

**Thursday 13 June 2013 – Morning**

**A2 GCE MATHEMATICS**

**4726/01 Further Pure Mathematics 2**

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4726/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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- 1 By using the substitution  $t = \tan \frac{1}{2}\theta$ , find  $\int_0^{\frac{1}{2}\pi} \frac{1}{1 + \cos \theta} d\theta$ . [5]
- 2 (i) Using the definitions for  $\cosh x$  and  $\sinh x$  in terms of  $e^x$  and  $e^{-x}$ , show that  $\cosh^2 x - \sinh^2 x \equiv 1$ . [3]  
(ii) Hence solve the equation  $\sinh^2 x = 5 \cosh x - 7$ , giving your answers in logarithmic form. [5]
- 3 It is given that  $f(x) = \tanh^{-1}\left(\frac{1-x}{3+x}\right)$  for  $x > -1$ .  
(i) Show that  $f''(x) = \frac{1}{2(x+1)^2}$ . [6]  
(ii) Hence find the Maclaurin series for  $f(x)$  up to and including the term in  $x^2$ . [4]
- 4 It is given that  $I_n = \int_0^{\frac{1}{2}\pi} \cos^n x dx$  for  $n \geq 0$ .  
(i) Show that  $I_n = \frac{n-1}{n} I_{n-2}$  for  $n \geq 2$ . [5]  
(ii) Hence find  $I_{11}$  as an exact fraction. [3]

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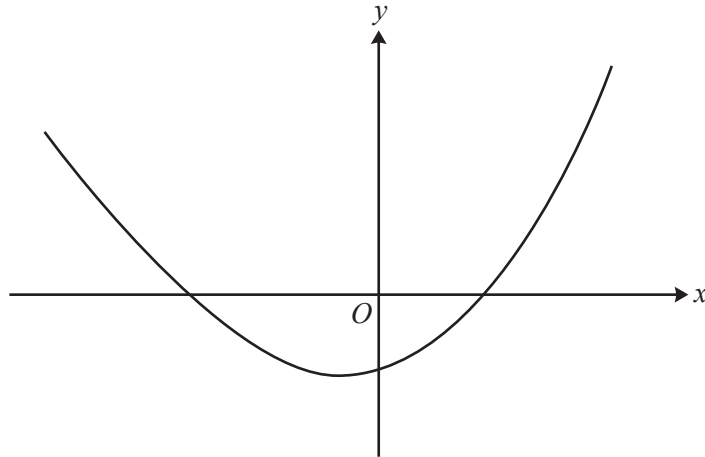
5 You are given that the equation  $x^3 + 4x^2 + x - 1 = 0$  has a root,  $\alpha$ , where  $-1 < \alpha < 0$ .

(i) Show that the Newton-Raphson iterative formula for this equation can be written in the form

$$x_{n+1} = \frac{2x_n^3 + 4x_n^2 + 1}{3x_n^2 + 8x_n + 1}. \quad [3]$$

(ii) Using the initial value  $x_1 = -0.7$ , find  $x_2$  and  $x_3$  and find  $\alpha$  correct to 5 decimal places. [3]

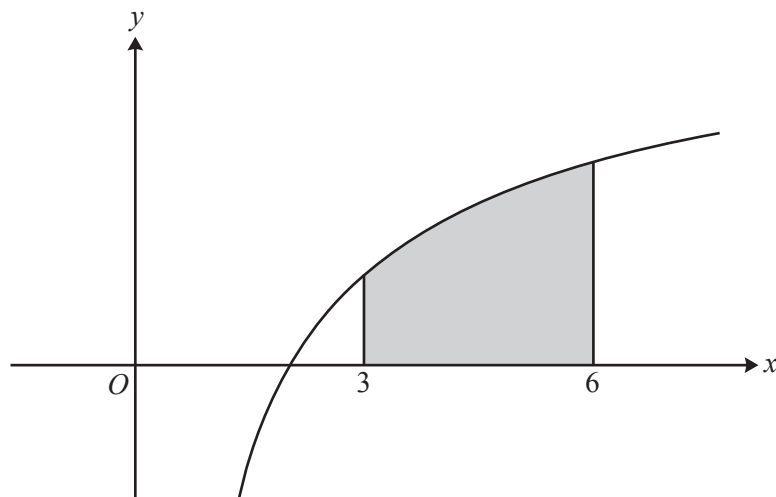
(iii) The diagram shows a sketch of the curve  $y = x^3 + 4x^2 + x - 1$  for  $-1.5 \leq x \leq 1$ .



Using the copy of the diagram in your answer book, explain why the initial value  $x_1 = 0$  will fail to find  $\alpha$ . [2]

[Questions 6, 7 and 8 are printed overleaf.]

6



The diagram shows part of the curve  $y = \ln(\ln(x))$ . The region between the curve and the  $x$ -axis for  $3 \leq x \leq 6$  is shaded.

(i) By considering  $n$  rectangles of equal width, show that a lower bound,  $L$ , for the area of the shaded region is  $\frac{3}{n} \sum_{r=0}^{n-1} \ln\left(\ln\left(3 + \frac{3r}{n}\right)\right)$ . [3]

(ii) By considering another set of  $n$  rectangles of equal width, find a similar expression for an upper bound,  $U$ , for the area of the shaded region. [1]

(iii) Find the least value of  $n$  for which  $U - L < 0.001$ . [4]

7 The equation of a curve is  $y = \frac{x^2 + 1}{(x + 1)(x - 7)}$ .

(i) Write down the equations of the asymptotes. [3]

(ii) Find the coordinates of the stationary points on the curve. [5]

(iii) Find the coordinates of the point where the curve meets one of its asymptotes. [3]

(iv) Sketch the curve. [3]

8 The equation of a curve is  $x^2 + y^2 - x = \sqrt{x^2 + y^2}$ .

(i) Find the polar equation of this curve in the form  $r = f(\theta)$ . [3]

(ii) Sketch the curve. [2]

(iii) The line  $x + 2y = 2$  divides the region enclosed by the curve into two parts. Find the ratio of the two areas. [6]