \end{aligned}$$

↪ -2π from the argument.

$$\begin{aligned}
 \text{b } 4e^{\frac{17\pi i}{5}} &= 4 \left(\cos\left(\frac{17\pi}{5}\right) + i \sin\left(\frac{17\pi}{5}\right) \right) \\
 &= 4 \left(\cos\left(\frac{7\pi}{5}\right) + i \sin\left(\frac{7\pi}{5}\right) \right) \\
 &= 4 \left(\cos\left(-\frac{3\pi}{5}\right) + i \sin\left(-\frac{3\pi}{5}\right) \right)
 \end{aligned}$$

$$\begin{aligned}
 \text{c } 5e^{\frac{9\pi i}{8}} &= 5 \left(\cos\left(-\frac{9\pi}{8}\right) + i \sin\left(-\frac{9\pi}{8}\right) \right) \\
 &= 5 \left(\cos\left(\frac{7\pi}{8}\right) + i \sin\left(\frac{7\pi}{8}\right) \right)
 \end{aligned}$$

$$4 \quad e^{i\theta} = \cos \theta + i \sin \theta \quad (1)$$

$$e^{-i\theta} = \cos(-\theta) + i \sin(-\theta) = \cos \theta - i \sin \theta \quad (2)$$

$$(1) - (2) \Rightarrow e^{i\theta} - e^{-i\theta} = 2i \sin \theta$$

$$\frac{1}{2i}(e^{i\theta} - e^{-i\theta}) = \sin \theta$$

$$\therefore \sin \theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta}) \text{ (as required)}$$