OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4726

Further Pure Mathematics 2

MARK SCHEME

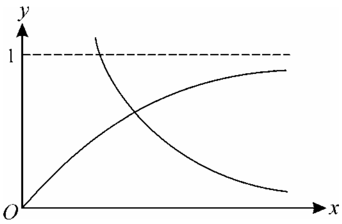
Specimen Paper

MAXIMUM MARK	72

This mark scheme consists of 4 printed pages.

1	**(i)**	$\text{RHS} = 2\left(\frac{1}{2}(e^x + e^{-x})\right)^2 - 1 = \frac{1}{2}(e^{2x} + e^{-2x}) = \text{LHS}$	M1	For correct squaring of $(e^x + e^{-x})$
			A1 **2**	For completely correct proof
	(ii)	$2\cosh^2 x - 1 = k \Rightarrow \cosh x = \sqrt{\left(\frac{1}{2}(1+k)\right)}$	M1	For use of **(i)** and solving for $\cosh x$
			A1	For correct positive square root only
		$2\sinh^2 x + 1 = k \Rightarrow \sinh x = \pm\sqrt{\left(\frac{1}{2}(k-1)\right)}$	M1	For use of $\cosh^2 x - \sinh^2 x = 1$, or equivalent
			A1 **4**	For both correct square roots
			6	
2	**(i)**	$x = -1$ is an asymptote	B1	For correct equation of vertical asymptote
		$y = 2x + 1 + \dfrac{2}{x+1}$	M1	For algebraic division, or equivalent
		Hence $y = 2x + 1$ is an asymptote	A1 **3**	For correct equation of oblique asymptote
	(ii)	*EITHER*: Quadratic $2x^2 + (3-y)x + (3-y) = 0$	M1	For using discriminant of relevant quadratic
		has no real roots if $(3-y)^2 < 8(3-y)$	A1	For correct inequality or equation in y
		Hence $(3-y)(-5-y) < 0$	M1	For factorising, or equivalent
		So required values are 3 and -5	A1	For given answer correctly shown
		OR: $\dfrac{dy}{dx} = 2 - \dfrac{2}{(x+1)^2} = 0$	M1	For differentiating and equating to zero
		Hence $(x+1)^2 = 1$	A1	For correct simplified quadratic in x
		So $x = -2$ and $0 \Rightarrow y = -5$ and 3	M1	For solving for x and substituting to find y
			A1 **4**	For given answer correctly shown
			7	
3	**(i)**	*EITHER*: If $f(x) = \ln(x+2)$, then $f'(x) = \dfrac{1}{2+x}$	M1	For at least one differentiation attempt
		and $f''(x) = -\dfrac{1}{(2+x)^2}$	A1	For correct first and second derivatives
		$f(0) = \ln 2,\ f'(0) = \frac{1}{2},\ f''(0) = -\frac{1}{4}$	A1√	For all three evaluations correct
		Hence $\ln(x+2) = \ln 2 + \frac{1}{2}x - \frac{1}{8}x^2 + \ldots$	A1	For three correct terms
		OR: $\ln(2+x) = \ln[2(1+\frac{1}{2}x)]$	M1	For factorising in this way
		$= \ln 2 + \ln(1+\frac{1}{2}x)$	A1	For using relevant log law correctly
		$= \ln 2 + \frac{1}{2}x - \dfrac{(\frac{1}{2}x)^2}{2} + \ldots$	M1	For use of standard series expansion
		$= \ln 2 + \frac{1}{2}x - \frac{1}{8}x^2 + \ldots$	A1 **4**	For three correct terms
	(ii)	$\ln(2-x) \approx \ln 2 - \frac{1}{2}x - \frac{1}{8}x^2$	B1√	For replacing x by $-x$
		$\ln\left(\dfrac{2+x}{2-x}\right) \approx (\ln 2 + \frac{1}{2}x - \frac{1}{8}x^2) - (\ln 2 - \frac{1}{2}x - \frac{1}{8}x^2)$	M1	For subtracting the two series
		$\approx x$, as required	A1 **3**	For showing given answer correctly
			7	

4	**(i)**	$r=0 \Rightarrow \cos 2\theta = 0 \Rightarrow \theta = \pm\frac{1}{4}\pi, \pm\frac{3}{4}\pi$	M1 A1 A1 **3**	For equating r to zero and solving for θ For any two correct values For all four correct values and no others
	(ii)	Area is $\frac{1}{2}\int_{-\frac{1}{4}\pi}^{\frac{1}{4}\pi} 4\cos^2 2\theta \, d\theta$ i.e. $\int_{-\frac{1}{4}\pi}^{\frac{1}{4}\pi} 1+\cos 4\theta \, d\theta = \left[\theta + \frac{1}{4}\sin 4\theta\right]_{-\frac{1}{4}\pi}^{\frac{1}{4}\pi} = \frac{1}{2}\pi$	M1 B1√ M1 A1 A1 **5**	For us of correct formula $\frac{1}{2}\int r^2 \, d\theta$ For correct limits from **(i)** For using double-angle formula For $\theta + \frac{1}{4}\sin 4\theta$ correct For correct (exact) answer
			8	
5	**(i)**	LHS is the total area of the four rectangles RHS is the corresponding area under the curve, which is clearly greater	B1 B1 **2**	For identifying rectangle areas (not heights) For correct explanation
	(ii)	$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5} > \int_0^4 \frac{1}{x+2} \, dx$	M1 A1 **2**	For attempt at relevant new inequality For correct statement
	(iii)	Sum is the area of 999 rectangles Bounds are $\int_0^{999} \frac{1}{x+2} \, dx$ and $\int_0^{999} \frac{1}{x+1} \, dx$ So lower bound is $\left[\ln(x+2)\right]_0^{999} = \ln(500.5)$ and upper bound is $\left[\ln(x+1)\right]_0^{999} = \ln(1000)$	M1 M1 A1 A1 **4**	For considering the sum as an area again For stating either integral as a bound For showing the given value correctly Ditto
			8	
6	**(i)**	$I_n = \left[-\frac{2}{3}x^n(1-x)^{\frac{3}{2}}\right]_0^1 + \frac{2}{3}n\int_0^1 x^{n-1}(1-x)^{\frac{3}{2}} \, dx$ $= \frac{2}{3}n\int_0^1 x^{n-1}(1-x)\sqrt{(1-x)} \, dx$ $= \frac{2}{3}n(I_{n-1} - I_n)$ Hence $(2n+3)I_n = 2nI_{n-1}$, as required	M1 A1 M1 M1 A1 A1 **6**	For using integration by parts For correct first stage result For use of limits in integrated term For splitting the remaining integral up For correct relation between I_n and I_{n-1} For showing given answer correctly
	(ii)	$I_2 = \frac{4}{7}I_1 = \frac{4}{7}\times\frac{2}{5}I_0$ Hence $I_2 = \frac{8}{35}\left[-\frac{2}{3}(1-x)^{\frac{3}{2}}\right]_0^1 = \frac{16}{105}$	M1 A1 M1 A1 **4**	For two uses of the recurrence relation For correct expression in terms of I_0 For evaluation of I_0 For correct answer
			10	

7 (i)	$\dfrac{dy}{dx} = \dfrac{\cosh x - x\sinh x}{\cosh^2 x}$	M1		For differentiating and equating to zero
	Max occurs when $\cosh x = x\sinh x$, i.e. $x\tanh x = 1$	A1	2	For showing given result correctly
(ii)		B1		For correct sketch of $y = \tanh x$
		B1		For identification of asymptote $y = 1$
		B1	3	For correct explanation of $\alpha > 1$ based on intersection $(1, 1)$ of $y = 1/x$ with $y = 1$
(iii)	$x_{n+1} = x_n - \dfrac{x_n \tanh x_n - 1}{\tanh x_n + x_n \operatorname{sech}^2 x_n}$	M1		For correct Newton-Raphson structure
		A1		For all details in $x - \dfrac{f(x)}{f'(x)}$ correct
	$x_1 = 1 \Rightarrow x_2 = 1.20177...$	M1		For using Newton-Raphson at least once
		A1		For x_2 correct to at least 3sf
	$x_3 = 1.1996785...$	A1	5	For x_3 correct to at least 4sf
(iv)	$e_1 \approx 0.2,\ e_2 \approx -0.002$	B1√		For both magnitudes correct
	$\dfrac{e_3}{e_2^2} \approx \dfrac{e_2}{e_1^2} \Rightarrow e_3 \approx -2\times10^{-7}$	M1		For use of quadratic convergence property
		A1	3	For answer of correct magnitude
			13	
8 (i)	$\dfrac{dt}{dx} = \frac{1}{2}(1+t^2)$	B1		For this relation, stated or used
	$\displaystyle\int_0^{\frac{1}{2}\pi} \sqrt{\frac{1-\cos}{1+\sin x}}\,dx = \int_0^1 \sqrt{\frac{1-\frac{1-t^2}{1+t^2}}{1+\frac{2t}{1+t^2}}}.\frac{2}{1+t^2}\,dt$	M1		For complete substitution for *x* in integrand
		B1		For justification of limits 0 and 1 for *t*
	$\displaystyle= \int_0^1 \sqrt{\frac{2t^2}{(1+t)^2}}.\frac{2}{1+t^2}\,dt = 2\sqrt{2}\int_0^1 \frac{t}{(1+t)(1+t^2)}\,dt$	A1	4	For correct simplification to given answer
(ii)	$\dfrac{t}{(1+t)(1+t^2)} = \dfrac{A}{1+t} + \dfrac{Bt+C}{1+t^2}$	B1		For statement of correct form of pfs
	Hence $t \equiv A(1+t^2) + (Bt+C)(1+t)$	M1		For any use of the identity involving *B* or *C*
	From which $A = -\frac{1}{2}, B = \frac{1}{2}, C = \frac{1}{2}$	B1		For correct value of *A*
		A1		For correct value of *B*
		A1	5	For correct value of *C*
(iii)	Int is $2\sqrt{2}\left[-\frac{1}{2}\ln(1+t) + \frac{1}{4}\ln(1+t^2) + \frac{1}{2}\tan^{-1} t\right]_0^1$	B1√		For both logarithm terms correct
		B1√		For the inverse tan term correct
	$= \frac{1}{4}(\pi - 2\ln 2)\sqrt{2}$	M1		For use of appropriate limits
		A1	4	For correct (exact) answer in any form
			13	