Mark Scheme 4726
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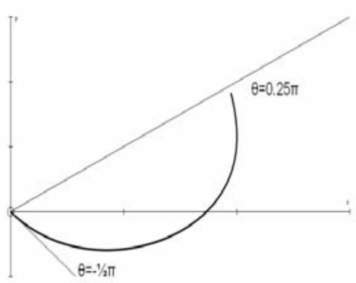
1 Correct expansion of sin x	B1	Quote or derive $x-\frac{1}{6}x^3$
Multiply their expansion by $(1 + x)$	M1	Ignore extra terms
Obtain $x + x^2 - x^3/6$	A1√	On their sin x; ignore extra terms; allow 3!
	SC	Attempt product rule M1 Attempt f(0), f ′ (0), f″(0) … (at least 3) M1 Use Maclaurin accurately cao A1
2 (i) Get $\sec^2 y \frac{dy}{dx} = 1$ or equivalent	M1	
Clearly use $1 + \tan^2 y = \sec^2 y$	M1	May be implied
Clearly arrive at A.G.	A1	
(ii) Reasonable attempt to diff. to $\frac{-2x}{(1+x^2)^2}$	M1	Use of chain/quotient rule
Substitute their expressions into D.E.	M1	Or attempt to derive diff. equ^n.
Clearly arrive at A.G.	A1	
	SC	Attempt diff. of $(1+x^2)\frac{dy}{dx} = 1$ M1,A1 Clearly arrive at A.G. B1
3 (i) State $y = 0$ (or seen if working given)	B1	Must be = ; accept x-axis; ignore any others
(ii) Write as quad. in x^2	M1	$(x^2y - x + (3y-1) = 0)$
Use for real x, $b^2-4ac\geq 0$	M1	Allow > ; or < for no real x
Produce quad. inequality in y	M1	$1\geq 12y^2 - 4y$; $12y^2 - 4y - 1 \leq 0$
Attempt to solve inequality	M1	Factorise/ quadratic formula
Justify A.G.	A1	e.g. diagram / table of values of y
	SC	Attempt diff. by product/quotient M1 Solve $dy/dx = 0$ for two real x M1 Get both $(-3,-\frac{1}{6})$ and $(1,\frac{1}{2})$ A1 Clearly prove min./max. A1 Justify fully the inequality e.g. detailed graph B1
4 (i) Correct definition of cosh x or cosh $2x$	B1	
Attempt to sub. in RHS and simplify	M1	or LHS if used
Clearly produce A.G.	A1	
(ii) Write as quadratic in cosh x	M1	$(2\cosh^2 x - 7\cosh x - 4 = 0)$
Solve their quadratic accurately	A1√	Factorise/quadratic formula
Justify one answer only	B1	State cosh $x\geq 1$/graph; allow ≥ 0
Give $\ln(4 + \sqrt{15})$	A1	cao; any one of $\pm\ln(4 \pm \sqrt{15})$ or decimal equivalent of ln ( )
5 (i) Get $(t + \frac{1}{2})^2 + \frac{3}{4}$	B1	cao
(ii) Derive or quote $dx = \frac{2}{1+t^2}dt$	B1	
Derive or quote $\sin x = 2t/(1 + t^2)$	B1	
Attempt to replace all x and dx	M1	
Get integral of form $A/(Bt^2+Ct+D)$	A1√	From their expressions, $C\neq 0$
Use complete square form as $\tan^{-1}(f(t))$	M1	From formulae book or substitution
Get A.G.	A1	

6 (i) Attempt to sum areas of rectangles	M1 ($h.3^h + h.3^{2h} + \ldots + h.3^{(n-1)h}$)
Use G.P. on $h(1+3^h+3^{2h}+\ldots+3^{(n-1)h})$	M1 All terms not required, but last term needed (or 3^{1-h}); or specify a, r and n for a G.P.
Simplify to A.G.	A1 Clearly use $nh = 1$
(ii) Attempt to find sum areas of different rect.	M1 Different from (i)
Use G.P. on $h(3^h+3^{2h}+\ldots+3^{nh})$	M1 All terms not required, but last term needed; G.P. specified as in (i), or deduced from (i)
Simplify to A.G.	A1
(iii) Get 1.8194(8), 1.8214(8) correct	B1,B1 Allow $1.81 \le A \le 1.83$
7 (i) Attempt to solve r=0, $\tan\theta = -\sqrt{3}$	M1 Allow $\pm\sqrt{3}$
Get $\theta = -\frac{1}{3}\pi$ only	A1 Allow – 60°
(ii) $r = \sqrt{3} + 1$ when $\theta = \frac{1}{4}\pi$	B1,B1 AEF for r, 45° for θ
(iii)	B1 Correct r at correct end-values of θ; Ignore extra θ used
	B1 Correct shape with r not decreasing
(iv) Formula with correct r used	M1 r^2 may be implied
Replace $\tan^2\theta = \sec^2\theta - 1$	B1
Attempt to integrate <u>their</u> expression	M1 Must be 3 different terms leading to any 2 of $a\theta + b\ln(\sec\theta/\cos\theta) + c\tan\theta$
Get $\theta + \sqrt{3}\ln\sec\theta + \frac{1}{2}\tan\theta$	A1 Condone answer x2 if ½ seen elsewhere
Correct limits to $\frac{1}{4}\pi + \sqrt{3}\ln\sqrt{2} + \frac{1}{2}$	A1 cao; AEF
8 (i) Attempt to diff. using product/quotient	M1
Attempt to solve $dy/dx = 0$	M1
Rewrite as A.G.	A1 Clearly gain A.G.
(ii) Diff. to $f'(x) = 1 \pm 2\,\text{sech}^2 x$	B1 Or $\pm 2\,\text{sech}^2 x - 1$
Use correct form of N-R with their expressions from correct f(x)	M1
Attempt N-R with x_1= 2 from previous M1	M1 To get an x_2
Get x_2 = 1.9162(2) (3 s.f. min.)	A1
Get x_3 = 1.9150(1) (3 s.f. min.)	A1 cao
(iii) Work out e_1 and e_2 (may be implied)	B1√ -0.083(8), -0.0012 (allow ± if both of same sign); e_1 from 0.083 to 0.085

Use $e_2 \approx k e_1^2$ and $e_3 \approx k e_2^2$	M1
Get $e_3 \approx e_2^3/e_1^2$ = - 0.0000002(or 3)	A1√ ± if same sign as B1√
	SC B1 only for x_4 - x_3
9 (i) Rewrite as quad. in e^y	M1 Any form
Solve to $e^y = (x \pm \sqrt{(x^2+1)})$	A1 Allow $y = \ln($ $)$
Justify one solution only	B1 $x - \sqrt{(x^2+1)} < 0$ for all real x
	SC Use $C^2 - S^2 = 1$ for $C = \pm\sqrt{(1+x^2)}$ M1
	Use/state $\cosh y + \sinh y = e^y$ A1
	Justify one solution only B1
(ii) Attempt parts on $\sinh x.\ \sinh^{n-1} x$	M1
Get correct answer	A1 $(\cosh x.\sinh^{n-1}x - \int \cosh^2 x.(n-1)\sinh^{n-2}x\ dx)$
Justify $\sqrt{2}$ by $\sqrt{(1+\sinh^2 x)}$ for $\cosh x$ when limits inserted	B1 Must be clear
Replace $\cosh^2 = 1 + \sinh^2$; tidy at this stage	M1
Produce I_{n-2}	A1
Gain A.G. clearly	A1
(iii) Attempt $4I_4 = \sqrt{2} - 3I_2$, $2I_2 = \sqrt{2} - I_0$	M1 Clear attempt at iteration (one at least seen)
Work out $I_0 = \sinh^{-1}1 = \ln(1+\sqrt{2}) = \alpha$	B1 Allow I_2
Sub. back completely for I_4	M1
Get $^1/_8(3\ln(1+\sqrt{2}) - \sqrt{2})$	A1 AEEF