ADVANCED GCE UNIT **4726/01**

MATHEMATICS

Further Pure Mathematics 2

TUESDAY 16 JANUARY 2007 Morning

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 It is given that $\mathrm{f}(x) = \ln(3 + x)$.

(i) Find the exact values of $\mathrm{f}(0)$ and $\mathrm{f}'(0)$, and show that $\mathrm{f}''(0) = -\frac{1}{9}$. [3]

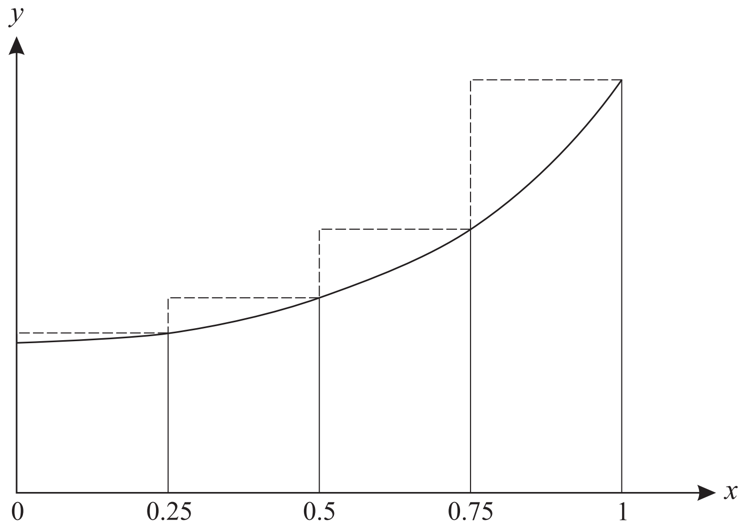
(ii) Hence write down the first three terms of the Maclaurin series for $\mathrm{f}(x)$, given that $-3 < x \leqslant 3$. [2]

2 It is given that $\mathrm{f}(x) = x^2 - \tan^{-1} x$.

(i) Show by calculation that the equation $\mathrm{f}(x) = 0$ has a root in the interval $0.8 < x < 0.9$. [2]

(ii) Use the Newton-Raphson method, with a first approximation 0.8, to find the next approximation to this root. Give your answer correct to 3 decimal places. [4]

3



The diagram shows the curve with equation $y = \mathrm{e}^{x^2}$, for $0 \leqslant x \leqslant 1$. The region under the curve between these limits is divided into four strips of equal width. The area of this region under the curve is A.

(i) By considering the set of rectangles indicated in the diagram, show that an upper bound for A is 1.71. [3]

(ii) By considering an appropriate set of four rectangles, find a lower bound for A. [3]

4 **(i)** On separate diagrams, sketch the graphs of $y = \sinh x$ and $y = \operatorname{cosech} x$. [3]

(ii) Show that $\operatorname{cosech} x = \dfrac{2\mathrm{e}^x}{\mathrm{e}^{2x} - 1}$, and hence, using the substitution $u = \mathrm{e}^x$, find $\displaystyle\int \operatorname{cosech} x \,\mathrm{d}x$. [6]

5 It is given that, for non-negative integers n,

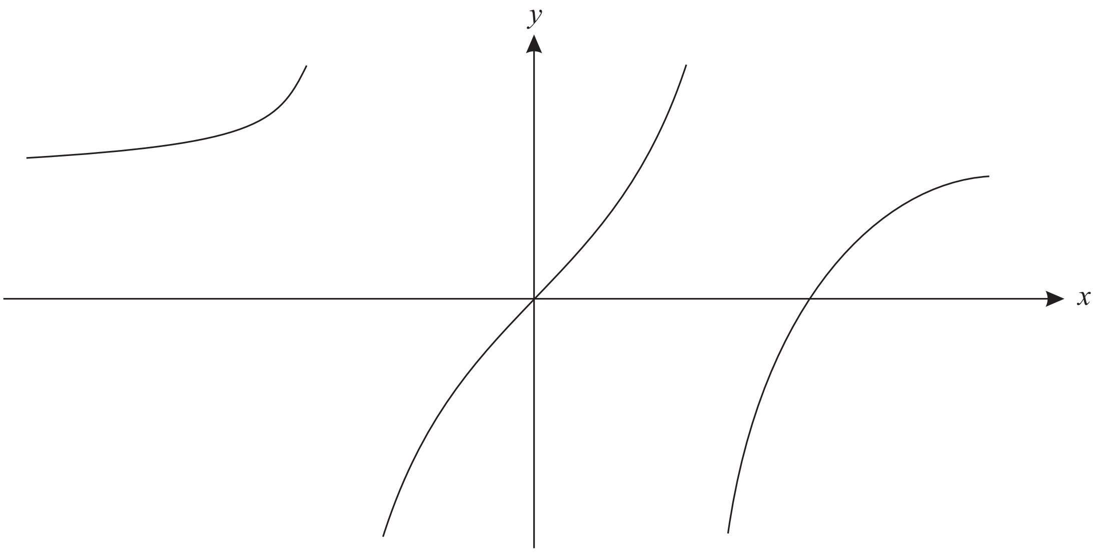
$$I_n = \int_0^{\frac{1}{2}\pi} x^n \cos x \, dx.$$

(i) Prove that, for $n \geqslant 2$,

$$I_n = \left(\tfrac{1}{2}\pi\right)^n - n(n-1)I_{n-2}.$$ [5]

(ii) Find I_4 in terms of π. [4]

6



The diagram shows the curve with equation $y = \dfrac{2x^2 - 3ax}{x^2 - a^2}$, where a is a positive constant.

(i) Find the equations of the asymptotes of the curve. [3]

(ii) Sketch the curve with equation

$$y^2 = \frac{2x^2 - 3ax}{x^2 - a^2}.$$

State the coordinates of any points where the curve crosses the axes, and give the equations of any asymptotes. [5]

7 **(i)** Express $\dfrac{1 - t^2}{t^2(1 + t^2)}$ in partial fractions. [4]

(ii) Use the substitution $t = \tan \frac{1}{2}x$ to show that

$$\int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi} \frac{\cos x}{1 - \cos x} \, dx = \sqrt{3} - 1 - \tfrac{1}{6}\pi.$$ [5]

8 **(i)** Define $\tanh y$ in terms of e^y and e^{-y}. [1]

(ii) Given that $y = \tanh^{-1} x$, where $-1 < x < 1$, prove that $y = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$. [3]

(iii) Find the exact solution of the equation $3\cosh x = 4\sinh x$, giving the answer in terms of a logarithm. [2]

(iv) Solve the equation

$$\tanh^{-1} x + \ln(1-x) = \ln\left(\tfrac{4}{5}\right).$$ [3]

9 The equation of a curve, in polar coordinates, is

$$r = \sec\theta + \tan\theta, \quad \text{for } 0 \leqslant \theta \leqslant \tfrac{1}{3}\pi.$$

(i) Sketch the curve. [2]

(ii) Find the exact area of the region bounded by the curve and the lines $\theta = 0$ and $\theta = \frac{1}{3}\pi$. [6]

(iii) Find a cartesian equation of the curve. [3]