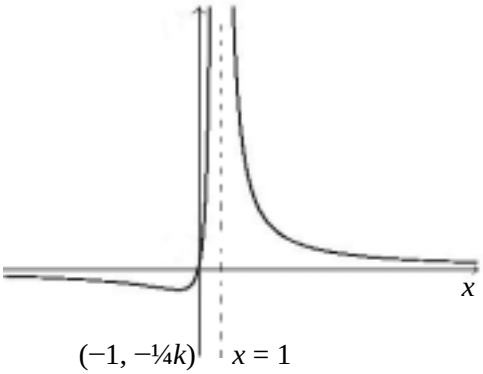
4726 Further Pure Mathematics 2

1	**(i)**	Get 0.876096, 0.876496, 0.876642	B1√ B1	For any one correct or √ from wrong answer; radians only All correct
	(ii)	Subtract correctly (0.00023(0), 0.000084) Divide their errors as e_4/e_3 only Get 0.365(21…)	B1√ M1 A1	On their answers May be implied Cao
2	**(i)**	Find $f'(x) = 1/(1+(1+x)^2)$ Get $f(0) = \frac{1}{4}\pi$ and $f'(0) = \frac{1}{2}$ Attempt $f''(x)$ Correctly get $f''(0) = -\frac{1}{2}$	M1 A1√ M1 A1	Quoted or derived; may be simplified or left as $\sec^2 y\ dy/dx = 1$ On their $f'(0)$; allow f(0)=0.785 but not 45 Reasonable attempt at chain/quotient rule or implicit differentiation A.G.
	(ii)	Attempt Maclaurin as $af(0)+bf'(0)+cf''(0)$ Get $\frac{1}{4}\pi + \frac{1}{2}x - \frac{1}{4}x^2$	M1 A1	Using their f(0) and $f'(0)$ Cao; allow 0.785
3	**(i)**	Attempt gradient as $\pm f(x_1)/(x_2 - x_1)$ Equate to gradient of curve at x_1 Clearly arrive at A.G. **SC** Attempt equation of tangent Put $(x_2, 0)$ into their equation Clearly arrive at A.G.	M1 M1 A1 M1 M1 A1	Allow reasonable y-step/x-step Allow ± Beware confusing use of ± As $y - f(x_1) = f'(x_1)(x - x_1)$
	(ii)	Diagram showing at least one more tangent Description of tangent meeting x-axis, used as next starting value	B1 B1	
	(iii)	Reasonable attempt at N-R Get 1.60	M1 A1	Clear attempt at differentiation Or answer which rounds
4	**(i)**	State $r = 1$ and $\theta = 0$.  	B1 B1	May be seen or implied Correct shape, decreasing r (not through O)
	(ii)	Use $\frac{1}{2}\int r^2\ d\theta$ with $r = e^{-2\theta}$ seen or implied Integrate correctly as $-\frac{1}{8}e^{-4\theta}$ Use limits in correct order Use $r_1^2 = e^{-4\theta}$ etc. Clearly get $k = \frac{1}{8}$	M1 A1 M1 M1 A1	Allow $\frac{1}{2}\int e^{4\theta}\ d\theta$ In their answer May be implied

5	**(i)**	Use correct definitions of cosh and sinh	B1	
		Attempt to square and subtract	M1	On their definitions
		Clearly get A.G.	A1	
		Show division by $\cosh^2$	B1	Or clear use of first result
	(ii)	Rewrite as quadratic in sech and attempt to solve	M1	Or quadratic in cosh
		Eliminate values outside $0 < \text{sech} \leq 1$	B1	Or eliminate values outside $\cosh \geq 1$ (allow positive)
		Get $x = \ln(2+\sqrt{3})$	A1	
		Get $x = -\ln(2+\sqrt{3})$ or $\ln(2-\sqrt{3})$	A1	
6	**(i)**	Attempt at correct form of P.F.	M1	Allow $Cx/(x^2+1)$ here; not $C = 0$
		Rewrite as 4= $A(1 + x)(1 + x^2) + B(1 - x)(1 + x^2) + (Cx + D)(1 - x)(1 + x)$	M1 √	From their P.F.
		Use values of x/equate coefficients	M1	
		Get $A = 1$, $B = 1$	A1	cwo
		Get $C = 0$, $D = 2$	A1	
				SC Use of cover-up rule for A,B M1
				If both correct A1 cwo
	(ii)	Get $A\ln(1 + x) - B\ln(1 - x)$	M1	Or quote from List of Formulae
		Get $D\tan^{-1}x$	B1	
		Use limits in their integrated expressions	M1	
		Clearly get A.G.	A1	
7	**(i)**	LHS = sum of areas of rectangles, area = 1x y-value from $x = 1$ to $x = n$	B1	
		RHS = Area under curve from $x = 0$ to n	B1	
	(ii)	Diagram showing areas required	B1	
		Use sum of areas of rectangles	B1	
		Explain/show area inequality with limits in integral clearly specified	B1	
	(iii)	Attempt integral as $kx^{4/3}$	M1	
		Limits gives 348(.1) and 352(.0)	A1	Allow one correct
		Get 350	A1	From two correct values only

8	**(i)**	Get $x = 1$, $y = 0$	B1,B1	
	(ii)	Rewrite as quadratic in x	M1	$(x^2y - x(2y + k) + y = 0)$
		Use $b^2 - 4ac \geq 0$ for all real x	M1	Allow >, = here
		Get correct inequality	A1	$4ky + k^2 \geq 0$
		State use of $k>0$ to A.G.	A1	
				SC Use differentiation (parts **(ii)** and **(iii)**)
				Attempt prod/quotient rule M1
				Solve = 0 for $x = -1$ A1
				Use $x = -1$ only (reject $x=1$), $y = -¼k$ A1
				Fully justify minimum B1
				Attempt to justify for all x M1
				Clearly get A.G. A1
	(iii)	Replace $y = -¼k$ in quadratic in x	M1	
		Get $x = -1$ only	A1	
			B1	Through origin with minimum at $(-1, -¼k)$ seen or given in the answer
			B1	Correct shape (asymptotes and approaches)
				SC (Start again)
				Differentiate and solve $dy/dx = 0$ for at least one x-value, independent of k M1
				Get $x = -1$ only A1
9	**(i)**	Rewrite $\tanh y$ as $(e^y - e^{-y})/(e^y + e^{-y})$	B1	Or equivalent
		Attempt to write as quadratic in e^{2y}	M1	
		Clearly get A.G.	A1	
	(ii)	**(a)** Attempt to diff. and solve = 0	M1	
		Get $\tanh x = b/a$	A1	
		Use $(-1) < \tanh x < 1$ to show $b < a$	B1	
				SC Use exponentials M1
				Get $e^{2x} = (a + b)/(a - b)$ A1
				Use $e^{2x} > 0$ to show $b < a$ B1
				SC Write $x = \tanh^{-1}(b/a)$ M1
				$= ½\ln((1 + b/a)/(1 - b/a))$ A1
				Use ( ) > 0 to show $b < a$ B1
		(b) Get $\tanh x = 1/a$ from part **(ii)(a)**	B1	
		Replace as ln from their answer	M1	
		Get $x = ½ \ln((a + 1)/(a - 1))$	A1	
		Use $e^{½\ln((a+1)/(a-1))} = \sqrt{((a + 1)/(a - 1))}$	M1	At least once
		Clearly get A.G.	A1	
		Test for minimum correctly	B1	
				SC Use of $y = \cosh x(a - \tanh x)$ and $\cosh x = 1/\text{sech}x = 1/\sqrt{(1 - \tanh^2 x)}$