OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4726

Further Pure Mathematics 2

Tuesday **6 JUNE 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

1 Find the first three non-zero terms of the Maclaurin series for

$$(1+x)\sin x,$$

simplifying the coefficients. [3]

2 **(i)** Given that $y = \tan^{-1} x$, prove that $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$. [3]

(ii) Verify that $y = \tan^{-1} x$ satisfies the equation

$$(1+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0. \quad [3]$$

3 The equation of a curve is $y = \dfrac{x+1}{x^2+3}$.

(i) State the equation of the asymptote of the curve. [1]

(ii) Show that $-\frac{1}{6} \leqslant y \leqslant \frac{1}{2}$. [5]

4 **(i)** Using the definition of $\cosh x$ in terms of e^x and e^{-x}, prove that

$$\cosh 2x = 2\cosh^2 x - 1. \quad [3]$$

(ii) Hence solve the equation

$$\cosh 2x - 7\cosh x = 3,$$

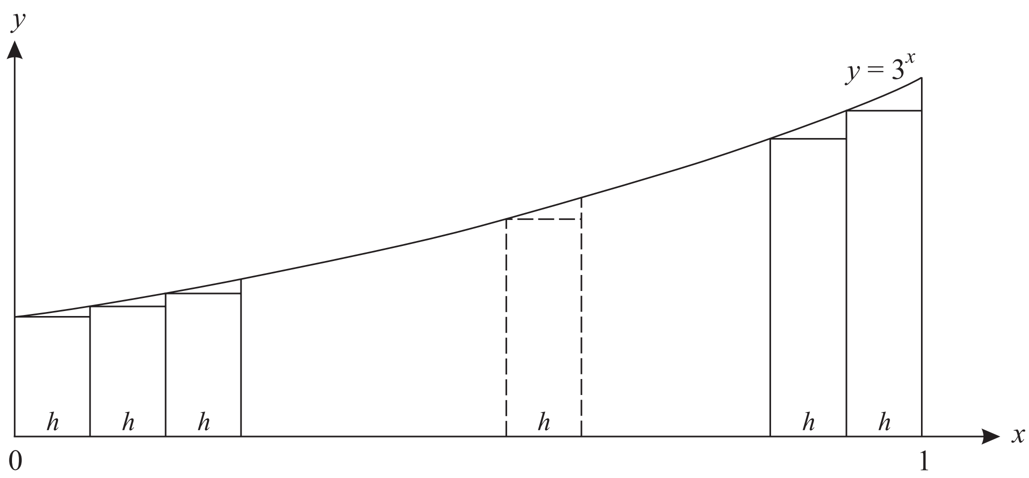
giving your answer in logarithmic form. [4]

5 **(i)** Express $t^2 + t + 1$ in the form $(t+a)^2 + b$. [1]

(ii) By using the substitution $\tan \frac{1}{2}x = t$, show that

$$\int_0^{\frac{1}{2}\pi} \frac{1}{2+\sin x}\, dx = \frac{\sqrt{3}}{9}\pi. \quad [6]$$

6



The diagram shows the curve with equation $y = 3^x$ for $0 \leqslant x \leqslant 1$. The area A under the curve between these limits is divided into n strips, each of width h where $nh = 1$.

(i) By using the set of rectangles indicated on the diagram, show that $A > \dfrac{2h}{3^h - 1}$. [3]

(ii) By considering another set of rectangles, show that $A < \dfrac{(2h)3^h}{3^h - 1}$. [3]

(iii) Given that $h = 0.001$, use these inequalities to find values between which A lies. [2]

7 The equation of a curve, in polar coordinates, is

$$r = \sqrt{3} + \tan\theta, \quad \text{for } -\tfrac{1}{3}\pi \leqslant \theta \leqslant \tfrac{1}{4}\pi.$$

(i) Find the equation of the tangent at the pole. [2]

(ii) State the greatest value of r and the corresponding value of θ. [2]

(iii) Sketch the curve. [2]

(iv) Find the exact area of the region enclosed by the curve and the lines $\theta = 0$ and $\theta = \frac{1}{4}\pi$. [5]

8 The curve with equation $y = \dfrac{\sinh x}{x^2}$, for $x > 0$, has one turning point.

(i) Show that the x-coordinate of the turning point satisfies the equation $x - 2\tanh x = 0$. [3]

(ii) Use the Newton-Raphson method, with a first approximation $x_1 = 2$, to find the next two approximations, x_2 and x_3, to the positive root of $x - 2\tanh x = 0$. [5]

(iii) By considering the approximate errors in x_1 and x_2, estimate the error in x_3. (You are not expected to evaluate x_4.) [3]

[Question 9 is printed overleaf.]

9 **(i)** Given that $y = \sinh^{-1} x$, prove that $y = \ln\left(x + \sqrt{x^2 + 1}\right)$. [3]

(ii) It is given that, for non-negative integers n,

$$I_n = \int_0^{\alpha} \sinh^n \theta \, \mathrm{d}\theta,$$

where $\alpha = \sinh^{-1} 1$. Show that

$$nI_n = \sqrt{2} - (n-1)I_{n-2}, \quad \text{for } n \geqslant 2.$$ [6]

(iii) Evaluate I_4, giving your answer in terms of $\sqrt{2}$ and logarithms. [4]