



**ADVANCED GCE  
MATHEMATICS**

Further Pure Mathematics 2

**4726**

Candidates answer on the answer booklet.

**OCR supplied materials:**

- 8 page answer booklet (sent with general stationery)
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Monday 10 January 2011  
Morning**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a scientific or graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Use the substitution  $t = \tan \frac{1}{2}x$  to find  $\int \frac{1}{1 + \sin x + \cos x} dx$ . [5]

2 It is given that  $f(x) = \tanh^{-1} x$ .

(i) Show that  $f'''(x) = \frac{2(1 + 3x^2)}{(1 - x^2)^3}$ . [5]

(ii) Hence find the Maclaurin series for  $f(x)$ , up to and including the term in  $x^3$ . [3]

3 The function  $f$  is defined by  $f(x) = \frac{5ax}{x^2 + a^2}$ , for  $x \in \mathbb{R}$  and  $a > 0$ .

(i) For the curve with equation  $y = f(x)$ ,

(a) write down the equation of the asymptote, [1]

(b) find the range of values that  $y$  can take. [4]

(ii) For the curve with equation  $y^2 = f(x)$ , write down

(a) the equation of the line of symmetry, [1]

(b) the maximum and minimum values of  $y$ , [2]

(c) the set of values of  $x$  for which the curve is defined. [1]

4 (i) Use the definitions of hyperbolic functions in terms of exponentials to prove that

$$8 \sinh^4 x \equiv \cosh 4x - 4 \cosh 2x + 3. \quad [4]$$

(ii) Solve the equation

$$\cosh 4x - 3 \cosh 2x + 1 = 0,$$

giving your answer(s) in logarithmic form. [5]

5 The equation

$$x^3 - 5x + 3 = 0 \quad (A)$$

may be solved by the Newton-Raphson method. Successive approximations to a root are denoted by  $x_1, x_2, \dots, x_n, \dots$

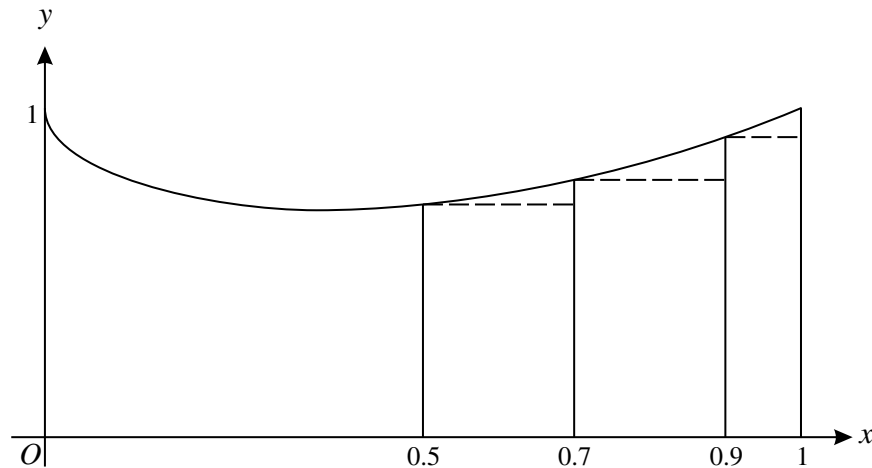
(i) Show that the Newton-Raphson formula can be written in the form  $x_{n+1} = F(x_n)$ , where

$$F(x) = \frac{2x^3 - 3}{3x^2 - 5}. \quad [3]$$

(ii) Find  $F'(x)$  and hence verify that  $F'(\alpha) = 0$ , where  $\alpha$  is any one of the roots of equation (A). [3]

(iii) Use the Newton-Raphson method to find the root of equation (A) which is close to 2. Write down sufficient approximations to find the root correct to 4 decimal places. [3]

6



The diagram shows the curve  $y = f(x)$ , defined by

$$f(x) = \begin{cases} x^x & \text{for } 0 < x \leq 1, \\ 1 & \text{for } x = 0. \end{cases}$$

- (i) By first taking logarithms, show that the curve has a stationary point at  $x = e^{-1}$ . [3]

The area under the curve from  $x = 0.5$  to  $x = 1$  is denoted by  $A$ .

- (ii) By considering the set of three rectangles shown in the diagram, show that a lower bound for  $A$  is 0.388. [2]

- (iii) By considering another set of three rectangles, find an upper bound for  $A$ , giving 3 decimal places in your answer. [2]

The area under the curve from  $x = 0$  to  $x = 0.5$  is denoted by  $B$ .

- (iv) Draw a diagram to show rectangles which could be used to find lower and upper bounds for  $B$ , using not more than three rectangles for each bound. (You are not required to find the bounds.) [3]

7 A curve has polar equation  $r = 1 + \cos 3\theta$ , for  $-\pi < \theta \leq \pi$ .

- (i) Show that the line  $\theta = 0$  is a line of symmetry. [2]

- (ii) Find the equations of the tangents at the pole. [3]

- (iii) Find the exact value of the area of the region enclosed by the curve between  $\theta = -\frac{1}{3}\pi$  and  $\theta = \frac{1}{3}\pi$ . [5]

8 (i) Without using a calculator, show that  $\sinh(\cosh^{-1} 2) = \sqrt{3}$ . [2]

- (ii) It is given that, for non-negative integers  $n$ ,

$$I_n = \int_0^\beta \cosh^n x \, dx, \quad \text{where } \beta = \cosh^{-1} 2.$$

Show that  $nI_n = 2^{n-1}\sqrt{3} + (n-1)I_{n-2}$ , for  $n \geq 2$ . [6]

- (iii) Evaluate  $I_5$ , giving your answer in the form  $k\sqrt{3}$ . [4]

There are no questions printed on this page.



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