OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4726

Further Pure Mathematics 2

Specimen Paper

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

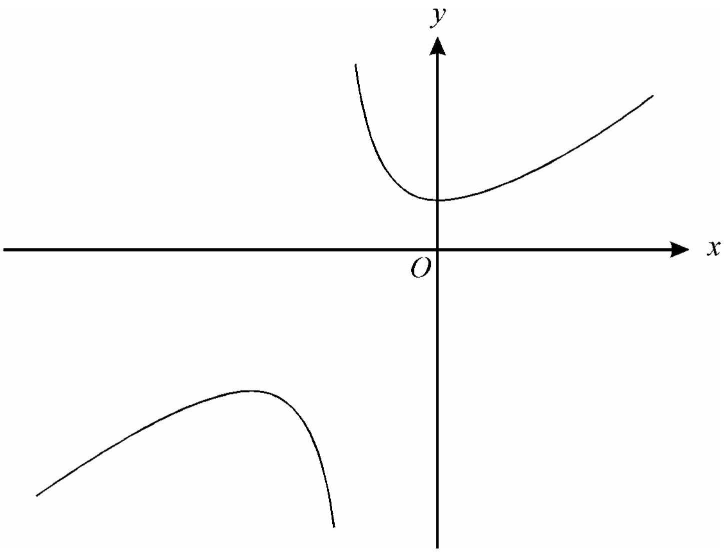
- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

1 **(i)** Starting from the definition of $\cosh x$ in terms of e^x, show that $\cosh 2x = 2\cosh^2 x - 1$. [2]

(ii) Given that $\cosh 2x = k$, where $k > 1$, express each of $\cosh x$ and $\sinh x$ in terms of k. [4]

2



The diagram shows the graph of

$$y = \frac{2x^2 + 3x + 3}{x + 1}.$$

(i) Find the equations of the asymptotes of the curve. [3]

(ii) Prove that the values of y between which there are no points on the curve are -5 and 3. [4]

3 **(i)** Find the first three terms of the Maclaurin series for $\ln(2 + x)$. [4]

(ii) Write down the first three terms of the series for $\ln(2 - x)$, and hence show that, if x is small, then

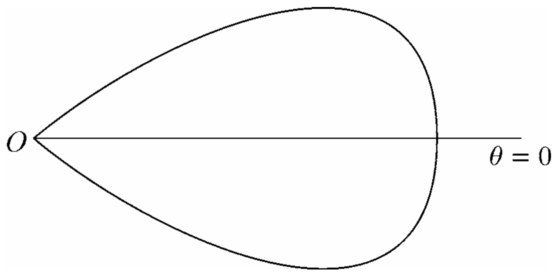
$$\ln\left(\frac{2 + x}{2 - x}\right) \approx x.$$ [3]

4 The equation of a curve, in polar coordinates, is

$$r = 2\cos 2\theta \qquad (-\pi < \theta \leqslant \pi).$$

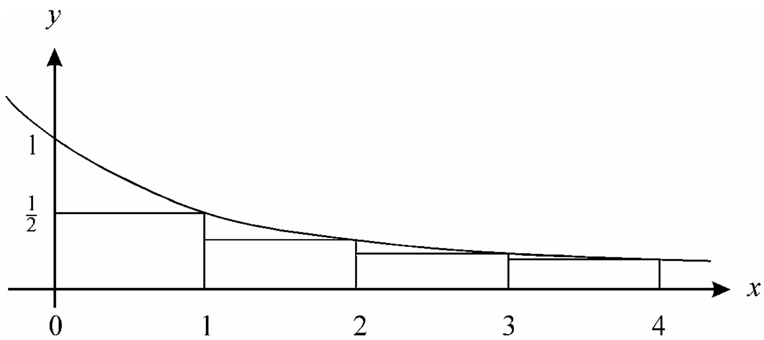
(i) Find the values of θ which give the directions of the tangents at the pole. [3]

One loop of the curve is shown in the diagram.



(ii) Find the exact value of the area of the region enclosed by the loop. [5]

5



The diagram shows the curve $y = \dfrac{1}{x+1}$ together with four rectangles of unit width.

(i) Explain how the diagram shows that

$$\tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4} + \tfrac{1}{5} < \int_0^4 \frac{1}{x+1}\,dx.$$ [2]

The curve $y = \dfrac{1}{x+2}$ passes through the top left-hand corner of each of the four rectangles shown.

(ii) By considering the rectangles in relation to this curve, write down a second inequality involving $\tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4} + \tfrac{1}{5}$ and a definite integral. [2]

(iii) By considering a suitable range of integration and corresponding rectangles, show that

$$\ln(500.5) < \sum_{r=2}^{1000} \frac{1}{r} < \ln(1000).$$ [4]

6 **(i)** Given that $I_n = \int_0^1 x^n \sqrt{(1-x)}\ \mathrm{d}x$, prove that, for $n \geqslant 1$,

$$(2n+3)I_n = 2nI_{n-1}.$$ [6]

(ii) Hence find the exact value of I_2. [4]

7 The curve with equation

$$y = \frac{x}{\cosh x}$$

has one stationary point for $x > 0$.

(i) Show that the x-coordinate of this stationary point satisfies the equation $x \tanh x - 1 = 0$. [2]

The positive root of the equation $x \tanh x - 1 = 0$ is denoted by α.

(ii) Draw a sketch showing (for positive values of x) the graph of $y = \tanh x$ and its asymptote, and the graph of $y = \frac{1}{x}$. Explain how you can deduce from your sketch that $\alpha > 1$. [3]

(iii) Use the Newton-Raphson method, taking first approximation $x_1 = 1$, to find further approximations x_2 and x_3 for α. [5]

(iv) By considering the approximate errors in x_1 and x_2, estimate the error in x_3. [3]

8 **(i)** Use the substitution $t = \tan \frac{1}{2}x$ to show that

$$\int_0^{\frac{1}{2}\pi} \sqrt{\frac{1-\cos x}{1+\sin x}}\ \mathrm{d}x = 2\sqrt{2}\int_0^1 \frac{t}{(1+t)(1+t^2)}\ \mathrm{d}t.$$ [4]

(ii) Express $\frac{t}{(1+t)(1+t^2)}$ in partial fractions. [5]

(iii) Hence find $\int_0^{\frac{1}{2}\pi} \sqrt{\frac{1-\cos x}{1+\sin x}}\ \mathrm{d}x$, expressing your answer in an exact form. [4]