OCR Maths FP2

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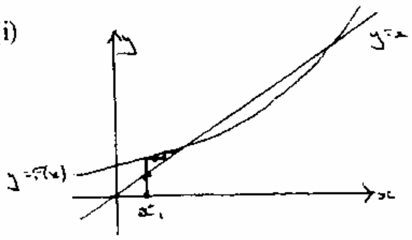
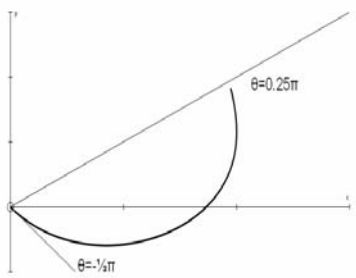
4726 FP2 MARK SCHEME January 2006 Final Draft

1(i) Use standard $\ln(1+3x) = 3x - \frac{(3x)^2}{2} + \frac{(3x)^3}{3}$	M1	Allow e.g. $3x^2$, 2! etc.
	M1	Attempt to simplify $(3x)^2$ etc.
$= 3x - 9x^2/2 + 9x^3$	A1	cao
(ii) Produce $(1 + x + x^2/2)$	B1	
	M1	Mult. 2 reasonable attempts, each of 3 terms (non-zero)
Get $3x - 3x^2/2 + 6x^3$	A1√	From their series
	SC M1	Reasonable attempt at diff. and replace $x = 0$ (2 correct)
	M1√	Put their values into correct Maclaurin expansion
	A1	cao
		(Applies to either/both parts)
2 Write as $f(x) = \pm(x - e^{-x})$	B1	Or equivalent
So $f'(x) = \pm(1 + e^{-x})$	B1	Correct from their $f(x)$
Use $x_{n+1} = x_n - f(x_n)/f'(x_n)$ with $x_0 = 0.5$	M1	Clear evidence of N-R on their f, f′
Get $x_1 = 0.56631$, $x_2 = 0.56714$	A1√	At least one to 4d.p.
Get $x_3 = 0.567(1)$	A1	cao to 3 d.p.
3 Use $A/x + (Bx + C)/(x^2 + 2)$	B1	
Equate $x+6$ to $A(x^2 + 2) + (Bx+C)x$ (or equiv.)	M1√	Equate to their P.F. (e.g. if B = 0 or C = 0 used)
Use $x = 0$ or equiv. for A (or equate coeff.etc.)	M1√	Include cover-up
Correctly find one of B,C	A1	
Get A=3, B=-3, C=1	A1	
4(i) 	B1	Line from x_1 to curve
	B1	Then to line
	B1	Clear explanation; allow use of step/staircase
(ii)(a) Converges to $x=\alpha$	B1, B1	
(b) Diverges (does not give either root)	B1	
5 (i) Give $x = -2$	B1	
Attempt to divide out	M1	Giving $y = x+k$; allow $k = 0$ here
Get $y = x + 1$	A1	Must be =
(ii) Write as quad. $x^2 + x(3 - y) + (3 - 2y) = 0$	M1	SC Differentiate M1
Use for real x, $b^2 - 4ac \geq 0$	M1	Solve $dy/dx = 0$ M1
Produce quad. inequality in y	M1	Get 2 x,y values correct A1
Attempt to solve quad. inequality	M1	Attempt at max/min M1
Get A.G. clearly e.g. graph	A1	Justify, e.g. graph, constraints on y A1

6 (i) Use parts to $(-e^{-x}.x^n - \int -e^{-x}.nx^{n-1}\,dx)$	M1	Reasonable attempt e.g. $+e^{-x}$
	A1	cao
Use limits to get e^{-1}	B1	Allow ±
Tidy correctly to A.G.	A1	
(ii) Use $I_3 = 3I_2 - e^{-1}$ $I_2 = 2I_1 - e^{-1}$ $I_1 = I_0 - e^{-1}$	B1	One such seen
Work out $I_0 = 1 - e^{-1}$ or $I_1 = 1 - 2e^{-1}$	M1,A1	
Get $6 - 16e^{-1}$	A1	
7 (i) Area under graph = $\int \sqrt{x}\,dx$	B1	Explain RHS (limits need not be specified)
> Sum of areas of rectangles from 1 to $N+1$	B1	
Area of each rect. = Width x Height = 1 x $\sqrt{x}$	B1	
(ii) Similarly, area under curve from 0 to N	B1	
< sum of areas of rect. from 0 to N	B1	
Clear explanation of A.G.	B1	
(iii) Integrate $x^{0.5}$ and use 2 different sets of limits	M1,M1	
Get area between $^2/_3((N+1)^{1.5}-1)$ and $^2/_3 N^{1.5}$	A1	
8 (i) Max. $r = 2$ at $\theta = 0$ and π	B1,B1	Two θ needed (rads only); ignore θ out of range
(ii) Solve $r = 0$ for θ, giving $\theta = \frac{1}{2}\pi$ and $^3/_2\pi$	M1,A1	Two θ needed (rads only); ignore θ out of range
(iii) Use correct formula with correct r	M1	
Expand r	M1	
Get $\int A + B\cos 2\theta + C\cos 4\theta\,d\theta$	M1	$C \neq 0$
Integrate their expression correctly	M1√	
Get $3\pi/8$	A1	cao
(iv) Express $\cos 2\theta = \cos^2\theta - \sin^2\theta$ or similar	M1	
Use $\cos\theta = x/r$ and/or $\sin\theta = y/r$	M1	
<u>Simplify</u> to $(x^2 + y^2)^{1.5} = 2x^2$ or similar	<u>M1</u>,A1	
9 (i) Correct def^n of cosh x and sinh x	B1,B1	
Expand $2.\frac{1}{2}(e^x - e^{-x}).\frac{1}{2}(e^x + e^{-x})$	M1	Reasonable attempt
Clearly get $\frac{1}{2}(e^{2x} - e^{-2x})$ to A.G.	A1	
(ii) Attempt to diff. and solve $dy/dx = 0$	M1	Reasonable attempt
Use (ii) to get A cosh x (B sinh x + C)=0	M1	
Clearly see cosh $x > 0$ or similar for one useable factor only	B1	
Attempt to solve sinh x = - C/B	M1	Quote or via e^{-x} correctly
Get $x = \ln((3+\sqrt{13})/2)$	A1	
Justify one answer only for sinh x = -C/B	B1	
Accurate test for MINIMUM	B1	First or second diff^t test with numeric evidence
	B1	Correct value(s) for min.

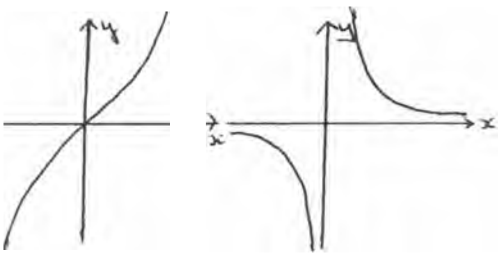
1 Correct expansion of sin x	B1	Quote or derive $x - \frac{1}{6}x^3$
Multiply their expansion by $(1 + x)$	M1	Ignore extra terms
Obtain $x + x^2 - x^3/6$	A1√	On their sin x; ignore extra terms; allow 3!
	SC	Attempt product rule M1 Attempt f(0), f ′(0), f″(0) … (at least 3) M1 Use Maclaurin accurately cao A1
2 (i) Get $\sec^2 y \frac{dy}{dx} = 1$ or equivalent	M1	
Clearly use $1 + \tan^2 y = \sec^2 y$	M1	May be implied
Clearly arrive at A.G.	A1	
(ii) Reasonable attempt to diff. to $\frac{-2x}{(1+x^2)^2}$	M1	Use of chain/quotient rule
Substitute their expressions into D.E.	M1	Or attempt to derive diff. equ^n.
Clearly arrive at A.G	A1	
	SC	Attempt diff. of $(1+x^2)\frac{dy}{dx} = 1$ M1,A1 Clearly arrive at A.G. B1
3 (i) State $y = 0$ (or seen if working given)	B1	Must be = ; accept x-axis; ignore any others
(ii) Write as quad. in x^2	M1	$(x^2y - x + (3y-1) = 0)$
Use for real x, $b^2-4ac\geq 0$	M1	Allow > ; or < for no real x
Produce quad. inequality in y	M1	$1\geq 12y^2 - 4y$; $12y^2 - 4y - 1 \leq 0$
Attempt to solve inequality	M1	Factorise/ quadratic formula
Justify A.G.	A1	e.g. diagram / table of values of y
	SC	Attempt diff. by product/quotient M1 Solve $dy/dx = 0$ for two real x M1 Get both $(-3, -\frac{1}{6})$ and $(1, ½)$ A1 Clearly prove min./max. A1 Justify fully the inequality e.g. detailed graph B1
4 (i) Correct definition of cosh x or cosh $2x$	B1	
Attempt to sub. in RHS and simplify	M1	or LHS if used
Clearly produce A.G.	A1	
(ii) Write as quadratic in cosh x	M1	$(2\cosh^2 x - 7\cosh x - 4 = 0)$
Solve their quadratic accurately	A1√	Factorise/quadratic formula
Justify one answer only	B1	State cosh $x\geq 1$/graph; allow ≥ 0
Give $\ln(4 + \sqrt{15})$	A1	cao; any one of $\pm\ln(4 \pm \sqrt{15})$ or decimal equivalent of ln ( )
5 (i) Get $(t + ½)^2 + ¾$	B1	cao
(ii) Derive or quote $dx = \frac{2}{1+t^2}dt$	B1	
Derive or quote $\sin x = 2t/(1 + t^2)$	B1	
Attempt to replace all x and dx	M1	
Get integral of form $A/(Bt^2+Ct+D)$	A1√	From their expressions, C≠ 0
Use complete square form as $\tan^{-1}(f(t))$	M1	From formulae book or substitution
Get A.G.	A1	

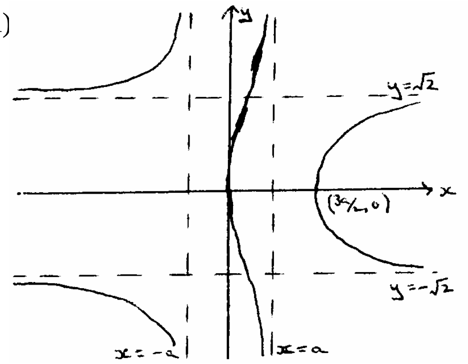
6 (i) Attempt to sum areas of rectangles	M1 ($h.3^h + h.3^{2h} + \ldots + h.3^{(n-1)h}$)
Use G.P. on $h(1+3^h+3^{2h}+\ldots+3^{(n-1)h})$	M1 All terms not required, but last term needed (or 3^{1-h}); or specify a, r and n for a G.P.
Simplify to A.G.	A1 Clearly use $nh = 1$
(ii) Attempt to find sum areas of different rect.	M1 Different from (i)
Use G.P. on $h(3^h+3^{2h}+\ldots+3^{nh})$	M1 All terms not required, but last term needed; G.P. specified as in (i), or deduced from (i)
Simplify to A.G.	A1
(iii) Get 1.8194(8), 1.8214(8) correct	B1,B1 Allow $1.81 \le A \le 1.83$
7 (i) Attempt to solve r=0, $\tan\theta = -\sqrt{3}$	M1 Allow $\pm\sqrt{3}$
Get $\theta = -{}^1/_3\,\pi$ only	A1 Allow – 60°
(ii) $r = \sqrt{3} + 1$ when $\theta = \frac{1}{4}\pi$	B1,B1 AEF for r, 45° for θ
(iii)	B1 Correct r at correct end-values of θ; Ignore extra θ used

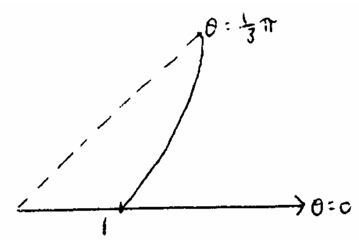


	B1 Correct shape with r not decreasing
(iv) Formula with correct r used	M1 r^2 may be implied
Replace $\tan^2\theta = \sec^2\theta - 1$	B1
Attempt to integrate their expression	M1 Must be 3 different terms leading to any 2 of $a\theta + b\ln(\sec\theta/\cos\theta) + c\tan\theta$
Get $\theta + \sqrt{3}\ln\sec\theta + \frac{1}{2}\tan\theta$	A1 Condone answer x2 if ½ seen elsewhere
Correct limits to $\frac{1}{4}\pi + \sqrt{3}\ln\sqrt{2} + \frac{1}{2}$	A1 cao; AEF
8 (i) Attempt to diff. using product/quotient	M1
Attempt to solve $dy/dx = 0$	M1
Rewrite as A.G.	A1 Clearly gain A.G.
(ii) Diff. to $f'(x) = 1 \pm 2\,\text{sech}^2 x$	B1 Or $\pm 2\,\text{sech}^2 x - 1$
Use correct form of N-R with their expressions from correct f(x)	M1
Attempt N-R with x_1= 2 from previous M1	M1 To get an x_2
Get x_2 = 1.9162(2) (3 s.f. min.)	A1
Get x_3 = 1.9150(1) (3 s.f. min.)	A1 cao
(iii) Work out e_1 and e_2 (may be implied)	B1√ -0.083(8), -0.0012 (allow ± if both of same sign); e_1 from 0.083 to 0.085

Use $e_2 \approx ke_1^2$ and $e_3 \approx ke_2^2$	M1
Get $e_3 \approx e_2^3/e_1^2$ = - 0.0000002(or 3)	A1√ ± if same sign as B1√
	SC B1 only for x_4 - x_3
9 (i) Rewrite as quad. in e^y	M1 Any form
Solve to $e^y = (x \pm \sqrt{(x^2 + 1)})$	A1 Allow y = ln( )
Justify one solution only	B1 $x - \sqrt{(x^2 + 1)} < 0$ for all real x
	SC Use $C^2 - S^2 = 1$ for $C = \pm\sqrt{(1+x^2)}$ M1
	Use/state cosh y + sinh y = e^y A1
	Justify one solution only B1
(ii) Attempt parts on sinh x. $\sinh^{n-1}x$	M1
Get correct answer	A1 $(\cosh x.\sinh^{n-1}x - \int \cosh^2 x.(n-1)\sinh^{n-2}x\,dx)$
Justify √2 by $\sqrt{(1+\sinh^2 x)}$ for cosh x when limits inserted	B1 Must be clear
Replace $\cosh^2 = 1 + \sinh^2$; tidy at this stage	M1
Produce I_{n-2}	A1
Gain A.G. clearly	A1
(iii) Attempt $4I_4 = \sqrt{2} - 3I_2$, $2I_2 = \sqrt{2} - I_0$	M1 Clear attempt at iteration (one at least seen)
Work out $I_0 = \sinh^{-1}1 = \ln(1 + \sqrt{2}) = \alpha$	B1 Allow I_2
Sub. back completely for I_4	M1
Get $^1/_8(3 \ln(1+\sqrt{2}) - \sqrt{2})$	A1 AEEF

1 (i) f(O) = In 3 f	Bl
f'(0) = $^1/_3$	Bl
f"(O) = -'/$_9$ *A.G.*	B1 Clearly derived
(ii) Reasonable attempt at Maclaurin	Ml Form In3 + $ax + bx^2$, with a,b related to f " f'
$f(x) = \ln 3 + {}^1/_3x - {}^1/_{18}x^2$	*Al* √*J* On their values off' and f"
	SR Use *In(3+x)* = In3 + In(1 + $^1/_3$ *x)* Ml Use Formulae Book to get In3 + *Y3X - Y2(VJX)2* = In3 + *Y3X - 1/lgX2* Al
2 (i) f(0.8) = - 0.03, f(0.9) = +0.077 (accurately e.g. accept -0.02 t0 -0.04)	B1
Explain (change of sign, graph etc.)	B1
	SR Use *x* = √*J(tan^{-1}x)* and compare *x* to √J(tan^{-1} x) for *x=0.8*, 0.9 B 1
	Explain "change in sign" B 1
(ii) Differentiate two terms	B1 Get *2x* - I *I(* 1 +x^2*)*
Use correct form of Newton-Ra ph son with 0.8, using their f *'(x)*	Ml 0.8 - *f(0.8)/f '(0.8)*
Use their N-R to give one more approximation to 3 d.p. minimum	Ml√
Get *x* = 0.835	Al 3d.p. - accept answer which rounds
3 (i) Show area of rect. = $^1/_4$ ($e^{1/16} + e^{1/4} + e^{9/16} + e^1$)	Ml Or numeric equivalent
Show area = 1.7054	Al At least 3 d.p. correct
Explain the < 1.71 in terms of areas	Bl AG. Inequality required
(ii) Identify areas for > sign	B1 Inequality or diagram required
Show area of rect. = $^1/_4$ ($e^o + e^{1/16} + e^{1/4} + e^{9/16}$)	Ml Or numeric evidence
Get *A* > 1.27	Al cao; or answer which rounds down
4 (i) 	BI Correct shape for sinh *x* B1 Correct shape for cosech *x* B1 Obvious point *(dy/dx* ≠O)/asymptotes clear
(ii) Correct definition of sinh *x*	B1 May be implied
Invert and mult. by e^x to AG.	B1 Must be clear; allow $2/(e^x - e^{-x})$ as mimimum simplification
Sub. $u = e^x$ and $du = e^x\,dx$	M1 Or equivalent, all *x* eliminated and not *dx* = *du*
Replace to $2/(u^2 - 1)\,du$	Al
Integrate to *aln((u - I)/(u* + 1)	A1√ Use formulae book, PT, or atanh^{-1}u
Replace *u*	Al No need for *c*

5 (i) Reasonable attempt at parts Get	M1 Involving second integral Al
xnsin x - ∫ sin x. nx^{n-1} dx	M1
Attempt parts again Accurately	Al
Clearly derive AG.	A1 Indicate $(^1/_2\pi)^n$ and 0 from limits
(ii) Get $I_4 = (^1/_2\pi)^4 - 12I_2$ or $I_2 = (^1/_2\pi)^2 - 2I_0$	B1
Show clearly $I_0 = 1$	B1 May use I_2
Replace their values in relation Get	M1
$I_4 = {}^1/_{16}\pi^4 - 3\pi^2 + 24$	A1 cao
6 (i) $x = \pm a$, $y = 2$	B1, B1, B1 Must be =; no working needed
(ii) 	B1 Two correct labelled asymptotes ‖ Ox and approaches B1 Two correct labelled asymptotes ‖ Oy and approaches B1 Crosses at $(^3/_2a, 0)$ (and (0,0) - may be implied B1 90° where it crosses Ox; smoothly B1 Symmetry in Ox
7 (i) Write as $A/t + B/t^2 + (Ct + D)/(t^2 + 1)$	M1 Allow $(At+B)/t^2$; justify $B/t^2 + D/(l + t^2)$ if only used
Equate $At(t^2 + 1) + B(t^2 + 1) + (Ct+D)t^2$ to $1 - t^2$	M1√
Insert t values / equate coeff.	M1 Lead to at least two constant values
Get A = C = 0, B = L D =-2	Al
	SR Other methods leading to correct PF can earn 4 marks; 2 M marks for reasonable method going wrong
(ii) Derive or quote cos x in terms of t	Bl
Derive or quote dx = 2 $dt/(1 + t^2)$	B1
Sub. in to correct P.F.	M1 Allow $k\,(l-t^2)/((t^2(l + t^2))$ or equivalent
Integrate to $-1/t$ -2 $\tan^{-1}t$	Al√ From their k
Use limits to clearly get AG.	Al
8 (i) Get $(e^y - e^{-y})/(e^y + e^{-y})$	B1 Allow $(e^{2Y}-1)/(e^{2y}+ 1)$ or if x used
(ii) Attempt quad. in e^Y	M1 Multiply by e^Y and tidy
Solve for e^Y	M1
Clearly get AG.	Al
(iii) Rewrite as tanh $x = k$	M1 SR Use hyp def^n to get quad. in e^X M I
Use (ii) for x = ½ 1n 7 or equivalent	Al Solve $e^{2x} = 7$ for x to $^1/_2$ 1n 7 Al
(iv) Use of log laws	Bl One used correctly
Correctly equate 1n A = 1n B to $A = B$	M1 Or $1n(^A/_B) = 0$
Get $x = \pm\, ^3/_5$	Al

9 (i)		B1 Shape for correct θ; ignore other θ Used; start at $(r,0)$ B1 θ=0, r=1 and increasing r
(ii)	Use correct formula with correct r	B1
	$\int \sec^2 x\, dx = \tan x$ used	B1
	Quote $\int 2 \sec x \tan x\, dx = 2\sec x$	B1 Or sub. correctly
	Replace $\tan^2 x$ by $\sec^2 x - 1$ to integrate	M1
	Reasonable attempt to integrate 3 terms And to use limits correctly	M1
	Get $\sqrt{3} + 1 - \frac{1}{6}\pi$	Al Exact only
(iii)	Use $x = r\cos\theta$, $y = r\sin\theta$, $r = (x^2+y^2)^{1/2}$	M1
	Reasonable attempt to eliminate r,θ	M1
	Get $y = (x-1)\sqrt{(x^2+y^2)}$	A1 Or equivalent

1		Correct formula with correct r	M1	Allow $r^2 = 2\sin^2 3\theta$
		Rewrite as $a + b\cos 6\theta$	M1	$a, b \neq 0$
		Integrate their expression correctly	A1√	From $a + b\cos 6\theta$
		Get $\frac{1}{3}\pi$	A1	cao
2	(i)	Expand to $\sin 2x \cos\frac{1}{4}\pi + \cos 2x \sin\frac{1}{4}\pi$	B1	
		Clearly replace $\cos\frac{1}{4}\pi$, $\sin\frac{1}{4}\pi$ to A.G.	B1	
	(ii)	Attempt to expand $\cos 2x$	M1	Allow $1 - 2x^2/2$
		Attempt to expand $\sin 2x$	M1	Allow $2x - 2x^3/3$
		Get $\frac{1}{2}\sqrt{2}\,(1 + 2x - 2x^2 - 4x^3/3)$	A1	Four correct unsimplified terms in any order; allow bracket; AEEF SR Reasonable attempt at $f^{\,n}(0)$ for $n = 0$ to 3 M1 Attempt to replace their values in Maclaurin M1 Get correct answer only A1
3	(i)	Express as $A/(x-1) + (Bx+C)/(x^2+9)$	M1	Allow C=0 here
		Equate (x^2+9x) to $A(x^2+9) + (Bx+C)(x-1)$	M1√	May imply above line; on their P.F.
		Sub. for x or equate coeff.	M1	Must lead to at least 3 coeff.; allow cover-up method for A
		Get A=1, B=0, C=9	A1	cao from correct method
	(ii)	Get $A\ln(x-1)$	B1√	On their A
		Get $C/3 \tan^{-1}(x/3)$	B1√	On their C; condone no constant; ignore any $B \neq 0$
4	(i)	Reasonable attempt at product rule	M1	Two terms seen
		Derive or quote diff. of $\cos^{-1}x$	M1	Allow +
		Get $-x^2(1 - x^2)^{-\frac{1}{2}} + (1 - x^2)^{\frac{1}{2}} + (1 - x^2)^{-\frac{1}{2}}$	A1	
		Tidy to $2(1 - x^2)^{\frac{1}{2}}$	A1	cao
	(ii)	Write down integral from (i)	B1	On any $k\sqrt{(1-x^2)}$
		Use limits correctly	M1	In any reasonable integral
		Tidy to $\frac{1}{2}\pi$	A1	
			SR	Reasonable sub. B1 Replace for new variable and attempt to integrate (ignore limits) M1 Clearly get $\frac{1}{2}\pi$ A1

5	(i)	Attempt at parts on $\int 1 (\ln x)^n \, dx$	M1	Two terms seen
		Get $x (\ln x)^n - \int n (\ln x)^{n-1} \, dx$	A1	
		Put in limits correctly in line above	M1	
		Clearly get A.G.	A1	ln e =1, ln1 =0 seen or implied
	(ii)	Attempt I_3 to I_2 as $I_3 = e - 3I_2$	M1	
		Continue sequence in terms of In	A1	$I_2 = e-2I_1$ and/or $I_1 = e-I_0$
		Attempt I_0 or I_1	M1	($I_0 = e-1$, $I_1=1$)
		Get 6 – 2e	A1	cao
6	(i)	Area under graph (= $\int 1/x^2 \, dx$, 1 to n+1) < Sum of rectangles (from 1 to n)	B1	Sum (total) seen or implied eg diagram; accept areas (of rectangles)
		Area of each rectangle = Width x Height = 1 x $1/x^2$	B1	Some evidence of area worked out – seen or implied
	(ii)	Indication of new set of rectangles	B1	
		Similarly, area under graph from 1 to n > sum of areas of rectangles from 2 to n	B1	Sum (total) seen or implied
		Clear explanation of A.G.	B1	Diagram; use of left-shift of previous areas
	(iii)	Show complete integrations of RHS, using correct, different limits	M1	Reasonable attempt at $\int x^{-2} \, dx$
		Correct answer, using limits, to one integral	A1	
		Add 1 to their second integral to get complete series	M1	
		Clearly arrive at A.G.	A1	
	(iv)	Get one limit	B1	Quotable
		Get both 1 and 2	B1	Quotable; limits only required

7	(i)	Use correct definition of cosh or sinh x	B1	Seen anywhere in (i)
		Attempt to mult. their cosh/sinh	M1	
		Correctly mult. out and tidy	A1√	
		Clearly arrive at A.G.	A1	Accept e^{x-y} and e^{y-x}
	(ii)	Get $\cosh(x-y) = 1$	M1	
		Get or imply $(x-y) = 0$ to A.G.	A1	
	(iii)	Use $\cosh^2 x = 9$ or $\sinh^2 x = 8$	B1	
		Attempt to solve cosh x = 3 (not –3) or sinh $x = \pm\sqrt{8}$ (allow $+\sqrt{8}$ or $-\sqrt{8}$ only)	M1	$x = \ln(3 + \sqrt{8})$ from formulae book or from basic cosh definition
		Get at least one x solution correct	A1	
		Get both solutions correct, x and y	A1	x, y = $\ln(3 \pm 2\sqrt{2})$; AEEF SR Attempt tanh = sinh/cosh B1 Get tanh x = $\pm\sqrt{8}/3$ (+ or -) M1 Get at least one sol. correct A1 Get both solutions correct A1 SR Use exponential definition B1 Get quadratic in e^x or e^{2x} M1 Solve for one correct x A1 Get both solutions, x and y A1
8	(i)	$x_2 = 0.1890$	B1	
		$x_3 = 0.2087$	B1√	From their x_1 (or any other correct)
		$x_4 = 0.2050$	B1√	Get at least two others correct, all to a minimum of 4 d.p.
		$x_5 = 0.2057$		
		$x_6 = 0.2055$		
		$x_7 (= x_8) = 0.2056$ (to x_7 minimum)		
		$\alpha = 0.2056$	B1	cao; answer may be retrieved despite some errors
	(ii)	Attempt to diff. f(x)	M1	$k/(2+x)^3$
		Use α to show $f'(\alpha) \neq 0$	A1√	Clearly seen, or explain $k/(2+x)^3 \neq 0$ as $k \neq 0$; allow ± 0.1864
			SR	Translate $y=1/x^2$ M1 State/show $y=1/x^2$ has no TP A1
	(iii)	δ_3 = -0.0037 (allow –0.004)	B1√	Allow ±, from their x_4 and x_3
	(iv)	Develop from $\delta_{10} = f'(\alpha)\,\delta_9$ etc. to get δ_i or quote $\delta_{10} = \delta_3\, f'(\alpha)^7$	M1	Or any δ_i eg use $\delta_9 = x_{10} - x_9$
		Use their δ_i and $f'(\alpha)$	M1	
		Get 0.000000028	A1	Or answer that rounds to ± 0.00000003

9	(i)	Quote $x = a$	B1	
		Attempt to divide out	M1	Allow M1 for y=x here; allow
			A1	$(x\text{-}a) + k/(x\text{-}a)$ seen or implied
		Get $y = x - a$	A1	Must be equations
	(ii)	Attempt at quad. in x (=0)	M1	
		Use $b^2 - 4ac \geq 0$ for real x	M1	Allow >
		Get $y^2 + 4a^2 \geq 0$	A1	
		State/show their quad. is always >0	B1	Allow ≥
	(iii)		B1√	Two asymptotes from (i) (need not be labelled)
			B1	Both crossing points
			B1√	Approaches – correct shape
			SR	Attempt diff. by quotient/product rule M1
				Get quadratic in x for $dy/dx = 0$ and note $b^2 - 4ac < 0$ A1
				Consider horizontal asymptotes B1
				Fully justify answer B1

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1	**(i)**	Get $f'(x) = \pm \sin x/(1+\cos x)$	M1	Reasonable attempt at chain at any stage
		Get $f''(x)$ using quotient/product rule	M1	Reasonable attempt at quotient/product
		Get $f(0) = \ln 2$, $f'(0) = 0$, $f''(0) = -½$	B1	Any one correct from correct working
			A1	All three correct from correct working
	(ii)	Attempt to use Maclaurin correctly	M1	Using their values in $af(0)+bf'(0)x+cf''(0)x^2$; may be implied
		Get $\ln 2 - ¼\,x^2$	A1√	From their values; must be quadratic
2	**(i)**	Clearly verify in $y = \cos^{-1}x$	B1	i.e. $x=½\sqrt{3}$, $y=\cos^{-1}(½\sqrt{3})= {}^1/_6\pi$, or similar
		Clearly verify in $y = ½\sin^{-1}x$	B1	Or solve $\cos y = \sin 2y$
			SR	Allow one B1 if not sufficiently clear detail
	(ii)	Write down at least one correct diff'al	M1	Or reasonable attempt to derive; allow ±
		Get gradient of –2	A1	cao
		Get gradient of 1	A1	cao
3	**(i)**	Get y- values of 3 and $\sqrt{28}$	B1	
		Show/explain areas of two rectangles equal y- value x 1, and relate to A	B1	Diagram may be used
	(ii)	Show $A>0.2(\sqrt{(1+2^3)} + \sqrt{(1+2.2^3)} + \ldots$ ..$\sqrt{(1+2.83)}$)	M1	Clear areas attempted below curve (5 values)
		= 3.87(28)	A1	To min. of 3 s.f.
		Show $A<0.2(\sqrt{(1+2.2^3)} + \sqrt{(1+2.4^3)} + \ldots$ …+ $\sqrt{(1+3^3)}$)	M1	Clear areas attempted above curve (5 values)
		= 4.33(11) <4.34	A1	To min. of 3 s.f.
4	**(i)**	Correct formula with correct r	M1	May be implied
		Expand r^2 as A + B$\sec\theta$ + C$\sec^2\theta$	M1	Allow B = 0
		Get C tanθ	B1	
		Use correct limits in their answer	M1	Must be 3 terms
		Limits to ${}^1/_{12}\pi + 2\ln(\sqrt{3}) + {}^{2\sqrt{3}}/_3$	A1	AEEF; simplified
	(ii)	Use $x=r\cos\theta$ and $r^2 = x^2+ y^2$	B1	Or derive polar form from given equation
		Eliminate r and θ	M1	Use their definitions
		Get $(x – 2)\sqrt{(x^2 + y^2)} = x$	A1	A.G.

5	**(i)**	Attempt use of product rule	M1	
		Clearly get x =1	A1	Allow substitution of x=1
	(ii)	Explain use of tangent for next approx.	B1	Not use of G.C. to show divergence
		Tangents at successive approx. give x>1	B1	Relate to crossing x-axis; allow diagram
	(iii)	Attempt correct use of N-R with their derivative	M1	
		Get x_2 = -1	A1√	
		Get –0.6839, -0.5775, (-0.5672…)	A1	To 3 d.p. minimum
		Continue until correct to 3 d.p.	M1	May be implied
		Get –0.567	A1	cao
6	**(i)**	Attempt division/equate coeff.	M1	To lead to some ax+b (allow b=0 here)
		Get a = 2, b = -9	A1	
		Derive/quote x = 1	B1	Must be equations
	(ii)	Write as quadratic in x	M1	$(2x^2-x(11+y)+(y-6)=0)$
		Use $b^2 \geq 4ac$ (for real x)	M1	Allow <, >
		Get $y^2 + 14y + 169 \geq 0$	A1	
		Attempt to justify positive/negative	M1	Complete the square/sketch
		Get $(y+7)^2 + 120 \geq 0$ – true for all y	A1	
			SC	Attempt diff; quot./prod. rule M1 Attempt to solve dy/dx = 0 M1 Show $2x^2 - 4x + 17 = 0$ has no real roots e.g. $b^2 - 4ac < 0$ A1 Attempt to use no t.p. M1 Justify all y e.g. consider asymptotes and approaches A1
7	**(i)**	Get $x(1+x^2)^{-n} - \int x.(-n(1+x^2)^{-n-1}.2x)\,dx$	M1	Reasonable attempt at parts
		Accurate use of parts	A1	
		Clearly get A.G.	B1	Include use of limits seen
	(ii)	Express x^2 as $(1+x^2) - 1$		
		Get $\dfrac{x^2}{(1+x^2)^{n+1}} = \dfrac{1}{(1+x^2)^n} - \dfrac{1}{(1+x^2)^{n+1}}$	B1	Justified
		Show $I_n = 2^{-n} + 2n(I_n - I_{n+1})$	M1	Clear attempt to use their first line above
		Tidy to A.G.	A1	
	(iii)	See $2I_2 = 2^{-1} + I_1$	B1	
		Work out $I_1 = \frac{1}{4}\pi$	M1	Quote/derive $\tan^{-1}x$
		Get $I_2 = \frac{1}{4} + \frac{1}{8}\pi$	A1	

8	**(i)**	Use correct exponential for sinh x	B1	
		Attempt to expand cube of this	M1	Must be 4 terms
		Correct cubic	A1	
		Clearly replace in terms of sinh	B1	(Allow RHS→ LHS or RHS = LHS separately)
	(ii)	Replace and factorise	M1	Or state sinh $x \neq 0$
		Attempt to solve for $\sinh^2 x$	M1	(= ¼(k-3)) or for k and use $\sinh^2 x>0$
		Get $k>3$	A1	Not ≥
	(iii)	Get $x = \sinh^{-1} c$	M1	($c= \pm½$); allow sinh $x = c$
		Replace in ln equivalent	A1√	As $\ln(½+\sqrt{^5/_4})$; their x
		Repeat for negative root	A1√	May be given as neg. of first answer (no need for x=0 implied)
			SR	Use of exponential definitions Express as cubic in $e^{2x} = u$ M1 Factorise to $(u-1)(u^2-3u+1)=0$ A1 Solve for x =0, $½\ln(^3/_2 \pm ^{\sqrt{5}}/_2)$ A1
9	**(i)**	Get sinh y $^{dy}/_{dx}$ =1	M1	Or equivalent; allow ± Allow use of ln equivalent with Chain Rule
		Replace sinh $y = \sqrt{(\cosh^2 y - 1)}$	A1	
		Justify positive grad. to A.G.	B1	e.g. sketch
	(ii)	Get $k \cosh^{-1} 2x$	M1	No need for c
		Get k=½	A1	
	(iii)	Sub. $x = k \cosh u$	M1	
		Replace all x to $\int k_1 \sinh^2 u \, du$	A1	
		Replace as $\int k_2(\cosh 2u - 1) \, du$	M1	Or exponential equivalent
		Integrate correctly	A1√	No need for c
		Attempt to replace u with x equivalent	M1	In their answer
		Tidy to reasonable form	A1	cao $(½x\sqrt{(4x^2 - 1)} - ¼ \cosh^{-1} 2x \ (+c))$

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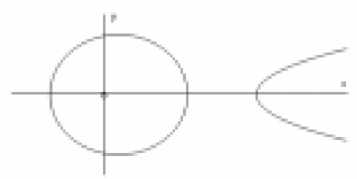
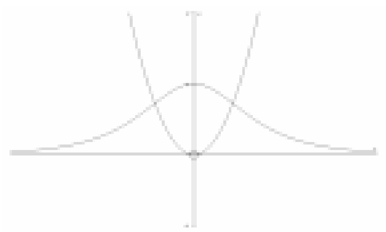
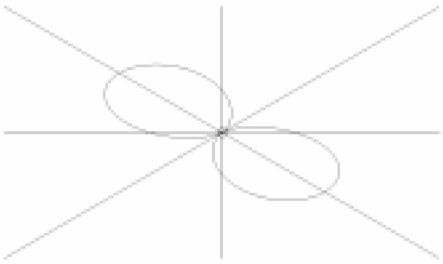
1		Write as $\frac{A}{x-2a}+\frac{Bx+C}{x^2+a^2}$	M1	Accept C=0
		Get $2ax = A(x^2+a^2) + (Bx+C)(x-2a)$	A1√	Follow-on for C=0
		Choose values of x and/or equate coeff.	M1	Must lead to at least one of their A,B,C
		Get $A = {}^4/_5$, $B = -{}^4/_5$, $C = {}^2/_5a$	A1	For two correct from correct working only
			A1	For third correct
			5	
2			B1	Get (4,0), (3,0), (-2,0) only
			B1	Get (0,√5) as "maximum"
			B1	Meets x-axis at 90^0 at all crossing points
			B1	Use $-2\le x\le 3$ and $x\ge 4$ only
			B1	Symmetry in Ox
			5	
3		Quote/derive $dx=\frac{2}{1+t^2}\,dt$	B1	
		Replace all x and dx from their expressions	M1	Not dx=dt; ignore limits
		Tidy to $2/(3t^2+1)$	A1	Not $a/(3t^2+1)$
		Get $k\tan^{-1}(At)$	M1	Allow A=1 if from $p/(t^2+1)$ only
		Get $k = ⅔\sqrt{3}$, $A = \sqrt{3}$	A1√	Allow $k=a/\sqrt{3}$ from line 3; AEEF
		Use limits correctly to ${}^2/_9\sqrt{3}\pi$	A1	AEEF
			6	
4	(i)		B1	Correct $y = x^2$
			B1	Correct shape/asymptote
			B1	Crossing (0,1)
			3	
	(ii)	Define sech $x = 2/(e^x + e^{-x})$	B1	AEEF
		Equate their expression to x^2 and attempt to simplify	M1	
		Clearly get A.G.	A1	
			3	
	(iii)	Cobweb	B1	
		Values > and then < root	B1	Only from cobweb
			2	

5	(i)	Factorise to $\tan^{n-2}x(1+\tan^2 x)$	B1	Or use $\tan^n x = \tan^{n-2}x.\tan^2 x$
		Clearly use $1+\tan^2 = \sec^2$	M1	Allow wrong sign
		Integrate to $\tan^{n-1}x/(n-1)$	A1	Quote or via substitution
		Use limits and tidy to A.G.	A1	Must be clearly derived
			4	
	(ii)	Get $3(I_4 + I_2) = 1$, $I_2 + I_0 = 1$	B1	Write down one correct from reduction formula
		Attempt to evaluate I_0 (or I_2)	M1	$I_2 = a\tan x + b$, $a,b \neq 0$
		Get $\frac{1}{4}\pi$ (or $1 - \frac{1}{4}\pi$)	A1	
		Replace to $\frac{1}{4}\pi - \frac{2}{3}$	A1	
			4	
6	(i)	Attempt to use N-R of correct form with clear $f'(x)$ used	M1	
		Get 2.633929, 2.645672	A1	For one correct to minimum of 6 d.p.
			A1√	For other correct from their x_2 in correct NR
			3	
	(ii)	$\sqrt{7}$	B1	Allow ±
			1	
	(iii)	Get $e_1 = 0.14575$, $e_2 = 0.01182$	B1√	From their values
		Get $e_3 = 0.00008$	B1√	
		Verify both ≈ 0.00008	B1	From 0.000077.. or $0.01182^3/0.14575^2$
			3	
7	(i)	Attempt quotient/product on bracket	M1	
		Get $-3/(2+x)^2$	A1	May be implied
		Use Formulae Booklet or derive from $\tanh y = (1-x)/(2+x)$	M1	Attempt $\tanh^{-1}$ part in terms of x
		Get $\frac{-3}{(2+x)^2}.\frac{1}{1-((1-x)/(2+x))^2}$	A1√	From their results above
		Clearly tidy to A.G.	A1	
		Get $f''(x) = 2/(1+2x)^2$	B1	cao
			6	
			SC	Use reasonable ln definition **M1** Get $y = \frac{1}{2}\ln((1-k)/(1+k))$ for $k = (1-x)/(1+2x)$ **A1** Tidy to $y = \frac{1}{2}\ln(3/(1+2x))$ **A1** Attempt chain rule **M1** Clearly tidy to A.G. **A1** Get $f''(x)$ **B1**
	(ii)	Attempt $f(0)$, $f'(0)$ and $f''(0)$	M1	From their differentiation
		Get $\tanh^{-1}\frac{1}{2}$, -1 and 2	A1√	
		Replace $\tanh^{-1}\frac{1}{2} = \frac{1}{2}\ln 3$ ($=\ln\sqrt{3}$)	B1	Only
		Get $\ln\sqrt{3} - x + x^2$	A1	
			4	
			SC	Use standard expansion from $\frac{1}{2}\ln 3 - \frac{1}{2}\ln(1+2x)$

Question	Answer	Mark	Guidance
8 (i)	Attempt to solve $r = 0$	**M1**	
	Get $\alpha = \frac{1}{4}\pi$	**A1**	From correct method; ignore others; allow θ
		2	
(ii) (a)	Get $1 - \sin((2k+1)\pi - 2\theta)$	**M1**	Attempt $f(\frac{1}{2}(2k+1)\pi - \theta)$, leading to 2θ here
	Expand as $\sin(A+B)$	**M1**	Or discuss periodicity for general k
	Use k as integer so $\sin(2k+1)\pi = 0$,		
	And $\cos(2k+1)\pi = -1$	**A1**	Needs a clear explanation
		3	
(b)	Quote $\frac{1}{4}(2k+1)\pi$	**B1**	For general answer or 2 correct (ignore other answers given)
	Select or give $k = 0,1,2,3$	**B1**	For all 4 correct in $0 \le \theta < 2\pi$
		2	
(iii)		**B1**	Correct shape; 2 branches only, roughly as shown
		B1	Clear symmetry in correct rays
		B1	Get max. $r = 2$
		B1	At $\theta = \frac{3}{4}\pi$ and $^{7}/_{4}\pi$; both required (allow correct answers not in $0 \le \theta < 2\pi$ here)
		4	
9 (i)	Attempt to use parts	**M1**	Two terms, one yet to be integrated
	Divide out $x/(1+x)$	**M1**	Or use substitution
	Correct answer $x\ln(1+x) - x + \ln(1+x)$	**A1**	
	Limits to correct A.G.	**A1**	
		4	
		SC	Quote $\int \ln x \, dx$ **M1**
			Clear use of limits to A.G. **A1**
		SC	Attempt to diff'ate by product rule **M1**
			Clear use of limits to A.G. **A1**
(ii) (a)	Use sum of areas of rect.< Area under curve (between limits 0 and 70)	**B1**	
	Areas = 1x heights = 1(ln2 + ln3+…ln70)	**B1**	<u>Areas</u> to be specified
		2	
(b)	Explain use of 69	**B1**	Allow diagram or use of left shift of 1 unit
	Explain first rectangle	**B1**	
	Areas as above > area under curve	**B1**	
		3	
(c)	Show/quote ln2 + ln3 + … ln70 = ln 70!	**B1**	
	Use N = 69, 70 in (i)	**M1**	No other numbers; may be implied by 228.39.. or 232.65.. seen; allow 228.4, 232.6 or 232.7
	Get 228.3, 232.7	**A1**	
		3	

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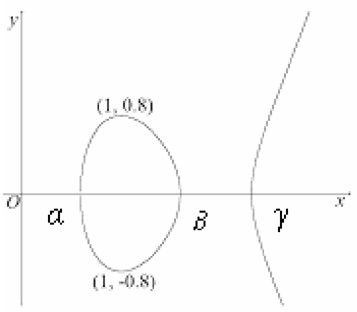
1	**(i)**	Give $1 + 2x + (2x)^2/2$	M1	Reasonable 3 term attempt e.g. allow $2x^2/2$
		Get $1 + 2x + 2x^2$	A1	cao
				SC Reasonable attempt at f′(0) and f″(0) M1
				Get $1+2x+2x^2$ cao A1
	(ii)	$\ln((1+2x+2x^2)$ $+ (1-2x+2x^2)) =$	M1	Attempt to sub for e^{2x} and e^{-2x}
		$\ln(2+4x^2) =$	A1√	On their part **(i)**
		$\ln 2 + \ln(1 + 2x^2)$	M1	Use of log law in reasonable expression
		$\ln 2 + 2x^2$	A1	cao
				SC Use of Maclaurin for f ′(x) and f″(x) M1
				One correct A1
				Attempt f(0), f ′(0) and f″(0) M1
				Get cao A1
2	**(i)**	$x_2 = 1.8913115$	B1	x_2 correct; allow answers which round
		$x_3 = 1.8915831$	B1√	For any other from their working
		$x_4 = 1.8915746$	B1	For all three correct
	(ii)	$e_3/e_2 = -0.031(1)$	M1	Subtraction and division on their values; allow ±
		$e_4/e_3 = -0.036(5)$	A1	Or answers which round to −0.031 and −0.037
		State $f'(\alpha) \approx e_3/e_2 \approx e_4/e_3$	B1√	Using their values but only if approx. equal; allow differentiation if correct conclusion; allow gradient for f′
3	**(i)**	Diff. $\sin y = x$	M1	Implicit diff. to $dy/dx = \pm(1/\cos y)$
		Use $\sin^2 + \cos^2 = 1$ to A.G.	A1	Clearly derived; ignore ±
		Justify +	B1	e.g graph/ principal values
	(ii)	Get $2/(\sqrt{(1-4x^2)})$ $+ 1/(\sqrt{(1-y^2)}) dy/dx = 0$	M1	Attempt implicit diff. and chain rule; allow e.g. $(1-2x^2)$ or $a/\sqrt{(1-4x^2)}$
			A1	
		Find $y = \sqrt{3}/2$	M1	Method leading to y
		Get $-2\sqrt{3}/3$	A1√	AEEF; from their a above
				SC Write $\sin(\frac{1}{2}\pi - \sin^{-1}2x) = \cos(\sin^{-1}2x)$ B1
				Attempt to diff. as above M1
				Replace x in reasonable dy/dx and attempt to tidy M1
				Get result above A1

4	(i)	Let x=cosh θ such that dx = sinh θ dθ	M1	
		Clearly use $\cosh^2 - \sinh^2 = 1$	A1	Clearly derive A.G.
	(ii)	Replace $\cosh^2\theta$	M1	Allow a (cosh $2\theta \pm 1$)
		Attempt to integrate their expression	M1	Allow $b$sinh $2\theta \pm a\theta$
		Get ¼sinh2θ + ½θ (+c)	A1	
		Clearly replace for x to A.G.	B1	Condone no +c
				SC Use expo. def^n; three terms M1 Attempt to integrate M1 Get $^1/_8(e^{2\theta}-e^{-2\theta})$ + ½θ (+c) A1 Clearly replace for x to A.G. B1
5	(i)	(**a**) State (x=) α	B1	
		None of roots	B1	No explanation needed
		(**b**) Impossible to say	B1	
		All roots can be derived	B1	Some discussion of values close to 1 or 2 or central leading to correct conclusion
	(ii)		B1	Correct x for y=0; allow 0.591, 1.59, 2.31
			B1	Turning at (1,0.8) and/or (1,-0.8)
			B1	Meets x-axis at 90^0
			B1	Symmetry in x-axis; allow
6	(i)	Correct definitions used	B1	
		Attempt at $(e^x-e^{-x})^2/4 + 1$	M1	Allow $(e^x+e^{-x})^2+1$; allow /2
		Clearly derive A.G.	A1	
	(ii)	Form a quadratic in sinh x	M1	
		Attempt to solve	M1	Factors or formula
		Get sinh x = -½ or 3	A1	
		Use correct ln expression	M1	On their answer(s) seen once
		Get ln(-½+$^{\sqrt{5}}/_2$) and ln(3+√10)	A1	
7	(i)	OP= 3 + 2cos α		
		OQ=3 + 2cos(½π+α) =3 – 2sin α	M1	Any other unsimplified value
		Similarly OR=3 – 2cos α OS=3 + 2sin α	M1	Attempt at simplification of at least two correct expressions
		Sum = 12	A1	cao
	(ii)	Correct formula with attempt at r^2	M1	Need not be expanded, but three terms if it is
		Square r correctly	A1	
		Attempt to replace $\cos^2\theta$ with $a(\cos 2\theta \pm 1)$	M1	
		Integrate their expression	A1√	Need three terms
		Get $^{11\pi}/_4 - 1$	A1	cao

8	(i)	Area = $\int 1/(x+1)\,dx$	B1	Include or imply correct limits
		Use limits to $\ln(n+1)$	B1	
		Compare area under curve to areas of rectangles	B1	Justify inequality
		Sum of areas = $1\times(\frac{1}{2} + \frac{1}{3} + \ldots + 1/(n+1))$	M1	Sum seen or implied as 1 x y values
		Clear detail to A.G.	A1	Explanation required e.g. area of last rectangle at $x=n$, area under curve to $x=n$
	(ii)	Show or explain areas of rectangles above curve	M1	
		Areas of rectangles (as above) > area under curve	A1	First and last heights seen or implied; A.G.
	(iii)	Add 1 to both sides in **(i)** to make $\sum(^1/r)$	B1	Must be clear addition
		Add $^1/_{(n+1)}$ to both sides in **(ii)** to make $\sum(^1/r)$	B1	Must be clear addition; A.G.
	(iv)	State divergent	B1	Allow not convergent
		Explain e.g. $\ln(n+1)\to\infty$ as $n\to\infty$	B1	
9	(i)	Require denom. = 0	B1	
		Explain why denom. $\neq 0$	B1	Attempt to solve, explain always > 0 etc.
	(ii)	Set up quadratic in x	M1	
		Get $2yx^2-4x+(2a^2y+3a)=0$	A1	
		Use $b^2 \geq 4ac$ for real x	M1	Produce quadratic inequality in y from their quad.; allow use of = or <
		Attempt to solve their inequality	M1	Factors or formula
		Get $y > {}^1/_{2a}$ and $y < {}^{-2}/_a$	A1	Justified from graph SC Attempt diff. by quot./product rule M1 Solve $dy/dx=0$ for two values of x M1 Get $x=2a$ and $x=-a/2$ A1 Attempt to find two y values M1 Get correct inequalities (graph used to justify them) A1
	(iii)	Split into two separate integrals	M1	
		Get $k\ln(x^2+a^2)$	A1	Or $p\ln(2x^2+2a^2)$
		Get $k_1\tan^{-1}(x/a)$	A1	k_1 not involving a
		Use limits and attempt to simplify	M1	
		Get $\ln 2.5 - 1.5\tan^{-1}2 + 3\pi/8$	A1	AEEF SC Sub. $x = a\tan\theta$ and $dx = a\sec^2\theta\, d\theta$ M1 Reduce to $\int p\tan\theta - p_1\, d\theta$ (ignore limits here) A1 Integrate to $p\ln(\sec\theta)-p_1\theta$ A1 Use limits (old or new) and attempt to simplify M1 Get answer above A1

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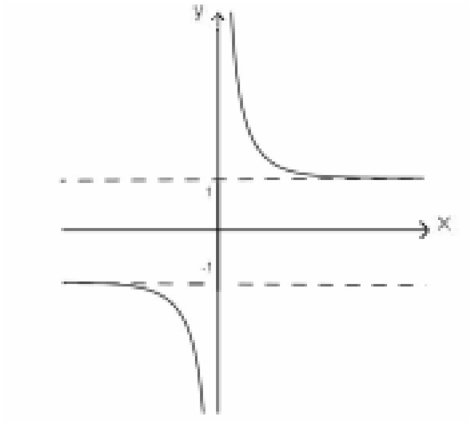
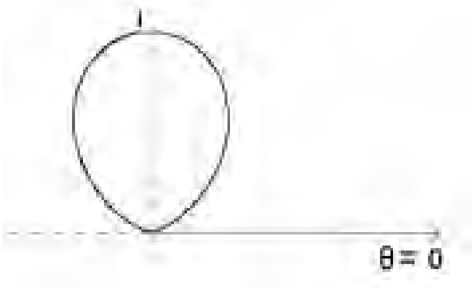
1(i)	Attempt area = $\pm\Sigma(0.3y)$ for at least three y values	M1	May be implied
	Get 1.313(1..) or 1.314	A1	Or greater accuracy
(ii)	Attempt ± sum of areas (4 or 5 values)	M1	May be implied
	Get 0.518(4..)	A1	Or greater accuracy **SC** If answers only seen, 1.313(1..) or 1.314 B2 0.518(4..) B2 −1.313(1..) or −1.314 B1 −0.518(4..) B1
	Or Attempt answer to part **(i)**−final rectangle Get 0.518(4..)	M1 A1	
(iii)	Decrease width of strips	B1	Use more strips or equivalent
2	Attempt to set up quadratic in x	M1	Must be quadratic; = 0 may be implied
	Get $x^2(y-1) - x(2y+1) + (y-1)=0$	A1	
	Use $b^2 \geq 4ac$ for real x on their quadratic	M1	Allow =,>,<,≤ here; may be implied
	Clearly solve to AG	A1	If other (in)equalities used, the step to AG must be clear **SC** Reasonable attempt to diff. using prod/quot rule M1 Solve correct dy/dx=0 to get $x = -1$, $y = \frac{1}{4}$ A1 Attempt to justify inequality e.g. graph or to show $d^2y/dx^2>0$ M1 Clearly solve to AG A1
3(i)	Reasonable attempt at chain rule	M1	Product in answer
	Reasonable attempt at product/quotient rule	M1	Sum of two parts
	Correctly get $f'(0) = 1$	A1	
	Correctly get $f''(0) = 1$	A1	**SC** Use of ln$y$ = sin$x$ follows same scheme
(ii)	Reasonable attempt at Maclaurin with their values	M1	In $af(0) + bf'(0)x + cf''(0)x^2$
	Get $1 + x + \frac{1}{2}x^2$	A1√	From their f(0), f′(0), f″(0) in a correct Maclaurin; all non-zero terms
4	Attempt to divide out.	M1	Or $A+B/(x-2)+(Cx(+D))/(x^2+4)$; allow A=1 and/or B=1 quoted
	Get $x^3 = A(x-2)(x^2+4)+B(x^2+4)+(Cx+D)(x-2)$	M1	Allow √ mark from their Part Fract; allow D=0 but not C=0
	State/derive/quote A=1	A1	
	Use x values and/or equate coeff	M1	To potentially get all their constants

	Get B=1, C=1, D=-2	A1 A1	For one other correct from cwo For all correct from cwo
5(i)	Derive/quote $d\theta=2dt/(1+t^2)$	B1	May be implied
	Replace their cos θ and their dθ, both in terms of t	M1	Not $d\theta = dt$
	Clearly get $\int(1-t^2)/(1+t^2)\,dt$ or equiv	A1	Accept limits of t quoted here
	Attempt to divide out	M1	Or use AG to get answer above
	Clearly get/derive AG	A1	
			SC Derive $d\theta = 2\cos^2\frac{1}{2}\theta\, dt$ B1 Replace cosθ in terms of half-angles and their dθ ($\neq dt$) M1 Get $\int 2\cos^2\frac{1}{2}\theta - 1\, dt$ or $\int 1 - 1/2\cos^2\frac{1}{2}\theta\, .2/(1+t^2)\, dt$ A1 Use $\sec^2\frac{1}{2}\theta = 1+t^2$ M1 Clearly get/derive AG A1
(ii)	Integrate to $a\tan^{-1}bt - t$	M1	
	Get $\frac{1}{2}\pi - 1$	A1	
6	Get $k\sinh^{-1}k_1x$	M1	For either integral; allow attempt at ln version here
	Get $\frac{1}{3}\sinh^{-1}\frac{3}{4}x$	A1	Or ln version
	Get $\frac{1}{2}\sinh^{-1}\frac{2}{3}x$	A1	Or ln version
	Use limits in their answers	M1	
	Attempt to use correct ln laws to set up a solvable equation in a	M1	
	Get $a = 2^{\frac{1}{3}}.\ 3^{\frac{1}{2}}$	A1	Or equivalent

7(i)		B1 B1 B1	y-axis asymptote; equation may be implied if clear Shape $y = \pm 1$ asymptotes; may be implied if seen as on graph
(ii)	Reasonable attempt at product rule, giving two terms	M1	
	Use correct Newton-Raphson at least once with their f $'(x)$ to produce an x_2	M1	May be implied
	Get $x_2 = 2.0651$	A1√	One correct at any stage if reasonable
	Get $x_3 = 2.0653$, $x_4 = 2.0653$	A1	cao; or greater accuracy which rounds
(iii)	Clearly derive coth x=½x	B1	AG; allow derivation from AG Two roots only
	Attempt to find second root e.g. symmetry Get ± 2.0653	M1 A1√	± their iteration in part **(ii)**
8(i)	**(a)** Get ½($e^{\ln a} + e^{-\ln a}$) Use $e^{\ln a} = a$ and $e^{-\ln a} = {}^1/_a$ Clearly derive AG	M1 M1 A1	
	(b) Reasonable attempt to multiply out their attempts at exponential definitions of cosh and sinh	M1	4 terms in each
	Correct expansion seen as $e^{(x+y)}$ etc.	A1	
	Clearly tidy to AG	A1	With $e^{-(x-y)}$ seen or implied
(ii)	Use $x = y$ and cosh0 = 1 to get AG	B1	
(iii)	Attempt to expand and equate coefficients	M1	($13 = R \cosh \ln a = R(a^2+1)/2a$ $5 = R \sinh \ln a = R(a^2-1)/2a$)
	Attempt to eliminate R (or a) to set up a solvable equation in a (or R)	M1	**SC** If exponential definitions used, $8e^x + 18e^{-x} = Re^x/a + Rae^{-x}$ and same scheme follows
	Get $a = {}^3/_2$ (or $R = 12$)	A1	
	Replace for a (or R) in relevant equation to set up solvable equation in R (or a)	M1	
	Get R=12 (or $a = {}^3/_2$)	A1	Ignore if $a={}^2/_3$ also given
(iv)	Quote/derive ($\ln{}^3/_2$, 12)	B1√ B1√	On their R and a
9(i)	Use $\sin\theta.\sin^{n-1}\theta$ and parts	M1	Reasonable attempt with 2 parts, one yet to be integrated

	Get $-\cos\theta.\sin^{n-1}\theta+(n-1)\int\sin^{n-2}\theta.\cos^2\theta\ d\theta$	A1	Signs need to be carefully considered
	Replace $\cos^2 = 1 - \sin^2$	M1	
	Clearly use limits and get AG	A1	
(ii)	**(a)** Solve for r=0 for at least one θ	M1	θ need not be correct
	Get (θ) = 0 and π	A1	Ignore extra answers out of range
		B1	General shape (symmetry stated or approximately seen)
		B1	Tangents at θ=0, π and max r seen
	(b) Correct formula used; correct r	M1	May be $\int r^2\ d\theta$ with correct limits
	Use $6I_6 = 5I_4$, $4I_4 = 3I_2$	M1	At least one
	Attempt I_0 (or I_2)	M1	(I_0 = ½π)
	Replace their values to get I_6	M1	
	Get 5π/32	A1	
	Use symmetry to get 5π/32	A1	May be implied but correct use of limits must be given somewhere in answer
	Or		
	Correct formula used; correct r	M1	
	Reasonable attempt at formula $(2i\sin\theta)^6 = (z - {}^1/z)^6$	M1	
	Attempt to multiply out both sides (7 terms)	M1	
	Get correct expansion	A1	
	Convert to trig. equivalent and integrate their expression	M1	cwo
	Get 5π/32	A1	
	Or		
	Correct formula used; correct r	M1	
	Use double-angle formula and attempt to cube (4 terms)	M1	
	Get correct expression	A1	
	Reasonable attempt to put $\cos^2 2\theta$ into integrable form and integrate	M1	
	Reasonable attempt to integrate $\cos^3 2\theta$ as e.g. $\cos^2 2\theta.\cos 2\theta$	M1	cwo
	Get 5π/32	A1	

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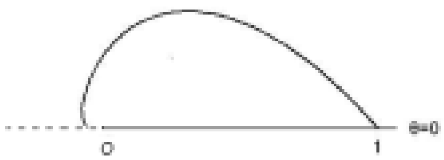
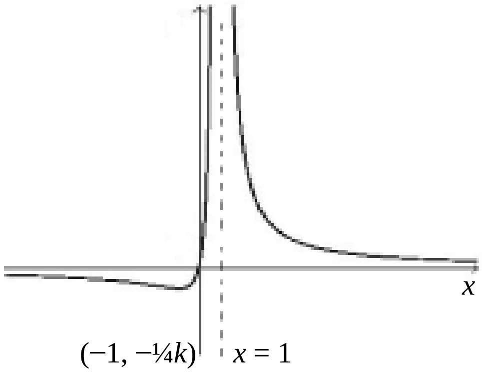
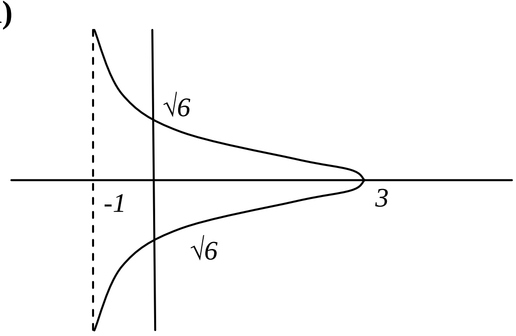
1	**(i)**	Get 0.876096, 0.876496, 0.876642	B1√ B1	For any one correct or √ from wrong answer; radians only All correct
	(ii)	Subtract correctly (0.00023(0), 0.000084) Divide their errors as e_4/e_3 only Get 0.365(21…)	B1√ M1 A1	On their answers May be implied Cao
2	**(i)**	Find $f'(x) = 1/(1+(1+x)^2)$ Get $f(0) = \frac{1}{4}\pi$ and $f'(0) = \frac{1}{2}$ Attempt $f''(x)$ Correctly get $f''(0) = -\frac{1}{2}$	M1 A1√ M1 A1	Quoted or derived; may be simplified or left as $\sec^2 y \, dy/dx = 1$ On their f′(0); allow f(0)=0.785 but not 45 Reasonable attempt at chain/quotient rule or implicit differentiation A.G.
	(ii)	Attempt Maclaurin as $af(0)+bf'(0)+cf''(0)$ Get $\frac{1}{4}\pi + \frac{1}{2}x - \frac{1}{4}x^2$	M1 A1	Using their f(0) and f′(0) Cao; allow 0.785
3	**(i)**	Attempt gradient as $\pm f(x_1)/(x_2 - x_1)$ Equate to gradient of curve at x_1 Clearly arrive at A.G. **SC** Attempt equation of tangent Put $(x_2, 0)$ into their equation Clearly arrive at A.G.	M1 M1 A1 M1 M1 A1	Allow reasonable y-step/x-step Allow ± Beware confusing use of ± As $y - f(x_1) = f'(x_1)(x - x_1)$
	(ii)	Diagram showing at least one more tangent Description of tangent meeting x-axis, used as next starting value	B1 B1	
	(iii)	Reasonable attempt at N-R Get 1.60	M1 A1	Clear attempt at differentiation Or answer which rounds
4	**(i)**	State $r = 1$ and $\theta = 0$. 	B1 B1	May be seen or implied Correct shape, decreasing r (not through O)
	(ii)	Use $\frac{1}{2}\int r^2 \, d\theta$ with $r = e^{-2\theta}$ seen or implied Integrate correctly as $-\frac{1}{8}e^{-4\theta}$ Use limits in correct order Use $r_1^2 = e^{-4\theta}$ etc. Clearly get $k = \frac{1}{8}$	M1 A1 M1 M1 A1	Allow $\frac{1}{2}\int e^{4\theta} \, d\theta$ In their answer May be implied

5	**(i)**	Use correct definitions of cosh and sinh	B1	
		Attempt to square and subtract	M1	On their definitions
		Clearly get A.G.	A1	
		Show division by $\cosh^2$	B1	Or clear use of first result
	(ii)	Rewrite as quadratic in sech and attempt to solve	M1	Or quadratic in cosh
		Eliminate values outside $0 < \text{sech} \le 1$	B1	Or eliminate values outside $\cosh \ge 1$ (allow positive)
		Get $x = \ln(2+\sqrt{3})$	A1	
		Get $x = -\ln(2+\sqrt{3})$ or $\ln(2-\sqrt{3})$	A1	
6	**(i)**	Attempt at correct form of P.F.	M1	Allow $Cx/(x^2+1)$ here; not $C = 0$
		Rewrite as 4= $A(1 + x)(1 + x^2) + B(1 - x)(1 + x^2) + (Cx + D)(1 - x)(1 + x)$	M1 √	From their P.F.
		Use values of x/equate coefficients	M1	
		Get $A = 1$, $B = 1$	A1	cwo
		Get $C = 0$, $D = 2$	A1	
				SC Use of cover-up rule for A,B M1; If both correct A1 cwo
	(ii)	Get $A\ln(1 + x) - B\ln(1 - x)$	M1	Or quote from List of Formulae
		Get $D\tan^{-1}x$	B1	
		Use limits in their integrated expressions	M1	
		Clearly get A.G.	A1	
7	**(i)**	LHS = sum of areas of rectangles, area = 1x y-value from $x = 1$ to $x = n$	B1	
		RHS = Area under curve from $x = 0$ to n	B1	
	(ii)	Diagram showing areas required	B1	
		Use sum of areas of rectangles	B1	
		Explain/show area inequality with limits in integral clearly specified	B1	
	(iii)	Attempt integral as $kx^{4/3}$	M1	
		Limits gives 348(.1) and 352(.0)	A1	Allow one correct
		Get 350	A1	From two correct values only

8	**(i)**	Get $x = 1$, $y = 0$	B1,B1	
	(ii)	Rewrite as quadratic in x Use $b^2 - 4ac \geq 0$ for all real x Get correct inequality State use of $k>0$ to A.G.	M1 M1 A1 A1	$(x^2y - x(2y + k) + y = 0)$ Allow >, = here $4ky + k^2 \geq 0$ SC Use differentiation (parts **(ii)** and **(iii)**) Attempt prod/quotient rule M1 Solve = 0 for $x = -1$ A1 Use $x = -1$ only (reject $x=1$), $y = -¼k$ A1 Fully justify minimum B1 Attempt to justify for all x M1 Clearly get A.G. A1
	(iii)	Replace $y = -¼k$ in quadratic in x Get $x = -1$ only 	M1 A1 B1 B1	 Through origin with minimum at $(-1, -¼k)$ seen or given in the answer Correct shape (asymptotes and approaches) **SC** (Start again) Differentiate and solve $dy/dx = 0$ for at least one x-value, independent of k M1 Get $x = -1$ only A1
9	**(i)**	Rewrite tanh y as $(e^y - e^{-y})/(e^y + e^{-y})$ Attempt to write as quadratic in e^{2y} Clearly get A.G.	B1 M1 A1	Or equivalent
	(ii)	**(a)** Attempt to diff. and solve = 0 Get $\tanh x = b/a$ Use $(-1) < \tanh x < 1$ to show $b < a$	M1 A1 B1	SC Use exponentials M1 Get $e^{2x} = (a + b)/(a - b)$ A1 Use $e^{2x} > 0$ to show $b < a$ B1 SC Write $x = \tanh^{-1}(b/a)$ M1 $= ½\ln((1 + b/a)/(1 - b/a))$ A1 Use ( ) > 0 to show $b < a$ B1
		(b) Get $\tanh x = 1/a$ from part **(ii)(a)** Replace as ln from their answer Get $x = ½ \ln ((a + 1)/(a - 1))$ Use $e^{½\ln((a+1)/(a-1))} = \sqrt{((a + 1)/(a - 1))}$ Clearly get A.G. Test for minimum correctly	B1 M1 A1 M1 A1 B1	 At least once **SC** Use of $y = \cosh x(a - \tanh x)$ and $\cosh x = 1/\text{sech} x = 1/\sqrt{(1 - \tanh^2 x)}$

1	Derive/quote $g'(x)= p/(1+x^2)$	B1
	Attempt $f'(x)$ as $a/(1+bx^2)$	M1 Allow any a, b=2 or 4
	Use x =½ to set up a solvable equation in p, leading to at least one solution	M1
	Get $p = {}^5/_4$ only	A1 AEEF
2	Reasonable attempt at $e^{2x}(1+2x+2x^2)$	M1 3 terms of the form $1+2x+ax^2$, $a\neq0$
	Multiply out their expressions to get all terms up to x^2	M1 (3 terms) x (minimum of 2 terms)
	Get $1+3x+4x^2$	A1 cao
	Use binomial, equate coefficients to get 2 solvable equations in a and n	M1 Reasonable attempt at binomial, each term involving a and n (an=3, $a^2n(n-1)/2=4$)
	Reasonable attempt to eliminate a or n	M1
	Get n=9, a=⅓ cwo	A1 cao
		SC Reasonable $f'(x)$ and $f''(x)$ using product rule (2 terms) M1 Use their expressions to find $f'(0)$ and $f''(0)$ M1 Get $1+3x+4x^2$ cao A1
3	Quote/derive correct $dx=2dt/(1+t^2)$	B1
	Replace all x (not $dx=dt$)	M1 From their expressions
	Get $2/(t-1)^2$ or equivalent	A1
	Reasonable attempt to integrate their expression	M1
	Use correct limits in their correct integral	A1√ Must involve √3
	Clearly tidy to √3+1 from cwo	A1 A.G.
4 (i)	Get a = -2	B1 May be quoted (from correct working)
	Get b = 6	B1 May be quoted (from correct working)
	Get c = 1	B1 May be quoted (from correct working)

(ii)



B1 Correct shape in $-1<x\leq3$ only (allow just top or bottom half)

B1 90^0 (at x=3) (must cross x-axis i.e. symmetry)

B1 Asymptote at x= -1 only (allow −1 seen)

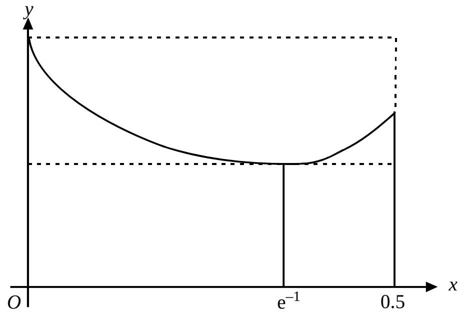
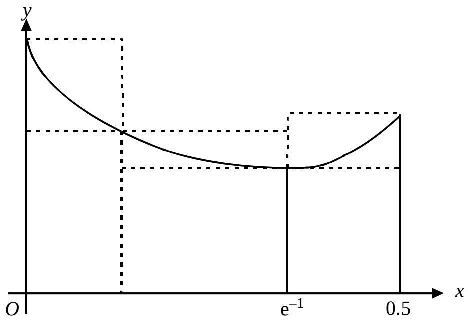
B1√ Correct crossing points; $\pm\sqrt{(b/c)}$ from their b,c

5 (i) Reasonable attempt at parts	M1	Leading to second integral
Get $e^x(1-2x)^n - \int e^x.n(1-2x)^{n-1}.-2\ dx$	A1	Or $(1-2x)^{n+1}/(-2(n+1))e^x$ $-\int (1-2x)^{n+1}/(-2(n+1))e^x dx$
Evidence of limits used in integrated part	M1	Should show ±1
Tidy to A.G.	A1	Allow $I_{n+1} = 2(n+1)I_n - 1$
(ii) Show any one of $I_3=6I_2-1$, $I_2=4I_1-1$, $I_1=2I_0-1$	B1	May be implied
Get $I_0(=e^{½}-1)$ or $I_1(=2e^{½}-3)$	B1	
Substitute their values back for their I_3	M1	Not involving n
Get $48e^{½} - 79$	A1	
6 (i) Reasonable attempt to differentiate $\sinh y = x$ to get dy/dx in terms of y	M1	Allow $\pm\cosh y\ dy/dx = 1$
Replace $\sinh y$ to A.G.	A1	Clearly use $\cosh^2 - \sinh^2 = 1$
		SC Attempt to diff. $y = \ln(x+\sqrt{(x^2+1)})$ using chain rule M1
		Clearly tidy to A.G. A1
(ii) Reasonable attempt at chain rule	M1	To give a product
Get $dy/dx = a\sinh(a\sinh^{-1}x)/\sqrt{(x^2+1)}$	A1	
Reasonable attempt at product/quotient	M1	Must involve sinh and cosh
Get d^2y/dx^2 correctly in some form	A1√	From $dy/dx = k\sinh(a\sinh^{-1}x)/\sqrt{(x^2+1)}$
Substitute in and clearly get A.G.	A1	
		SC Write $\sqrt{(x^2+1)}dy/dx = k\sinh(a\sinh^{-1}x)$ or similar
		Derive the A.G.
7 (i) Get 5.242, 5.239, 5.237	B1√	Any 3(minimum) correct from previous value
Get 5.24	B1	Allow one B1 for 5.24 seen if 2 d.p.used
(ii) Show reasonable staircase for any region	B1	Drawn curve to line
Describe any one of the three cases	B1	
Describe all three cases	B1	
(iii) Reasonable attempt to use log/expo. rules	M1	Allow derivation either way
Clearly get A.G.	A1	
Attempt f ′(x) and use at least once in correct N-R formula	M1	
Get answers that lead to 1.31	A1	Minimum of 2 answers; allow truncation/rounding to at least 3 d.p.
(iv) Show f ′(ln36) = 0	B1	
Explain why N-R would not work	B1	Tangent parallel to Ox would not meet Ox again or divide by 0 gives an error

8 (i)	Use correct definition of cosh x	B1	
	Attempt to cube their definition involving e^x and e^{-x} (or e^{2x} and e^x)	M1	Must be 4 terms
	Put their 4 terms into LHS and attempt to simplify	M1	
	Clearly get A.G.	A1	
			SC Allow one B1 for correct derivation from $\cosh 3x = \cosh(2x+x)$
(ii)	Rewrite as $k\cosh 3x = 13$	M1	
	Use ln equivalent on $13/k$	M1	Allow ±ln or $\ln(13/k \pm\sqrt{(13/k)^2 - 1})$ for their k or attempt to set up and solve quadratic via exponentials
	Get $x = (\pm)\ \tfrac{1}{3}\ln 5$	A1	
	Replace in cosh x for u	M1	
	Use $e^{a\ln b} = b^a$ at least once	M1	
	Get $\tfrac{1}{2}(5^{1/3} + 5^{-1/3})$	A1	
9 (i)	Attempt integral as $k(2x+1)^{1.5}$	M1	
	Get 9	A1	cao
	Attempt subtraction of areas	M1	Their answer – triangle
	Get 3	A1√	Their answer − 6 (>0)
(ii)	Use $r^2=x^2+y^2$ and $x=r\cos\theta$, $y=r\sin\theta$	B1	
	Eliminate x and y to produce quadratic equation (=0) in r (or $\cos\theta$)	M1	
	Solve their quadratic to get r in terms of θ (or vice versa)	A1√	
	Clearly get A.G.	A1	$r>0$ may be assumed
	Clearly show θ_1(at B)=$\tan^{-1}\tfrac{3}{4}$ and θ_2 (at A) =π	B1	
			SC Eliminate y to get r in terms of x only M1
			Get $r = x + 1$ A1
			SC Start with $r=1/(1-\cos\theta)$ and derive cartesian
(iii)	Use area = $\tfrac{1}{2}\int r^2\, d\theta$ with correct r	B1	cwo; ignore limits
	Rewrite as $k\,\text{cosec}^4(\tfrac{1}{2}\theta)$	M1	Not just quoted
	Equate to their part **(i)** and tidy	M1	To get ∫ = some constant
	Get 24	A1	A.G.

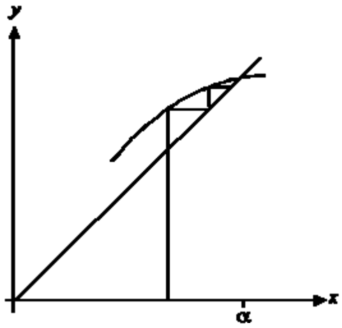
Question	Answer	Marks	Total	Guidance
1	$t = \tan\frac{1}{2}x \Rightarrow dt = \frac{1}{2}\sec^2\frac{1}{2}x\,dx = \frac{1}{2}(1+t^2)\,dx$	B1		For correct result **AEF** (may be implied)
	$\int \frac{1}{1+\sin x+\cos x}\,dx = \int \frac{1}{1+\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}} \cdot \frac{2}{1+t^2}\,dt$	M1		For substituting throughout for x
		A1		For correct unsimplified t integral
	$= \int \frac{1}{1+t}\,dt = \ln\lvert 1+t\rvert\ (+c)$	M1		For integrating (even incorrectly) to $a\ln\lvert f(t)\rvert$. Allow $\lvert\ \rvert$ or $(\ )$
	$= \ln k\left\lvert 1+\tan\frac{1}{2}x\right\rvert\ (+c)$	A1	5	For correct x expression k may be numerical, c is not required
			5	
2 (i)	$f(x) = \tanh^{-1}x,\ f'(x) = \frac{1}{1-x^2},\ f''(x) = \frac{2x}{(1-x^2)^2}$	M1		For quoting $f'(x) = \frac{1}{1 \pm x^2}$ and attempting to differentiate $f'(x)$
		A1		For $f''(x)$ correct **www**
	$f'''(x) =$ $\frac{2(1-x^2)^2 - 2x.2(1-x^2).-2x}{(1-x^2)^4}$ *OR* $\frac{2x.4x}{(1-x^2)^3} + \frac{2}{(1-x^2)^2}$	M1		For using quotient *OR* product rule on $f''(x)$
		A1		For correct unsimplified $f'''(x)$
	$= \frac{2(1-x^2)^2 + 8x^2(1-x^2)}{(1-x^2)^4}$ *OR* $\frac{8x^2}{(1-x^2)^3} + \frac{2(1-x^2)}{(1-x^2)^3}$			
	$= \frac{2(1+3x^2)}{(1-x^2)^3}$	A1	5	For simplified $f'''(x)$ **www AG**
(ii)	$f(0)=0,\ f'(0)=1,\ f''(0)=0$	B1√		For all values correct (may be implied) f.t. from **(i)**
	$f'''(0) = 2 \Rightarrow \tanh^{-1}x = x + \frac{1}{3}x^3$	M1		For evaluating $f'''(0)$ and using Maclaurin expansion
		A1	3	For correct series
			8	
3 (i)(a)	Asymptote $y = 0$	B1	1	For correct equation (allow x-axis)
(b)	METHOD 1 $y = \frac{5ax}{x^2+a^2} \Rightarrow yx^2 - 5ax + a^2y = 0$	M1		For expressing as a quadratic in x
		M1		For using $b^2 - 4ac \lesseqqgtr 0$
	$b^2 \geqslant 4ac \Rightarrow 25a^2 \geqslant 4a^2y^2 \Rightarrow -\frac{5}{2} \leqslant y \leqslant \frac{5}{2}$	A1		For $25a^2 - 4a^2y^2$ seen or implied
		A1	4	For correct range
	METHOD 2 $y = \frac{5ax}{x^2+a^2} \Rightarrow \frac{dy}{dx} = \frac{-5a\left(x^2-a^2\right)}{\left(x^2+a^2\right)^2}$	M1*		For differentiating y by quotient *OR* product rule
	$\frac{dy}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm\frac{5}{2}$	A1		For correct values of x
		M1		For finding y values and giving argument for range
	Asymptote, sketch etc $\Rightarrow -\frac{5}{2} \leqslant y \leqslant \frac{5}{2}$	A1 (*dep)		For correct range
(ii)(a)	$y = 0$	B1	1	For correct equation (allow x-axis)
(b)	Maximum $\sqrt{\frac{5}{2}}$, minimum $-\sqrt{\frac{5}{2}}$	B1√		For correct maximum f.t. from **(i)(b)**
		B1√	2	For correct minimum f.t. from **(i)(b)** Allow decimals
(c)	$x \geqslant 0$	B1	1	For correct set of values (allow in words)
			9	

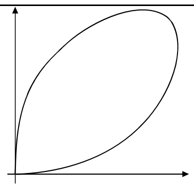
4 (i)	$8\sinh^4 x \equiv \frac{8}{16}\left(e^x - e^{-x}\right)^4$	B1		$\sinh x = \frac{1}{2}\left(e^x - e^{-x}\right)$ seen or implied
	$\equiv \frac{8}{16}\left(e^{4x} - 4e^{2x} + 6 - 4e^{-2x} + e^{-4x}\right)$	M1		For attempt to expand $(\ldots)^4$ by binomial theorem *OR* otherwise
	$\equiv \frac{1}{2}\left(e^{4x} + e^{-4x}\right) - \frac{4}{2}\left(e^{2x} + e^{-2x}\right) + \frac{6}{2}$	M1		For grouping terms for $\cosh 4x$ *or* $\cosh 2x$ *OR* using e^{4x} *or* e^{2x} expressions from RHS
	$\equiv \cosh 4x - 4\cosh 2x + 3$	A1	4	For correct expression **AG**
	SR may be done wholly from RHS to LHS	M1 M1 B1 A1		Evidence of factorising required for 2nd M1
(ii)	METHOD 1 $\cosh 4x - 3\cosh 2x + 1 = 0$			
	$\Rightarrow (8\sinh^4 x + 4\cosh 2x - 3) - 3\cosh 2x + 1 = 0$	M1		For using **(i)** and $\cosh 2x \equiv \pm 1 \pm 2\sinh^2 x$
	$\Rightarrow 8\sinh^4 x + 2\sinh^2 x - 1 = 0$	A1		For correct equation
	$\Rightarrow \left(4\sinh^2 x - 1\right)\left(2\sinh^2 x + 1\right) = 0 \Rightarrow \sinh x = \pm\frac{1}{2}$	M1 A1		For solving their quartic for $\sinh x$ For correct $\sinh x$ (ignore other roots)
	$\Rightarrow x = \ln\left(\pm\frac{1}{2} + \frac{1}{2}\sqrt{5}\right) = \pm\ln\left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)$	A1√	5	For correct answers (and no more) f.t. from their value(s) for $\sinh x$
	SR Similar scheme for $8\cosh^4 x - 14\cosh^2 x + 5 = 0 \Rightarrow \cosh x = \frac{1}{2}\sqrt{5} \Rightarrow x = \pm\ln\left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)$			
	METHOD 2 $\cosh 4x - 3\cosh 2x + 1 = 0$			
	$\Rightarrow (2\cosh^2 2x - 1) - 3\cosh 2x + 1 = 0$	M1		For using $\cosh 4x \equiv \pm 2\cosh^2 2x \pm 1$
	$\Rightarrow 2\cosh^2 2x - 3\cosh 2x = 0$	A1		For correct equation
	$\Rightarrow \cosh 2x = \frac{3}{2} \Rightarrow x = \frac{1}{2}\ln\left(\frac{3}{2} \pm \frac{1}{2}\sqrt{5}\right)$ $= \pm\frac{1}{2}\ln\left(\frac{3}{2} + \frac{1}{2}\sqrt{5}\right)$	M1 A1 A1√		For solving for $\cosh 2x$ For correct $\cosh 2x$ (ignore others) For correct answers (and no more) f.t. from value(s) for $\cosh 2x$
	METHOD 3 Put all into exponentials	M1		For changing to $e^{\pm kx}$
	$\Rightarrow e^{4x} - 3e^{2x} + 2 - 3e^{-2x} + e^{-4x} = 0$	A1		For correct equation
	$\Rightarrow \left(e^{4x} + 1\right)\left(e^{4x} - 3e^{2x} + 1\right) = 0$	M1 A1		For solving for e^{2x} For correct e^{2x} (ignore others)
	$\Rightarrow e^{2x} = \frac{1}{2}\left(3 \pm \sqrt{5}\right) \Rightarrow x = \frac{1}{2}\ln\left(\frac{3}{2} \pm \frac{1}{2}\sqrt{5}\right)$	A1√		For correct answers (and no more) f.t. from value(s) for e^{2x}
			9	
5 (i)	$x_{n+1} = x_n - \dfrac{x_n^3 - 5x_n + 3}{3x_n^2 - 5} = \dfrac{2x_n^3 - 3}{3x_n^2 - 5}$	M1 A1 A1	3	For attempt at N-R formula For correct N-R expression For correct answer (necessary details needed) **AG** Allow omission of suffixes
(ii)	$F'(x) =$	M1		For using quotient *OR* product rule to find $F'(x)$
	$\dfrac{6x^2\left(3x^2 - 5\right) - 6x\left(2x^3 - 3\right)}{\left(3x^2 - 5\right)^2} = \dfrac{6x\left(x^3 - 5x + 3\right)}{\left(3x^2 - 5\right)^2}$	M1		For factorising numerator to show $k\left(x^3 - 5x + 3\right)$
	$F'(\alpha) = \dfrac{6\alpha\left(\alpha^3 - 5\alpha + 3\right)}{\left(3\alpha^2 - 5\right)^2} = 0$ since $\alpha^3 - 5\alpha + 3 = 0$	A1	3	For correct explanation of **AG**
(iii)	$x_1 = 2 \Rightarrow 1.85714,\ 1.83479,\ 1.83424,\ 1.83424$ $(\alpha =)\ 1.8342$	B1 B1 B1	3	First iterate correct to at least 4 d.p. *OR* $\frac{13}{7}$ For 2 equal iterates to at least 4 d.p. For correct α to 4 d.p. Allow answer rounding to 1.8342 **SR** If not N-R, B0 B0 B0
	SR For starting value leading to another root allow up to B1 B1 B0			
			9	

6	(i)	$y = x^x \Rightarrow \ln y = x \ln x \Rightarrow \frac{1}{y}\frac{dy}{dx} = 1 + \ln x$	M1		For differentiating $\ln y$ *OR* $x \ln x$ w.r.t. x
		$\frac{dy}{dx} = x^x(1+\ln x) = 0 \Rightarrow \ln x = -1 \Rightarrow x = e^{-1}$	A1		For $(1+\ln x)$ seen or implied
			A1	3	For correct x-value from fully correct working **AG**
	(ii)	$A > 0.2 \times 0.5^{0.5} + 0.2 \times 0.7^{0.7} + 0.1 \times 0.9^{0.9}$	M1		For areas of 3 lower rectangles
		$\Rightarrow A > 0.3881(858) > 0.388$	A1	2	For lower bound rounding to **AG**
	(iii)	$A < 0.2 \times 0.7^{0.7} + 0.2 \times 0.9^{0.9} + 0.1 \times 1^1$	M1		For areas of 3 upper rectangles
		$\Rightarrow A < 0.4377(177) < 0.438$	A1	2	For upper bound rounding to 0.438
	(iv)		M1		Consider rectangle of height $f(e^{-1})$
			A1		Use at least 1 lower rectangle, height $f(e^{-1})$
			B1	3	Use at least 1 upper rectangle, height $f(0)$
					**SR** If more than one rectangle is used for either bound, they must be shown correctly
				10	
7	(i)	$\cos 3\theta = \cos(-3\theta)$ *OR* $\cos\theta = \cos(-\theta)$ for all θ	M1		For a correct procedure for symmetry related to the equation *OR* to $\cos 3\theta$
		$\Rightarrow$ equation is unchanged, so symmetrical about $\theta = 0$	A1	2	For correct explanation relating to equation **AG**
	(ii)	$r = 0 \Rightarrow \cos 3\theta = -1$	M1		For obtaining equation for tangents
		$\Rightarrow \theta = \pm\frac{1}{3}\pi, \pi$	A1		A1 for any 2 values
			A1	3	A1 for all, no extras (ignore outside range)
	(iii)	$\int_{-\frac{1}{3}\pi}^{\frac{1}{3}\pi} \frac{1}{2}(1+\cos 3\theta)^2 \, (d\theta)$	B1		For correct integral with limits soi (limits may be $\left[0, \frac{1}{3}\pi\right]$ at any stage)
		$= \frac{1}{2}\int_{-\frac{1}{3}\pi}^{\frac{1}{3}\pi} 1 + 2\cos 3\theta + \cos^2 3\theta \, d\theta$	M1*		For multiplying out, giving at least 2 terms
		$= \frac{1}{2}\int_{-\frac{1}{3}\pi}^{\frac{1}{3}\pi} 1 + 2\cos 3\theta + \frac{1}{2}(1+\cos 6\theta) \, d\theta$	M1		For integration to $A\theta + B\sin 3\theta + C\sin 6\theta$ **AEF**
		$= \frac{1}{2}\left[\theta + \frac{2}{3}\sin 3\theta + \left(\frac{1}{2}\theta + \frac{1}{12}\sin 6\theta\right)\right]_{-\frac{1}{3}\pi}^{\frac{1}{3}\pi}$	M1 (*dep)		For completing integration and substituting their limits into terms in $\frac{\cos}{\sin} n\theta$
		$= \frac{1}{2}\pi$	A1	5	For correct area **www**
				10	

8	(i)	METHOD 1 $\sinh(\cosh^{-1}2) =$ $\sinh\beta = \sqrt{\cosh^2\beta - 1} = \sqrt{2^2 - 1} = \sqrt{3}$	M1 A1	 2	For appropriate use of $\sinh^2\theta = \cosh^2\theta - 1$ For correct verification to **AG**
		METHOD 2 $\sinh^{-1}\sqrt{3} = \ln\left(\sqrt{3}+2\right)$, $\cosh^{-1}2 = \ln\left(2+\sqrt{3}\right)$ $\Rightarrow \sinh(\cosh^{-1}2) = \sqrt{3}$	M1 A1		For attempted use of ln forms of $\sinh^{-1}x$ and $\cosh^{-1}x$ For both ln expressions seen
		METHOD 3 $\cosh^{-1}2 = \ln\left(2+\sqrt{3}\right)$ $\sinh\left(\cosh^{-1}2\right) = \frac{1}{2}\left(e^{\ln\left(2+\sqrt{3}\right)} - e^{-\ln\left(2+\sqrt{3}\right)}\right)$ $= \frac{1}{2}\left(2+\sqrt{3} - \left(2-\sqrt{3}\right)\right) = \sqrt{3}$	M1 A1		For use of ln form of $\cosh^{-1}x$ and definition of $\sinh x$ For correct verification to **AG** **SR** Other similar methods may be used Note that $\ln\left(2+\sqrt{3}\right) = -\ln\left(2-\sqrt{3}\right)$
	(ii)	$I_n = \int_0^\beta \cosh^n x \, dx$	M1*		For attempt to integrate $\cosh x . \cosh^{n-1} x$ by parts
		$= \left[\sinh x . \cosh^{n-1} x\right]_0^\beta - \int_0^\beta \sinh^2 x . (n-1)\cosh^{n-2} x \, dx$	A1		For correct first stage of integration (ignore limits)
		$= \sinh\beta . \cosh^{n-1}\beta - (n-1)\int_0^\beta \left(\cosh^2 x - 1\right)\cosh^{n-2} x \, dx$	M1 (*dep)		For using $\sinh^2 x = \cosh^2 x - 1$
		$= 2^{n-1}\sqrt{3} - (n-1)\left(I_n - I_{n-2}\right)$	A1 B1		For $(n-1)\left(I_n - I_{n-2}\right)$ correct For $2^{n-1}\sqrt{3}$ correct at any stage
		$\Rightarrow nI_n = 2^{n-1}\sqrt{3} + (n-1)I_{n-2}$	A1	6	For correct result **AG**
	(iii)	$I_1 = \int_0^\beta \cosh x \, dx = \sinh\beta = \sqrt{3}$	B1		For correct value
		$I_3 = \frac{1}{3}\left(2^2\sqrt{3} + 2\sqrt{3}\right) = 2\sqrt{3}$	M1 A1		For using **(ii)** with $n = 3$ *OR* $n = 5$ For $I_3 = \frac{1}{3}\left(2^2\sqrt{3} + 2I_1\right)$ *OR* $I_5 = \frac{1}{5}\left(2^4\sqrt{3} + 4I_3\right)$
		$I_5 = \frac{1}{5}\left(2^4\sqrt{3} + 8\sqrt{3}\right) = \frac{24}{5}\sqrt{3}$	A1	4	For correct value
				12	

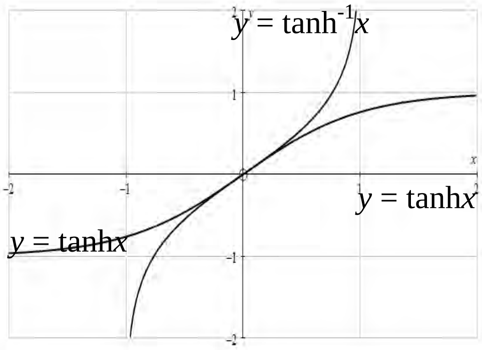
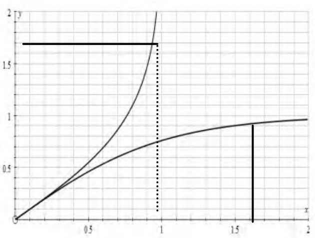
1	$\dfrac{2x+3}{(x+3)(x^2+9)} \equiv \dfrac{A}{x+3}+\dfrac{Bx+C}{x^2+9}$	**B1**	For correct form seen anywhere with letters or values
	$A=-\dfrac{1}{6}$	**B1**	For correct *A* (cover up or otherwise)
	$2x+3 \equiv A(x^2+9)+(Bx+C)(x+3)$	**M1**	For equating coefficients at least once.(or substituting values) into correct identity.
	$B=\dfrac{1}{6},\quad C=\dfrac{3}{2}$	**A1**	For correct *B* and *C*
	$\Rightarrow \dfrac{-1}{6(x+3)}+\dfrac{x+9}{6(x^2+9)}$	**A1**	For correct final statement cao, oe
		5	
2(i)	Asymptote $x = 2$	**B1**	For correct equation
	$y=x-4-\dfrac{13}{x-2}$	**M1**	For dividing out (remainder not required)
	$\Rightarrow$ asymptote $y=x-4$	**A1**	For correct equation of asymptote
		3	(ignore any extras)
(ii)	METHOD 1		**N.B. answer given**
	$x^2-(y+6)x+(2y-5)=0$	**M1**	For forming quadratic in x
	$b^2-4ac(\geq 0) \Rightarrow (y+6)^2-4(2y-5)(\geq 0)$	**M1**	For considering discriminant
	$\Rightarrow y^2+4y+56(\geq 0)$	**A1**	For correct simplified expression in y **soi**
	$\Rightarrow (y+2)^2+52\geq 0$: this is true $\forall y$ So y takes all values	**A1**	For completing square (or equivalent) and correct conclusion **www**
	METHOD 2		
	Obtain $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{x^2-4x+17}{(x-2)^2}$ OR $1+\dfrac{13}{(x-2)^2}$	**M1**	For finding $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ either by direct differentiation or dividing out first
		A1	For correct expression **oe.**
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}\geq 1\ \forall x$,	**M1**	For drawing a conclusion
	so y takes all values.	**A1**	For correct conclusion **www**
		4	
	Alternate scheme: Sketching graph		
	Graph correct approaching asymptotes from both side	**B1**	A graph with no explanation can only score 2
	Graph completely correct	**B1**	
	Explanation about no turning values	**B1**	
	Correct conclusion	**B1**	

3(i)	$x_1 = 3.1 \Rightarrow x_2 = 3.13140$, $x_3 = 3.14148$	**B1** **B1** **2**	For correct x_2 For correct x_3
(ii)	$F'(\alpha) \approx \frac{e_3}{e_2} = \frac{0.00471}{0.01479} = 0.318\ (0.31846)$ $F'(\alpha) = \frac{1}{\alpha} = 0.3178\ (0.31784)$	**M1** **A1** **B1** **3**	For dividing e_3 by e_2 For estimate of $F'(\alpha)$ For true $F'(\alpha)$ obtained from $\frac{d}{dx}(2 + \ln x)$ **TMDP anywhere in (i) (ii) deduct 1 once (but answers must round to given values or A0)**
(iii)	 Staircase	**B1** **B1** **B1** **3**	For $y = x$ and $y = F(x)$ drawn, crossing as shown For lines drawn to illustrate iteration (Min 2 horizontal and 2 vertical seen) For stating "staircase"

4(i)	$x = r\cos\theta,\ y = r\sin\theta$ $\Rightarrow r = \dfrac{a\cos\theta\sin\theta}{\cos^3\theta + \sin^3\theta}$ for $0 \le \theta \le \frac{1}{2}\pi$	**M1** **A1** **A1** **3**	For substituting for x and y For correct equation **oe** (Must be r = …..) For correct limits for θ (Condone <)
(ii)	$\mathrm{f}\left(\frac{1}{2}\pi - \theta\right) = \dfrac{a\cos\left(\frac{1}{2}\pi - \theta\right)\sin\left(\frac{1}{2}\pi - \theta\right)}{\cos^3\left(\frac{1}{2}\pi - \theta\right) + \sin^3\left(\frac{1}{2}\pi - \theta\right)}$ $= \dfrac{a\sin\theta\cos\theta}{\sin^3\theta + \cos^3\theta}$ $\mathrm{f}(\theta) = \mathrm{f}\left(\frac{1}{2}\pi - \theta\right) \Rightarrow \alpha = \frac{1}{4}\pi$	**M1** **A1** **A1** **3**	**N.B. answer given** For replacing θ by $\left(\frac{1}{2}\pi - \theta\right)$ in their $\mathrm{f}(\theta)$ For correct simplified form. (Must be convincing) For correct reason for $\alpha = \frac{1}{4}\pi$
(iii)	$r = \dfrac{a.\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}}}{\left(\frac{1}{\sqrt{2}}\right)^3 + \left(\frac{1}{\sqrt{2}}\right)^3} = \frac{1}{2}\sqrt{2}\,a$	**B1** **1**	For correct value of r.**oe**
(iv)		**B1** **B1** **2**	Closed curve in 1st quadrant only, symmetrical about $\theta = \frac{1}{4}\pi$ Diagram showing $\theta = 0, \frac{1}{2}\pi$ tangential at O

5(i)	$x=\sin y \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y}=\cos y$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\sqrt{1-\sin^2 y}}=\frac{1}{\sqrt{1-x^2}}$ $+\sqrt{\ }$ taken since $\sin^{-1}x$ has positive gradient	**M1** **A1** **B1** **3**	For implicit diffn to $\frac{\mathrm{d}y}{\mathrm{d}x}=\pm\frac{1}{\cos y}$ **oe** For using $\sin^2 y+\cos^2 y=1$ to obtain **N.B. Answer given** For justifying + sign
(ii)	$\mathrm{f}(0)=0,\ \mathrm{f}'(0)=1$ $\mathrm{f}''(x)=\frac{x}{\left(1-x^2\right)^{\frac{3}{2}}}$ $\mathrm{f}'''(x)=\frac{\left(1-x^2\right)^{\frac{3}{2}}+3x^2\left(1-x^2\right)^{\frac{1}{2}}}{\left(1-x^2\right)^3}$ $\Rightarrow \mathrm{f}''(0)=0,\quad \mathrm{f}'''(0)=1$ $\Rightarrow \sin^{-1}x=x+\tfrac{1}{6}x^3$	**B1** **M1** **M1** **A1** **A1** **5**	For correct values Use of chain rule to differentiate $\mathrm{f}'(x)$ Use of quotient or product rule to differentiate $\mathrm{f}''(0)$. For correct values **www, soi** For correct series (allow 3!) **www**
	Alternative Method: $\mathrm{f}(0)=0,\ \mathrm{f}'(0)=1$ $\mathrm{f}'(x)=\frac{1}{\sqrt{1-x^2}}=\left(1-x^2\right)^{-\frac{1}{2}}=1+\frac{1}{2}x^2+\frac{3}{8}x^4+\ldots$ $\mathrm{f}''(x)=x+\frac{3}{2}x^3+\ldots$ $\mathrm{f}'''(x)=1+\frac{9}{2}x^2+\ldots$ $\Rightarrow \mathrm{f}'(0)=1, \mathrm{f}''(0)=0, \mathrm{f}'''(0)=1$ $\Rightarrow \sin^{-1}x=x+\tfrac{1}{6}x^3$	**B1** **M1** **M1** **A1** **A1**	For correct values Correct use of binomial Differentiate twice Correct values Correct series
(iii)	$\left(\sin^{-1}x\right)\ln(1+x)$ $=\left(x+\tfrac{1}{6}x^3\right)\left(x-\tfrac{1}{2}x^2+\tfrac{1}{3}x^3\right)$ $=x^2-\tfrac{1}{2}x^3+\tfrac{1}{2}x^4$	**B1ft** **M1** **A1** **A1** **4**	For terms in both series to at least x^3 f.t. from their **(ii)** multiplied together For multiplying terms to at least x^3 For correct series up to x^3 **www** For correct term in x^4 **www**

6(i)	$I_n = \int_0^1 x^n (1-x)^{\frac{3}{2}}\ \mathrm{d}x$	**M1**	For integrating by parts (correct way round)
	$= \left[-\frac{2}{5} x^n (1-x)^{\frac{5}{2}} \right]_0^1 + \frac{2}{5} n \int_0^1 x^{n-1} (1-x)^{\frac{5}{2}}\ \mathrm{d}x$	**A1**	For correct first stage
	$\Rightarrow I_n = \frac{2}{5} n \int_0^1 x^{n-1} (1-x)^{\frac{5}{2}}\ \mathrm{d}x$	**A1**	
	$\Rightarrow I_n = \frac{2}{5} n \int_0^1 x^{n-1} (1-x)(1-x)^{\frac{3}{2}}\ \mathrm{d}x$	**M1**	For splitting $(1-x)^{5/2}$ suitably
	$\Rightarrow I_n = \frac{2}{5} n I_{n-1} - \frac{2}{5} n I_n$	**A1**	For obtaining correct relation between I_n and I_{n-1}
	$\Rightarrow I_n = \frac{2n}{2n+5} I_{n-1}$	**A1** **6**	For correct result (**N.B. answer given**)
(ii)	$I_0 = \left[-\frac{2}{5} (1-x)^{\frac{5}{2}} \right]_0^1 = \frac{2}{5}$	**M1**	For evaluating I_0 [*OR* I_1 by parts]
		M1	For using recurrence relation 3 [*OR* 2] times (may be combined together)
	$I_3 = \frac{6}{11} I_2 = \frac{6}{11} \times \frac{4}{9} I_1 = \frac{6}{11} \times \frac{4}{9} \times \frac{2}{7} I_0$	**A1**	For 3 [*OR* 2] correct fractions
	$I_3 = \frac{32}{1155}$	**A1** **4**	For correct exact result

7(i)	 y = tanh^{-1}x	**B1** **B1** **B1** **B1** **4**	Both curves of the correct shape (ignore overlaps) and labelled gradient = 1 at $x = 0$ stated For asymptotes $y = \pm1$ and $x = \pm1$ (or on sketch) Sketch all correct
(ii)	$\int_0^k \tanh x\,\mathrm{d}x = \left[\ln(\cosh x)\right]_0^k = \ln(\cosh k)$	**M1** **A1** **2**	For substituting limits into $\ln\cosh x$ For correct answer
(iii)	 Areas shown are equal: $x = \tanh k$ $\Rightarrow y = k$ $\Rightarrow \int_0^{\tanh k} \tanh^{-1} x\,\mathrm{d}x$ = rectangle ($k \times \tanh k$)– **(ii)** $= k\tanh k - \ln(\cosh k)$	**M1** **A1** **M1** **A1** **4**	For consideration of areas For sufficient justification For subtraction from rectangle For correct answer **N.B. answer given** **Alternative:** Otherwise by parts, as $1 \times \tanh^{-1} x$ *OR* $1 \times \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$

PTO for alternative schemes

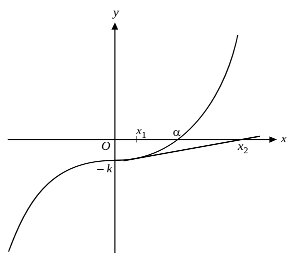
7(iii)	Alternative method 1 By parts: $I = \int_0^{\tanh k} \tanh^{-1} x \, dx$ $u = \tanh^{-1} x \quad dv = dx$ $du = \frac{1}{1-x^2} dx \quad v = x$ $\Rightarrow I = \left[x \tanh^{-1} x\right]_0^{\tanh k} - \int_0^{\tanh k} \frac{x}{1-x^2} \, dx$ $= k \tanh k + \frac{1}{2}\left[\ln(1-x^2)\right]_0^{\tanh k}$ $= k \tanh k + \frac{1}{2}\ln(1-\tanh^2 k)$ $= k \tanh k + \frac{1}{2}\ln(\text{sech}^2 k)$ $= k \tanh k + \ln(\text{sech} k)$	**M1** **A1** **M1** **A1**	For integrating by parts (correct way round) For getting this far Dealing with the resulting integral
	Alternative method 2 By substitution Let $y = \tanh^{-1} x \Rightarrow x = \tanh y$ $\Rightarrow dx = \text{sech}^2 y \, dy$ When $x = 0, \ y = 0$ When $x = \tanh k, \ y = k$ $\Rightarrow I = \int_0^{\tanh k} \tanh^{-1} x \, dx = \int_0^k y \text{sech}^2 y \, dy$ $u = y \quad dv = \text{sech}^2 y \, dy$ $du = dy \quad v = \tanh y$ $\Rightarrow I = \left[y \tanh y\right]_0^k - \int_0^k \tanh y \, dy$ $= k \tanh k - \ln \cosh k$	**M1** **A1** **M1** **A1**	For substitution to obtain equivalent integral Correct so far For integration by parts (correct way round) Final answer

8(i)	$x=\cosh^2 u \Rightarrow \mathrm{d}u = 2\cosh u \sinh u \,\mathrm{d}u$	**B1**	For correct result
	$\int \sqrt{\frac{x}{x-1}}\,\mathrm{d}x = \int \frac{\cosh u}{\sinh u} 2\cosh u \sinh u \,\mathrm{d}u$	**M1**	For substituting throughout for x
	$= \int 2\cosh^2 u \,\mathrm{d}u$	**A1**	For correct simplified u integral
	$= \int (\cosh 2u + 1)\,\mathrm{d}u = \sinh u \cosh u + u$	**M1**	For attempt to integrate $\cosh^2 u$
		A1	For correct integration
	$= x^{\frac{1}{2}}(x-1)^{\frac{1}{2}} + \ln\left(x^{\frac{1}{2}} + (x-1)^{\frac{1}{2}}\right) \; (+c)$	**M1**	For substituting for u
		A1 **7**	For correct result **oe** as $\mathrm{f}(x)+\ln(\mathrm{g}(x))$
(ii)	$2\sqrt{3} + \ln(2+\sqrt{3})$	**B1** **1**	
(iii)	$V = (\pi)\int_1^4 \frac{x}{x-1}\,\mathrm{d}x = (\pi)\left[x + \ln(x-1)\right]_1^4$	**M1**	For attempt to find $\int \frac{x}{x-1}\,\mathrm{d}x$
		A1	For correct integration (ignore π)
	$V \to \infty$	**B1** **3**	For statement that volume is infinite (independent of M mark)

Question			Answer	Marks	Guidance	
1			$f'(x)=\frac{-3\sin 3x}{\cos 3x}=-3\tan 3x \Rightarrow f'(0)=0$	M1	For differentiating f(x) twice (y' as a function of a function)	
			$f''(x)=-9\sec^2 3x \Rightarrow f''(0)=-9$	A1	For correct f '(0) and f "(0) **www** (soi by correct expansion)	
			$\Rightarrow f(x)=-\frac{9}{2}x^2$	M1	For use of Maclaurin soi	If f"(0) = f'(0) = f(0) = 0 then **M0**
				A1	For correct series (condone $a=-\frac{9}{2}x^2$)	
			ALT: $\ln(\cos 3x)=\ln\left(1-\frac{1}{2}(3x)^2\right)=-\frac{9}{2}x^2$	**[4]**	**SC** Use of standard cos and ln series can earn second M1 A1	
				[4]		
2			$=\int_{\frac{1}{2}}^{\frac{3}{2}}\frac{1}{(2x-1)^2+4}\,dx$ *OR* $\frac{1}{4}\int_{\frac{1}{2}}^{\frac{3}{2}}\frac{1}{\left(x-\frac{1}{2}\right)^2+1}\,dx$	B1	For correct denominator (in 2nd case must include $\frac{1}{4}$)	
				M1	For integration to $k\tan^{-1}(ax+b)$ or $k\ln\left(\frac{ax+b-c}{ax+b+c}\right)$	
			$=\frac{1}{2}\left[\frac{1}{2}\tan^{-1}\frac{2x-1}{2}\right]_{\frac{1}{2}}^{\frac{3}{2}}$ *OR* $\frac{1}{4}\left[\tan^{-1}\left(x-\frac{1}{2}\right)\right]_{\frac{1}{2}}^{\frac{3}{2}}$	A1	FT for $ax+b$ from their denominator For correct integration	
			$=\frac{1}{4}\left(\tan^{-1}1-\tan^{-1}0\right)=\frac{1}{16}\pi$	M1	For substituting limits in any $\tan^{-1}$ expression	
				A1 **[5]**	For correct value	

Question			Answer	Marks	Guidance	
3			$\frac{2x^3+x+12}{(2x-1)\left(x^2+4\right)} \equiv A+\frac{B}{2x-1}+\frac{Cx+D}{x^2+4}$	B1	For correct form soi (*A* can be P*x* + Q, but not 0)	
			$2x^3+x+12 \equiv$ $A(2x-1)\left(x^2+4\right)+B\left(x^2+4\right)+(Cx+D)(2x-1)$	M1	For multiplying out from their form	
			$A=1$, $B=3$	B1	For either *A* or *B* correct (dep on 1st B1)	
			$x^3:\ 2=2A \quad x^2:\ 0=-A+B+2C$ $x^1:\ 1=8A-C+2D \qquad x^0:\ 12=-4A+4B-D$	M1	For equating at least 2 coefficients (or substitute two values for *x* or one of each)	
			$C=-1$, $D=-4$	A1A1	For *C*, *D* correct	
			$\Rightarrow 1+\frac{3}{2x-1}+\frac{-x-4}{x^2+4}$	A1 **[7]**	For correct expression **www** **SC4** $\Rightarrow \frac{3}{2x-1}+\frac{x^2-x}{x^2+4}$	
			ALT: Divide out as not proper $\Rightarrow 1+\frac{x^2-7x+16}{(2x-1)\left(x^2+4\right)}$	B1	Divide out	
			$=1+\frac{A}{2x-1}+\frac{Bx+C}{x^2+4}$	B1	Writing in this form including 1	
			$x^2-7x+16 \equiv A(x^2+4)+(Bx+C)(2x-1)$	M1	For multiplying out from their form	
			$x^2:1=A+2B \qquad x:\ -7=-B+2C$ $1:16=4A-C$	M1	For equating at least 2 coefficients (or substitute two values for *x* or one of each)	
			$\Rightarrow A=3,\ B=-1,\ C=-4$	A1 A1	*B* correct *C* correct	
			$\Rightarrow 1+\frac{3}{2x-1}+\frac{-x-4}{x^2+4}$	A1	For correct expression **www**	

Question			Answer	Marks	Guidance	
4	(i)		Given expression is sum of areas of rectangles of width $\frac{1}{n}$, heights $e^{-1/x}$	B1	For identifying rectangle widths and heights	
			Given integral is area under the curve which is clearly greater	B1	For correct explanation of lower bound	
				[2]		
4	(ii)		Upper bound = $\frac{1}{n}\left(e^{-n}+e^{-\frac{n}{2}}+e^{-\frac{n}{3}}+\cdots+e^{-\frac{n}{n-1}}+e^{-1}\right)$	M1 A1	For using n upper rectangles soi by e^{-n} and e^{-1} For correct expression	
				[2]		
4	(iii)		Lower bound = 0.104(31) Upper bound = 0.196(28)	B1 B1 [2]	For correct value For correct value – accept 0.197	
4	(iv)		$\frac{1}{n}e^{-1} < 0.001$	B1	For a correct statement (includes <)	
			$\Rightarrow n > \frac{1000}{e} = 367.879$	M1	For rearranging (ignore < > = and allow RHS = $10^{\pm m}e^{\pm 1}$)	
			$\Rightarrow$ least N = 368	A1 [3]	For correct value	
5	(i)		$x_{n+1} = x_n - \frac{x_n^3 - k}{3x_n^2}$	M1	For correct $\frac{f(x)}{f'(x)}$ seen (x or x_n)	
			$\Rightarrow x_{n+1} = \frac{2x_n^3 + k}{3x_n^2}$	A1	For simplification to **AG** (x_n and x_{n+1} required)	
				[2]		

Question			Answer	Marks	Guidance	
5	**(ii)**			B1 M1 A1 **[3]**	For correct curve with α (or $\sqrt[3]{k}$) and $-k$ marked For a suitable tangent shown with x_1 and x_2 marked such that $\lvert\alpha - x_2\rvert > \lvert\alpha - x_1\rvert$	Curve looks like cubic with one pt of inflection (g not nec. 0) at y axis
5	**(iii)**		$\alpha = \sqrt[3]{100}$ $x_2 = 4.66667$ $x_3 = 4.64172$	B1 B1 B1 **[3]**	For correct α (Condone x =...) For correct x_2 (to at least 5dp) For correct x_3 (to at least 5dp)	
5	**(iv)**		$e_1 = -0.35841, \quad e_2 = -0.02508, \quad e_3 = -0.00013$ $\frac{{e_2}^3}{e_1^2} = -0.00012$	M1 A1 A1 **[3]**	For calculating e_1, e_2, e_3 from α or something better than x_3 All correct to 5 dp For obtaining –0.00012 **SC2** for consistently without –ve signs	
6	**(i)**		$\cos y = x \Rightarrow -\sin y\frac{dy}{dx} = 1$ $\Rightarrow \frac{dy}{dx} = -\frac{1}{\sin y} = -\frac{1}{\sqrt{1-x^2}}$ – sign since $\frac{dy}{dx} < 0$ (e.g. by graph)	M1 A1 B1 **[3]**	For differentiating $\cos y$ wrt x For using $\cos^2 y + \sin^2 y = 1$ to obtain **AG** For justification of $+\sqrt{\ }$ taken **SC1** if in fractions $\frac{14}{3}$ and $\frac{2047}{441}$	

Question			Answer	Marks	Guidance	
6	(ii)		$\frac{dy}{dx}=-\frac{-2x}{\sqrt{1-\left(1-x^2\right)^2}}$	M1	For differentiating $\cos^{-1}\left(1-x^2\right)$ (as a function of a function)	
				A1	For correct $\frac{dy}{dx}$ (unsimplified)	
			$=\frac{2x}{\sqrt{2x^2-x^4}}=\frac{2}{\sqrt{2-x^2}}$	A1	For correct $\frac{dy}{dx}$ (simplified)	
			$\frac{d^2y}{dx^2}=2.-\tfrac{1}{2}.-2x\left(2-x^2\right)^{-\frac{3}{2}}=\frac{2x}{\left(2-x^2\right)^{\frac{3}{2}}}$	M1	For differentiating $\frac{dy}{dx}$ using chain rule correctly (or product or quotient if y' is wrong)	
			$\Rightarrow\left(2-x^2\right)\frac{d^2y}{dx^2}=\frac{2x}{\sqrt{2-x^2}}=x\frac{dy}{dx}$	A1	For verification of **AG**	
				[5]		
7	(i)		$x=\sinh y=\frac{e^y-e^{-y}}{2}$	M1	For correct expression for sinh y and attempt to obtain quadratic	
			$\Rightarrow e^{2y}-2xe^y-1=0 \Rightarrow e^y=x\pm\sqrt{x^2+1}$	A1	For correct solution(s) for e^y	
			reject – sign as $e^y>0 \Rightarrow y=\ln\left(x+\sqrt{x^2+1}\right)$	A1	For justification of + sign to **AG**	
				[3]		
			Alt: $\sinh y+\cosh y=e^y$ $\sinh y=x\Rightarrow\cosh y=\pm\sqrt{x^2+1}$ reject -ve sign as $e^y>0$ $\Rightarrow e^y=x+\sqrt{x^2+1}\Rightarrow y=\ln\left(x+\sqrt{x^2+1}\right)$			

Question			Answer	Marks	Guidance	
7	(ii)		$\ln\left(x+\sqrt{x^2+1}\right)-\ln\left(x+\sqrt{x^2-1}\right)=\ln 2$	M1	For stating both ln expressions and attempting to exponentiate	Removing lns is not an attempt to exponentiate
			$\Rightarrow \dfrac{x+\sqrt{x^2+1}}{x+\sqrt{x^2-1}}=2$			
			$\Rightarrow \sqrt{x^2+1}-2\sqrt{x^2-1}=x$	A1	For correct equation **AG**	
				M1	For attempting to square once	
			$\Rightarrow 4x^2-3=4\sqrt{x^4-1}$	A1	For a correct equation with $\sqrt{\ }$ as subject	
			$\Rightarrow 24x^2=25\Rightarrow x=\dfrac{5}{\sqrt{24}}\left(=\dfrac{5}{12}\sqrt{6}\right)$	A1	For correct x and no others **isw**	
				[5]		
8	(i)		$2\cos^2\alpha=2\sin 2\alpha=4\sin\alpha\cos\alpha$	M1	For equation in $\cos\alpha$ and $\sin\alpha$ (only - ie dealing with $\sin 2\alpha$	
			$\Rightarrow \tan\alpha=\frac{1}{2}$	A1	leading to **AG**(θ may be used instead of α) **SR** Allow verification only if exact	
				[2]		
8	(ii)		Area $=\frac{1}{2}\int_0^{\alpha} r_2^2\,d\theta+\frac{1}{2}\int_{\alpha}^{\frac{1}{2}\pi} r_1^2\,d\theta$	M1	For both integrals added with limits soi Allow θ for α , and reversal of r^2 terms	
			$=\frac{1}{2}\int_0^{\alpha} 2\sin 2\theta\,d\theta+\frac{1}{2}\int_{\alpha}^{\frac{1}{2}\pi} 1+\cos 2\theta\,d\theta$	M1	For using $2\cos^2\theta=1+\cos 2\theta$ in 2nd integral	
			$=\left[-\frac{1}{2}\cos 2\theta\right]_0^{\alpha}+\left[\frac{1}{2}\theta+\frac{1}{4}\sin 2\theta\right]_{\alpha}^{\frac{1}{2}\pi}$	M1	For $k\cos 2\theta$ as first integrated term	
			$=\left(-\frac{1}{2}\cos 2\alpha+\frac{1}{2}\right)+\left(\frac{1}{4}\pi-\frac{1}{2}\alpha-\frac{1}{4}\sin 2\alpha\right)$	A1	For correct first area	
			$=\left(-\frac{1}{2}\left(1-2\sin^2\alpha\right)+\frac{1}{2}\right)+\left(\frac{1}{4}\pi-\frac{1}{2}\alpha-\frac{1}{2}\sin\alpha\cos\alpha\right)$	A1	For correct second area	
			$=\frac{1}{5}+\frac{1}{4}\pi-\frac{1}{2}\alpha-\frac{1}{2}.\frac{1}{\sqrt{5}}.\frac{2}{\sqrt{5}}$	M1	For using Pythagoras to find $\sin\alpha$ or $\cos\alpha$ ***OR*** t formula for $\cos 2\alpha$ or $\sin 2\alpha$	
			$=\frac{1}{4}\pi-\frac{1}{2}\alpha$	A1	For simplification to **AG**	
				[7]		

Question			Answer	Marks	Guidance	
9	(i)		$\tanh(\ln n) = \dfrac{e^{\ln n} - e^{-\ln n}}{e^{\ln n} + e^{-\ln n}}$	M1	For definition of $\tanh(\ln n)$ seen Or working with $\tanh(\ln n) = x$, definition of $\tanh^{-1} x$ seen	
			$= \dfrac{n - \frac{1}{n}}{n + \frac{1}{n}} = \dfrac{n^2 - 1}{n^2 + 1}$	A1	For simplification to **AG** **SC1** $\tanh(\ln n) = \dfrac{e^{\ln n} - e^{-\ln n}}{e^{\ln n} + e^{-\ln n}} = \dfrac{e^{2\ln n} - 1}{e^{2\ln n} + 1} = \dfrac{n^2 - 1}{n^2 + 1}$	
				[2]		
9	(ii)		$I_n - I_{n-2} = \int_0^{\ln 2} \left(\tanh^n u - \tanh^{n-2} u\right) du$ $= \int_0^{\ln 2} \tanh^{n-2} u \left(\tanh^2 u - 1\right) du = -\int_0^{\ln 2} \tanh^{n-2} u \operatorname{sech}^2 u \, du$	M1	For factorising and replacing $(\tanh^2 u - 1)$ by $\pm\operatorname{sech}^2 u$ (or similarly considering I_n)	
			$\Rightarrow I_n - I_{n-2} = -\left[\frac{1}{n-1} \tanh^{n-1} u\right]_0^{\ln 2}$	A1	For correct integrated term	
			$\Rightarrow I_n - I_{n-2} = -\frac{1}{n-1}\left(\frac{3}{5}\right)^{n-1}$	A1	For simplification to **AG**	
				[3]		
9	(iii)		$I_1 = \int_0^{\ln 2} \tanh u \, du = \left[\ln \cosh u\right]_0^{\ln 2}$	M1	For integration to $k \ln \dfrac{\cosh u}{\sinh u}$	
			$= \ln\left(\cosh(\ln 2)\right) = \ln \dfrac{e^{\ln 2} + e^{-\ln 2}}{2} = \ln \frac{5}{4}$	M1	For simplifying $\dfrac{\cosh}{\sinh}(\ln 2)$	
				A1	For correct value of I_1	
			$I_3 = I_1 - \frac{1}{2}\left(\frac{3}{5}\right)^2 = -\frac{9}{50} + \ln \frac{5}{4}$	B1ft	For correct I_3. FT from I_1 **SC** $I_3 = -\frac{9}{50} + \ln(\cosh(\ln 2))$ **M1 B1ft**	
				[4]		
9	(iv)		$\left(I_n - I_{n-2}\right) + \left(I_{n-2} - I_{n-4}\right) + \ldots + \left(I_3 - I_1\right)$ $= I_n - I_1 = -\left(\dfrac{1}{n-1}\left(\dfrac{3}{5}\right)^{n-1} + \dfrac{1}{n-3}\left(\dfrac{3}{5}\right)^{n-3} + \ldots + \dfrac{1}{2}\left(\dfrac{3}{5}\right)^2\right)$	M1	For attempting to sum equations of the form of **(ii)** and cancelling soi	
			$\Rightarrow \frac{1}{2}\left(\frac{3}{5}\right)^2 + \frac{1}{4}\left(\frac{3}{5}\right)^4 + \frac{1}{6}\left(\frac{3}{5}\right)^6 + \ldots = I_1 = \ln \frac{5}{4}$	A1ft	For correct answer ft from I_1	
				[2]		

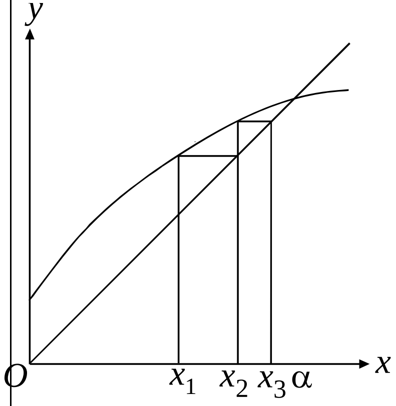
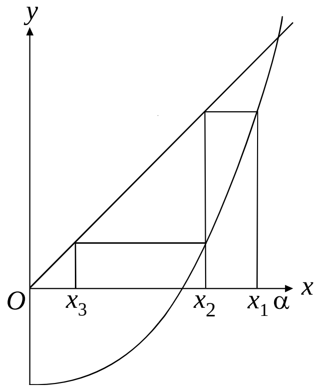
Alternative to Q9(ii)

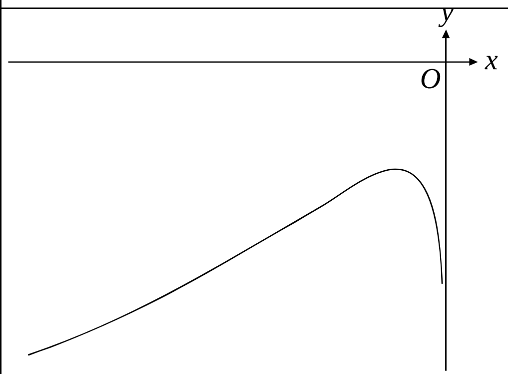
Question			Answer	Marks	Guidance	
9	(ii)		$I_n = \int_0^{\ln 2} \tanh^n u \, du = \int_0^{\ln 2} \tanh^{n-2} u . \tanh^2 u \, du$	M1	For attempt to integrate by parts.	
			$= \int_0^{\ln 2} \tanh^{n-2} u . \left(1 - \text{sech}^2 u\right) \, du$			
			$= \int_0^{\ln 2} \tanh^{n-2} u . \, du - \int_0^{\ln 2} \tanh^{n-2} u \, \text{sech}^2 u \, du$			
			$\Rightarrow I_n = I_{n-2} - \left[\dfrac{\tanh^{n-1} u}{n-1}\right]_0^{\ln 2}$	A1	For correct integrated term	
			$\Rightarrow I_n - I_{n-2} = -\dfrac{\tanh^{n-1}(\ln 2)}{n-1}$			
			$= -\dfrac{1}{n-1}\left(\dfrac{2^2-1}{2^2+1}\right)^{n-1} = -\dfrac{1}{n-1}\left(\dfrac{3}{5}\right)^{n-1}$	A1	For simplification to **AG**	
				[3]		

Question			Answer	Marks	Guidance	
1			$\operatorname{sech} 2x = \dfrac{2}{e^{2x} + e^{-2x}}$	B1	For sech2x expression **oe**	
			$u = e^{2x} \Rightarrow du = 2e^{2x}dx$ **or** $x = \frac{1}{2}\ln u \Rightarrow dx = \frac{1}{2u}du$	M1	For differentiating substitution correctly and substituting into *their* integral	
			$\Rightarrow I = \int \operatorname{sech} 2x\,dx = \int \dfrac{2}{e^{2x} + e^{-2x}}dx$			
			$= \int \dfrac{2}{\left(e^{2x} + e^{-2x}\right)} \cdot \dfrac{du}{2e^{2x}}$	A1	For correct integral	
			$= \int \dfrac{1}{u^2 + 1}du$			
			$= \tan^{-1} u\ (+c) = \tan^{-1}\left(e^{2x}\right) + c$	M1 A1 **[5]**	For integration to $\tan^{-1}(\ )$ For correct expression (c required)	

Question			Answer	Marks	Guidance	
2	(i)		$r=0 \Rightarrow \cos\theta=0, \sin 2\theta=0$ $\Rightarrow \quad \theta=0, \frac{1}{2}\pi$	M1 A1 **[2]**	For $r = 0$ **(soi)** and attempt to solve for θ For both values and no others (ignore values outside range)	
2	(ii)		$\frac{\mathrm{d}r}{\mathrm{d}\theta}=-\sin\theta\sin 2\theta+2\cos 2\theta\cos\theta$ $=0$ ***Alternatively:*** $r=2\cos^2\theta\sin\theta \Rightarrow \frac{\mathrm{d}r}{\mathrm{d}\theta}=2\cos^3\theta-4\cos\theta\sin^2\theta$ $\Rightarrow 2\sin^2\theta\cos\theta=2(1-2\sin^2\theta)\cos\theta$ $\Rightarrow \quad \sin\theta=\frac{1}{\sqrt{3}}\left(\cos\theta=\frac{\sqrt{2}}{\sqrt{3}},\ \tan\theta=\frac{1}{\sqrt{2}}\right)$ $\Rightarrow r=\frac{4}{3\sqrt{3}}=\frac{4}{9}\sqrt{3}$	M1 A1 A1 A1 **[4]**	For attempt to find $\frac{\mathrm{d}r}{\mathrm{d}\theta}$ using product rule For correct $\frac{\mathrm{d}r}{\mathrm{d}\theta}$ set = 0 **soi** For correct value of $\sin\theta$ (**OR** $\cos\theta$ *OR* $\tan\theta$) or decimal equivalent; $\sin\theta = 0.546$ or $\cos\theta = 0.816$ or $\tan\theta = 0.707$ For correct r or anything that rounds to 0.77	
2	(iii)		$x=r\cos\theta,\ y=r\sin\theta$ $\Rightarrow r=\frac{x}{r}.2\frac{y}{r}\frac{x}{r}$ $\Rightarrow \left(x^2+y^2\right)^2=2x^2y$	M1 M1 A1 **[3]**	For substituting $x=r\cos\theta$ **OR** $y=r\sin\theta$ For $r^2=x^2+y^2$ soi For a correct cartesian equation **Any equivalent form** without fractions	

Question			Answer	Marks	Guidance	
3	(i)		$\tanh 2x \equiv \dfrac{\sinh 2x}{\cosh 2x} \equiv \dfrac{2\sinh x\cosh x}{\cosh^2 x+\sinh^2 x}$	M1	For $\dfrac{\sinh 2x}{\cosh 2x}$ and use double angle formulae	
			$\equiv \dfrac{2\tanh x}{1+\tanh^2 x}$	A1	For division by $\cosh^2 x$ seen	N.B. Tanh(A + B) not in formula book
				[2]		
3	(ii)		$\dfrac{10t}{(t^2+1)} = (1+6t)$	M1	For using (i) to obtain equation in t.	
			$\Rightarrow 6t^3+t^2-4t+1=0$	A1	Correct cubic equation	
			$\Rightarrow (t+1)(3t-1)(2t-1)=0$	M1	Attempt to solve cubic (calculator OK)	
			$\Rightarrow t=(-1), \frac{1}{3}, \frac{1}{2}$	A1	Solution. Ignore any extra values at this stage	
			$x=\frac{1}{2}\ln\dfrac{1+t}{1-t} \Rightarrow x=\frac{1}{2}\ln 2, \frac{1}{2}\ln 3$	M1 A1	For using ln form for $\tanh^{-1}$ Correct 2 values (only) **oe**	
				[6]		
			Alternative: M1		Use exponentials to obtain a quadratic in e^{2x}	
			$e^{4x}-5e^{2x}+6=0$ A1		Correct	
			$\Rightarrow (e^{2x}-2)(e^{2x}-3)=0$ M1		Solve quadratic	
			$\Rightarrow e^{2x}=2,\ 3$ A1		Soln	
			$\Rightarrow 2x=\ln 2,\ \ln 3$ M1		Take logs	
			$\Rightarrow x=\dfrac{1}{2}\ln 2,\ \dfrac{1}{2}\ln 3$ A1			

Question			Answer	Marks	Guidance	
4	(i)		 $x_2 = 1.3869...$ $x_3 = 1.3938$	B1 B1 B1 **[3]**	For correct value (4 d.p. or better) For correct value. For sketch showing staircase towards α. (Vertical lines do not need to be labelled)	
4	(ii)			B1 B1 **[2]**	For sketch like $y = \frac{1}{2}\left(x^4 - 1\right)$ and $y = x$ (curve or continuation of curve cuts - y axis.) For sketch showing staircase away from α.("Away" means labelling or arrows required.) Labelling means x_1, x_2,... in right place or numeric values.	
4	(iii)		$x_{n+1} = x_n - \frac{x_n^4 - 2x_n - 1}{4x_n^3 - 2}$ $1.35 \rightarrow 1.398268$ $\rightarrow 1.395348 \rightarrow 1.395337$ $\Rightarrow 1.3953$	M1 A1 A1 A1 **[4]**	For deriving the iterative formula For correct formula For 1st value For correct 4dp α with 2 iterates equal to 4 dp. (i.e. last two iterates agree to 4dp) **www**	

Question			Answer	Marks	Guidance	
5	(i)		$f'(x)=\frac{1}{\sqrt{1+x^2}}+\frac{1}{\sqrt{1+\frac{1}{x^2}}}\cdot\frac{-1}{x^2}$	M1 B1	For attempt to differentiate using chain rule. First term correct	
			$=\frac{1}{\sqrt{1+x^2}}\left(1-\frac{1}{x}\right)$			
			$=0\Rightarrow x=1$	M1 A1	For attempt to solve their f '(x) = 0 For correct value of x (ignore x = –1)**www**	
			$f(1)=2\sinh^{-1}1=2\ln\left(1+\sqrt{2}\right)$	A1	For correct value obtained **www AG**	
				[5]		
5	(ii)			B1	For correct shape in 3rd quadrant only(condone inclusion of the 1st quadrant part given)	
			$\left\{f(x)\geqslant 2\ln\left(1+\sqrt{2}\right),\ f(x)\leqslant -2\ln\left(1+\sqrt{2}\right)\right\}$	B1 B1	For one part of range For other part of range **SC** B1 Both ranges correct but < and > used	
				[3]		

Question			Answer	Marks	Guidance	
6	(i)		$I_n = \left[-x^n \cos x\right]_0^{\pi} + n\int_0^{\pi} x^{n-1}\cos x \, dx$	M1 A1	For attempt to integrate by parts For correct result before limits	
			$= \pi^n + n\left\{\left[x^{n-1}\sin x\right]_0^{\pi} - (n-1)\int_0^{\pi} x^{n-2}\sin x \, dx\right\}$	M1 A1	For attempt at second integration by parts For correct result before limits	
			$\Rightarrow I_n = \pi^n - n(n-1)I_{n-2}$	A1 **[5]**	For correct result **www AG**	
6	(ii)		$I_1 = \left[-x\cos x\right]_0^{\pi} + \int_0^{\pi} \cos x \, dx$	M1	For integrating by parts for I_1	
			$\Rightarrow I_1 = \pi + \left[\sin x\right]_0^{\pi} = \pi$	A1	For correct I_1 SC B1 $I_1 = \pi$ with no working	
			$I_3 = \pi^3 - 6I_1$, $I_5 = \pi^5 - 20I_3$	M1	For substituting n = 3 or 5 in reduction formula	
			$\Rightarrow I_5 = \pi^5 - 20\pi^3 + 120\pi$	A1 **[4]**	For correct result	

Question			Answer	Marks	Guidance	
7	(i)		$a = 2,\ b = n$ $c = 1,\ \ d = n - 1$	B1 B1 B1 **[3]**	for any 2 correct for the third correct for all four correct. Allow values inserted in series. **SC** treat $a = \frac{1}{2}$ etc as MR –1 once	
7	(ii)		$\int_1^n \frac{1}{x}\,dx = \ln n$ $1 + \frac{1}{2} + \ldots + \frac{1}{n} < 1 + \ln n$ $\Rightarrow\ f(n) < 1$ (upper bound) $\Rightarrow\ f(n) > \frac{1}{n}$ (lower bound)	B1 M1 A1 A1 **[4]**	For integral evaluated **soi** (Definite integral between 1 and *n*) For adding 1 *OR* $\frac{1}{n}$ to series For correct upper bound For correct lower bound	
7	(iii)		$f(n+1) - f(n) = \frac{1}{n+1} - \ln(n+1) + \ln n$ $= \frac{1}{n+1} - \ln\left(\frac{n+1}{n}\right) \approx \frac{1}{n+1} - \left(\frac{1}{n} - \frac{1}{2n^2}\right)$ $\approx \frac{1}{n+1} - \frac{2n-1}{2n^2}$ $\approx -\frac{n-1}{2n^2(n+1)}$	B1 M1 M1 A1 A1 **[5]**	For correct expression For combining ln terms For attempt to expand $\ln\left(1 + \frac{1}{n}\right)$ Correct expansion of $\ln\left(1 + \frac{1}{n}\right)$ For correct expression **AG**	Any expansion of ln(1+*n*) oe is 0

Alternative answer to 7(iii)

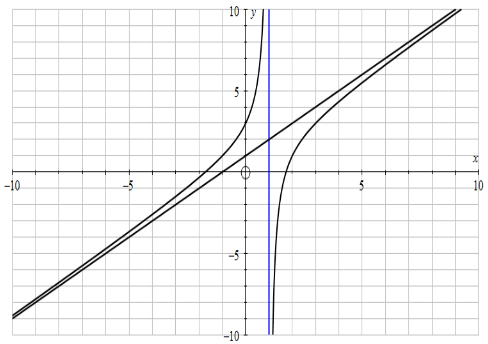
Question			Answer	Marks	Guidance	
7	(iii)		$\mathrm{f}\,(n+1)-\mathrm{f}\,(n)=\dfrac{1}{n+1}-\ln(n+1)+\ln n$	B1	For correct expression	
			$=\dfrac{1}{n+1}-\ln\left(\dfrac{n+1}{n}\right)$			
			$=\dfrac{1}{n+1}+\ln\left(\dfrac{n}{n+1}\right)$	M1	For combining ln terms and attempt to expand	
			$=\dfrac{1}{n+1}+\ln\left(1-\dfrac{1}{(n+1)}\right)$	M1	For attempt to expand $\ln\left(1-\dfrac{1}{(n+1)}\right)$	
			$=\dfrac{1}{n+1}+\left(-\dfrac{1}{(n+1)}-\dfrac{1}{2(n+1)^2}\right)$	A1	Correct expansion of $\ln\left(1-\dfrac{1}{(n+1)}\right)$	
			$=-\dfrac{1}{2(n+1)^2}$		**Max 4**	

Question			Answer	Marks	Guidance	
8	(i)		$\mathrm{q}(x) = x+2$	B1	For correct $\mathrm{q}(x)$ **soi oe**	
			$y = \dfrac{A}{x+2} + \frac{1}{2}x + 1$	M1	For expressing *y* in this form. Allow $cx+d$ for *A*	
			$\left(-1, \frac{17}{2}\right) \Rightarrow A = 8$	A1	For correct *A*	
			$y = \dfrac{\frac{1}{2}x^2 + 2x + 10}{x+2} \Rightarrow \mathrm{p}(x) = \frac{1}{2}x^2 + 2x + 10$	A1 **[4]**	For correct $\mathrm{p}(x)$ Allow equal multiples of $\mathrm{p}(x)$ and $\mathrm{q}(x)$	
			Alternative: $\mathrm{q}(x) = x+2$ B1		For correct $\mathrm{q}(x)$ **soi oe**	
			$y = \dfrac{ax^2+bx+c}{\mathrm{q}(x)} = ax + (b-2a) + \dfrac{c-2b+4a}{x+2}$ M1		For division by *their* $\mathrm{q}(x)$	1st line of division and 1st term in quotient should be seen for correct method
			$y = \frac{1}{2}x + 1 \Rightarrow a = \frac{1}{2}, b = 2$ A1		For correct *a* and *b* **oe**	
			$\left(-1, \frac{17}{2}\right) \Rightarrow c - 2b + 4a = 8 \Rightarrow c = 10$ A1		For correct *c* **oe**	
8	(ii)		$\frac{1}{2}x^2 + (2-y)x + 10 - 2y = 0$	M1	For attempt to rearrange as quadratic in *x*	
			$b^2 - 4ac \geqslant 0 \Rightarrow (2-y)^2 \geqslant 2(10-2y)$	M1	For use of $b^2 - 4ac$ ($\le$ or $\ge$ or $=$ or $<$ or $>$)	
			$\Rightarrow y^2 \ge 16 \Rightarrow \{y \le -4,\ y \ge 4\}$	A1	For critical values ± 4	
			(pto for alternative)	A1 **[4]**	For correct range. (Must be $\le$ and $\ge$) **www**	
8	(iii)		$\left(\frac{1}{2}x+1\right)^2 = \dfrac{\frac{1}{2}x^2 + 2x + 10}{x+2}$ **OR** $y^2 = \dfrac{4}{y} + y$	B1ft	For a correct equation derived from intersection of C_2 with $y = \frac{1}{2}x + 1$ FT from (i)	
			$\Rightarrow x^3 + 4x^2 + 4x - 32 = 0$ **OR** $y^3 - y^2 - 4 = 0$	M1 A1	For obtaining a cubic Correct cubic	
			$\Rightarrow (2, 2)$	A1 **[4]**	Coordinates correct **www**	

Alternative to 8(ii)

Question			Answer	Marks	Guidance	
8	(ii)		$y=\dfrac{\frac{1}{2}x^2+2x+10}{x+2}$ $\Rightarrow \dfrac{dy}{dx}=\dfrac{(x+2)(x+2)-\left(\frac{1}{2}x^2+2x+10\right)}{(x+2)^2}$	M1	Diffn using quotient rule	
			$=0$ when $(x+2)(x+2)=\left(\frac{1}{2}x^2+2x+10\right)$ $\Rightarrow \frac{1}{2}x^2+2x-6=0 \Rightarrow x^2+4x-12=0$ $\Rightarrow (x+6)(x-2)=0$	M1	Attempt to find soln using $\dfrac{dy}{dx}=0$	
			$\Rightarrow x=2,\ y=4$ $\quad x=-6,\ y=-4$	A1		
			$\{y\le -4,\ \ y\ge 4\}$	A1	For correct range. (Must be $\le$ and $\ge$) **www**	
			Alternatively: $y=\frac{1}{2}x+1+\dfrac{8}{x+2}$			
			$\Rightarrow \dfrac{dy}{dx}=\dfrac{1}{2}-\dfrac{8}{(x+2)^2}$ M1		Diffn using chain rule	
			$=0$ when $\dfrac{1}{2}-\dfrac{8}{(x+2)^2}\Rightarrow (x+2)^2=16$ M1		Attempt to find soln using $\dfrac{dy}{dx}=0$	
			$\Rightarrow x+2=\pm 4\Rightarrow x=2$ or -6 $\Rightarrow y=4$ or -4 A1			
			$\{y\le -4,\ \ y\ge 4\}$ A1		For correct range. (Must be $\le$ and $\ge$) **www**	

Question			Answer	Marks	Guidance	
1			$\frac{A}{x-1}+\frac{Bx+C}{x^2+4}$ $\Rightarrow 5x \equiv A(x^2+4)+(Bx+C)(x-1) \quad [+D(x-1)(x^2+4)]$	B1	Sight of expression	Allow addition of constant
			Equate coefficients or substitute values for x	M1	For Equating 3 coeffs or sub 3 times	
			$\Rightarrow A=1$ $B=-1$	A1	For one value (not D)	
			$C=4$	A1	For 2nd and 3rd values (not D)	
			$\Rightarrow \frac{5x}{(x-1)(x^2+4)}=\frac{1}{(x-1)}+\frac{4-x}{(x^2+4)}$	A1	For final answer expressed properly	
				[5]		

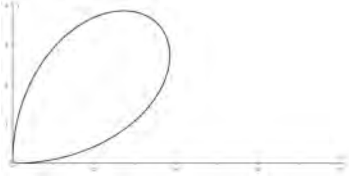
Question			Answer	Marks	Guidance	
2	(i)		$x = 1$	B1		
			$y=\frac{x^2-3}{x-1}=\frac{(x-1)(x+1)-2}{x-1}=x+1\left[-\frac{2}{x-1}\right]$	M1	Or long division with quotient x +...	
			$\Rightarrow y=x+1$	A1	Must be stated	
				[3]		
2	(ii)		(0,3) $(\sqrt{3},0)$ and $(-\sqrt{3},0)$	B1 **[1]**	All three	Allow when x = 0, y = 3, etc but do NOT allow y = 3, etc
2	(iii)		$\frac{dy}{dx}=\frac{2x(x-1)-(x^2-3)}{(x-1)^2}=\frac{x^2-2x+3}{(x-1)^2}$	M1 A1	Differentiate Gradient function	Alternative method: Diffn final expression from (i)
			$=\frac{(x-1)^2+2}{(x-1)^2}>0$ for all x.	A1	Conclusion	$\frac{dy}{dx}=1+\frac{2}{(x-1)^2}$
			So no turning points.	**[3]**		>1 so no turning points. Or "b^2 – 4ac"=-8 < 0 so no roots.
2	(iv)			B1	Correct shape going through axes at correct points which must be stated.	Allow omission of (0, 3) if not in (ii). Oblique asymptote can be $y= x + c$ with $c \neq 1$
				B1	Correct asymptotes included	
				B1	Approaches correct asymptotes correctly	
				[3]		

Question			Answer	Marks	Guidance	
3			$3\frac{e^x+e^{-x}}{2}-4\frac{e^x-e^{-x}}{2}=7$	M1	Use of formulae	
			$\Rightarrow 3\left(e^x+e^{-x}\right)-4\left(e^x-e^{-x}\right)=14$	A1	Correct equation	
			$\Rightarrow -e^x+7e^{-x}=14$ $\Rightarrow e^{2x}+14e^x-7=0$	A1	Correct quadratic equation in e^x	
			$\Rightarrow e^x=\frac{-14\pm\sqrt{196+28}}{2}$	M1	Solve quadratic	
			$\left[e^x>0\right]$ so $e^x=\frac{-14+\sqrt{196+28}}{2}$ $=-7+\sqrt{56}$	A1	Correct value for e^x (ignore -ve value)	
			$\Rightarrow x=\ln\left(2\sqrt{14}-7\right)$	A1	One value only with statement of rejection of invalid value for e^x	
				[6]		
			Alternative Make sinh or cosh the subject, square, use $c^2 - s^2 = 1$	M1 A1		
			Gives $7s^2 + 56s + 40 = 0$ Or $7c^2 + 42c - 65 = 0$	A1		

Question			Answer	Marks	Guidance	
4	(i)		$I_n = \int_0^1 x^n.e^{2x}\,dx.$			
			Set $u = x^n \quad du = nx^{n-1}dx$	M1	Integration by parts	
			$dv = e^{2x}\,dx \quad v = \frac{1}{2}e^{2x}$	A1	Correct way round and correct diffn	
			$\Rightarrow I_n = \int_0^1 x^n e^{2x}\,dx = \left[\frac{1}{2}x^n e^{2x}\right]_0^1 - \frac{1}{2}n\int_0^1 x^{n-1}e^{2x}\,dx$	A1	Indefinite form acceptable	
			$I_n = \frac{1}{2}e^2 - \frac{1}{2}nI_{n-1}$	A1	Using limits	
				[4]		
4	(ii)		$I_0 = \int_0^1 e^{2x}\,dx = \frac{1}{2}\left[e^{2x}\right]_0^1 = \frac{1}{2}\left(e^2 - 1\right)$	M1 A1	Attempt to find I_0 or I_1.	
			$I_1 = \frac{1}{2}e^2 - \frac{1}{2}I_0 = \frac{1}{2}e^2 - \frac{1}{2}\left(\frac{1}{2}\left(e^2 - 1\right)\right) = \frac{1}{4}e^2 + \frac{1}{4}$	M1	Using this to progress, dep	
			$I_2 = \frac{1}{2}e^2 - I_1 = \frac{1}{2}e^2 - \left(\frac{1}{4}e^2 + \frac{1}{4}\right) = \frac{1}{4}e^2 - \frac{1}{4}$			
			$I_3 = \frac{1}{2}e^2 - \frac{3}{2}I_2 = \frac{1}{2}e^2 - \frac{3}{2}\left(\frac{1}{4}e^2 - \frac{1}{4}\right) = \frac{1}{8}e^2 + \frac{3}{8}$	A1		
				[4]		

Question			Answer	Marks	Guidance	
5	(i)		$f'(x) = -\sin x.e^{-x} + \cos x.e^{-x}$ $\Rightarrow f'(0) = 1$ $f(0) = 0$	M1 A1 A1 **[3]**	Diffn using product correctly. For **both** values www	
5	(ii)		$f'(x) = \cos x.e^{-x} - \sin x.e^{-x} = \cos x.e^{x} - f(x)$ $f''(x) = -f'(x) - \cos x.e^{-x} - f(x)$ $= -f'(x) - f'(x) - f(x) - f(x)$ $f''(x) = -2f'(x) - 2f(x)$ OR $-2\cos x.e^{-x}$ Showing the two equal $f''(0) = -2$	M1 A1 A1 A1 **[4]**	Diffn	
5	(iii)		$f''(x) = -2f'(x) - 2f(x)$ $\Rightarrow f'''(x) = -2f''(x) - 2f'(x)$ **oe** $\Rightarrow f'''(0) = 4 - 2 = 2$	B1 B1 **[2]**	Not involving trig or exp fns	=-f''+2f Or 2f' + 4f
5	(iv)		$f(x) = x - x^2 + \frac{x^3}{3}$	M1 A1 **[2]**		
			Alternative: Write down correct series expansion for e^{-x} and sin$x$ and multiply	M1 A1		

Question			Answer	Marks	Guidance	
6			$x^2+4x+8=(x+2)^2+4$	M1 A1	Complete the square in order to use standard form	
			$\int_0^1 \frac{1}{\sqrt{x^2+4x+8}}\,dx=\int_0^1 \frac{1}{\sqrt{(x+2)^2+4}}\,dx$	M1	Use correct standard form in integration	
			$=\left[\sinh^{-1}\frac{x+2}{2}\right]_0^1=\sinh^{-1}\left(\frac{3}{2}\right)-\sinh^{-1}1$	A1	Answer in $\sinh^{-1}$ form	
			$=\ln\left(\frac{3}{2}+\sqrt{1+\frac{9}{4}}\right)-\ln\left(1+\sqrt{2}\right)=\ln\left(\frac{3}{2}+\sqrt{\frac{13}{4}}\right)-\ln\left(1+\sqrt{2}\right)$	M1	Attempt to turn into log form	
			$=\ln\left(\frac{3+\sqrt{13}}{2+2\sqrt{2}}\right)$	A1	www isw	
				[6]		
			Alternative for last 4 marks			
			$\int_0^1 \frac{1}{\sqrt{(x+2)^2+4}}\,dx=\left[\ln\left((x+2)+\sqrt{(x+2)^2+4}\right)\right]_0^1$	M1 A1 M1	Attempt to use Standard form Limits	
			$=\ln\left(3+\sqrt{13}\right)-\ln\left(2+\sqrt{8}\right)=\ln\left(\frac{3+\sqrt{13}}{2+2\sqrt{2}}\right)$	A1	www isw	
			Alternative for last 4 marks			
			$x+2=2\tan\theta \Rightarrow I=\left[\ln\left(\sec\theta+\tan\theta\right)\right]_{\pi/4}^{\tan^{-1}3/2}$	M1 A1	Substitution Indefinite integral	
			$=\ln\left(\frac{3}{2}+\frac{\sqrt{13}}{2}\right)-\ln\left(1+\sqrt{2}\right)=\ln\left(\frac{3+\sqrt{13}}{2+2\sqrt{2}}\right)$	M1 A1	Deal with limits www isw	

Question			Answer	Marks	Guidance	
7	(i)			B1 B1 B1	Enclosed loop with axes tangential $\theta=\frac{\pi}{4}$ is a line of symmetry drawn and named	Ignore anything in other quadrants
			P is at $r=5,\ \theta=\frac{\pi}{4}$	B1 **[4]**	For both	
7	(ii)		Area $=\frac{1}{2}\int_0^{\pi/2} r^2\ \mathrm{d}\theta=\frac{1}{2}\int_0^{\pi/2} 25\sin^2 2\theta\ \mathrm{d}\theta$	M1	Correct formula with *r* substituted.	
			$=\frac{25}{4}\int_0^{\pi/2}(1-\cos 4\theta)\ \mathrm{d}\theta=\frac{25}{4}\left[\theta-\frac{1}{4}\sin 4\theta\right]_0^{\pi/2}$	M1	Correct method of integration including limits	
			$=\frac{25}{4}\left(\left(\frac{\pi}{2}-0\right)-(0)\right)=\frac{25\pi}{8}$	A1 **[3]**	www	
7	(iii)		Equation is of the form $x+y=c$	B1		
			P is $\left(\frac{5}{\sqrt{2}},\frac{5}{\sqrt{2}}\right)$ **oe**	B1		
			$\Rightarrow x+y=5\sqrt{2}$	B1 **[3]**	Ft. $x+y=c$ where *c* comes from their P.	
7	(iv)		$r=5\sin 2\theta=10\sin\theta\cos\theta$	M1	Square and convert r^2	
			$\Rightarrow r^2=100\sin^2\theta\cos^2\theta=100\left(\frac{y}{r}\right)^2\left(\frac{x}{r}\right)^2$	M1	Substitute for *r* and θ	
			$\Rightarrow\left(x^2+y^2\right)^3=100x^2y^2$	A1 **[3]**	***NB Answer given***	

Question			Answer	Marks	Guidance	
8	(i)	(a)	$x_1 = 4.15,\quad x_2 = 4.1474....$ $x_3 = 4.1465....,\quad x_4 = 4.1463....$ $\beta = 4.146$	M1 A1 **[2]**	Using iterative formula at least once using at least 4dp www	All iterates must be seen
8	(i)	(b)	Staircase diagram will always move to upper root	B1 B1 B1 **[3]**	Sketch showing an example $x_1 > \alpha$ Example with $x_1 < \alpha$ Statement Dep on 1st two B	Ignore any statement when $x_1 > \beta$
8	(ii)	(a)	$\ln(x-1) = x-3 \Rightarrow \ln(x-1)-(x-3)=0$ $\Rightarrow f(x) = \ln(x-1)-(x-3)$ $\Rightarrow f'(x) = \dfrac{1}{x-1}-1$ $\Rightarrow x_{n+1} = x_n - \dfrac{\ln(x_n-1)-(x_n-3)}{\dfrac{1}{x_n-1}-1}$ $= x_n - \dfrac{(x_n-1)(\ln(x_n-1)-(x_n-3))}{1-(x_n-1)}$ $= \dfrac{x_n(2-x_n)+(x_n-1)(x_n-3)-(x_n-1)\ln(x_n-1)}{2-x_n}$ $= \dfrac{2x_n - x_n^2 + x_n^2 - 4x_n + 3 - (x_n-1)\ln(x_n-1)}{2-x_n}$ $\Rightarrow x_{n+1} = \dfrac{3-2x_n-(x_n-1)(\ln(x_n-1)}{2-x_n}$	M1 M1 M1 A1 A1 **[5]**	Get equation in correct form Differentiate Use correct formula Mult by $(x-1)$ **soi**	

Question			Answer	Marks	Guidance	
8	(ii)	(b)	1.2 1.152(359) 1.152359 1.158448 1.158448 1.158594 1.158594 1.158594 Root = 1.159	B1 B1 B1 [3]	For x_2 For enough iterates to determine 3dp www	Allow 3 dp x_2 must be right for last B1. Any error is likely to be self-correcting

Annotations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

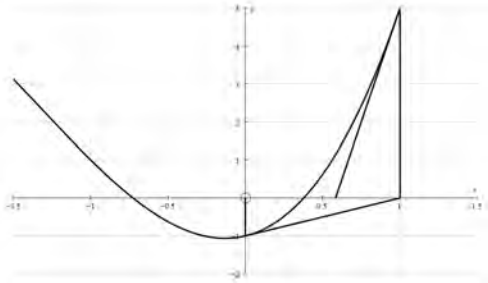
h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

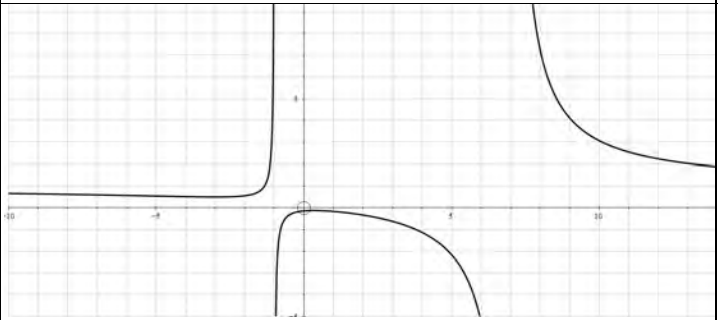
Question			Answer	Marks	Guidance	
1			$\cos\theta = \frac{1-t^2}{1+t^2}$	M1	Using t substitution for **both** $\cos\theta$ and $\mathrm{d}\theta$	
			$\frac{\mathrm{d}t}{\mathrm{d}\theta} = \frac{1}{2}\sec^2\frac{1}{2}\theta = \frac{1}{2}\left(1+\tan^2\frac{1}{2}\theta\right)$	A1	Subs correct	
			$\Rightarrow \quad \mathrm{d}t = \frac{1+t^2}{2}.\,\mathrm{d}\theta \Rightarrow \mathrm{d}\theta = \frac{2\mathrm{d}t}{1+t^2}$	M1	Dealing with limits and attempting integration.	
			$\Rightarrow I = \int_0^1 \frac{1}{1+\frac{1-t^2}{1+t^2}}\,\frac{2\mathrm{d}t}{1+t^2} = \int_0^1 \frac{1+t^2}{1+t^2+1-t^2}\,\frac{2\mathrm{d}t}{1+t^2}$	A1	Correct integral	
			$\int_0^1 \frac{2\mathrm{d}t}{2} = [t]_0^1 = 1$	A1 **[5]**	Answer	
			Alternative $1+\cos\theta = 2\cos^2\tfrac{1}{2}\theta$ $\Rightarrow \int_0^{\frac{\pi}{2}} \frac{1}{1+\cos\theta}\mathrm{d}\theta = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{1}{\cos^2\tfrac{1}{2}\theta}\mathrm{d}\theta = \frac{1}{2}\int_0^{\frac{\pi}{2}} \sec^2\tfrac{1}{2}\theta\,\mathrm{d}\theta$ $= \frac{1}{2}\left[2\tan\frac{1}{2}\theta\right]_0^{\frac{\pi}{2}} = \tan\frac{\pi}{2} - \tan 0 = 1$	**SC3**		
2	(i)		$\cosh x = \frac{e^x+e^{-x}}{2},\ \sinh x = \frac{e^x-e^{-x}}{2}$	B1	Correct formulae	
			$\Rightarrow \cosh^2 x - \sinh^2 x = \left(\frac{e^x+e^{-x}}{2}\right)^2 - \left(\frac{e^x-e^{-x}}{2}\right)^2$	M1	Dealing with squaring correctly	Difference of squares can be used
			$= \frac{1}{4}\left(e^{2x}+2+e^{-2x}-e^{2x}+2-e^{-2x}\right) = \frac{1}{4}.4 = 1$	A1 **[3]**	**www** All steps seen	

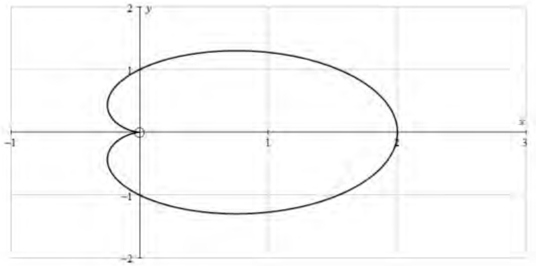
Question			Answer	Marks	Guidance	
2	(ii)		$\Rightarrow \cosh^2 x - 1 = 5\cosh x - 7$			
			$\Rightarrow \cosh^2 x - 5\cosh x + 6 = 0$	M1	Use **(i)**	
			$\Rightarrow (\cosh x - 2)(\cosh x - 3) = 0$	M1	Attempt to solve quadratic	E.g. correct formula or expansion of their brackets gives 2 out of 3 terms correct
			$\Rightarrow \cosh x = 2, 3$	A1		
			$\Rightarrow x = \cosh^{-1} 2 = \pm\ln\left(2 \pm \sqrt{3}\right)$	A1	Use correct ln formula	Condone lack of ±
			and $x = \cosh^{-1} 3 = \pm\ln\left(3 \pm \sqrt{8}\right)$	A1	Use correct ln formula	Condone lack of ±
				[5]		
3	(i)		$\frac{dy}{dx} = \frac{1}{1 - \left(\frac{1-x}{3+x}\right)^2} \times \frac{-(3+x)-(1-x)}{(3+x)^2}$	B1	Sight of standard diffn for $\tanh^{-1} x$	
				M1	Fn of fn **and** diffn of quotient	
				A1	Soi correct quotient (i.e. correct expression for 2nd part)	
			$\Rightarrow \frac{dy}{dx} = \left(\frac{-4}{(3+x)^2 - (1-x)^2}\right) = \frac{k}{1+x}$	A1		
			$\Rightarrow \frac{dy}{dx} = \frac{-1}{2(1+x)}$	A1	Correct for y'	
			$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{2(1+x)^2}$	A1	2^{nd} diffn (NB AG)	
				[6]		

Question			Answer	Marks	Guidance	
3	(ii)		When $x=0, y=\tanh^{-1}\frac{1}{3}$ or $\frac{1}{2}\ln 2$ or $\ln\sqrt{2}$	B1	For 1st value (needs to be exact)	
			$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2}$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{1}{2}$	B1	For both	
			$\Rightarrow y=\tanh^{-1}\frac{1}{3}+\left(-\frac{1}{2}\right)x+\left(\frac{1}{2}\right)\frac{x^2}{2}$	M1	Use of correct Maclaurin's series	
			$=\tanh^{-1}\frac{1}{3}-\frac{1}{2}x+\frac{x^2}{4}$	A1	Accept 0.347	
				[4]		
4	(i)		$u=\cos^{n-1}x,\ \mathrm{d}v=\cos x\,\mathrm{d}x$	M1*	By parts the right way round	
			$\mathrm{d}u=-(n-1)\cos^{n-2}x\sin x,\quad v=\sin x$	A1		
			$\Rightarrow I_n=\left[\cos^{n-1}x\sin x\right]_0^{\frac{\pi}{2}}+(n-1)\int_0^{\frac{\pi}{2}}\cos^{n-2}x\sin^2x\ \mathrm{d}x$	A1	Integral so far	
			$=0+(n-1)\left(I_{n-2}-I_n\right)$	*M1	Correct use of $\sin^2x=1-\cos^2x$ Dependent on 1st M	
			$\Rightarrow nI_n=(n-1)I_{n-2}\Rightarrow I_n=\frac{n-1}{n}I_{n-2}$	A1	**www** AG	
				[5]		
4	(ii)		$I_1=1$	B1	For I_1 soi	
			$I_{11}=\frac{10}{11}I_9=....=\frac{10}{11}.\frac{8}{9}.\frac{6}{7}.\frac{4}{5}.\frac{2}{3}I_1$	M1	Use of **(i)** to give product of 5 fractions	
			$\Rightarrow I_{11}=\frac{3840}{10395}=\frac{256}{693}$ oe	A1	Correct fraction	
				[3]		

Question		Answer	Marks	Guidance	
5	(i)	$f(x) = x^3 + 4x^2 + x - 1$ $f'(x) = 3x^2 + 8x + 1$ $\Rightarrow x_{n+1} = x_n - \dfrac{x_n^3 + 4x_n^2 + x_n - 1}{3x_n^2 + 8x_n + 1}$ $= \dfrac{x_n(3x_n^2 + 8x_n + 1) - (x_n^3 + 4x_n^2 + x_n - 1)}{3x_n^2 + 8x_n + 1}$ $= \dfrac{2x_n^3 + 4x_n^2 + 1}{3x_n^2 + 8x_n + 1}$	B1 M1 A1 **[3]**	Diffn Correct application of N-R formula And completed with suffices on last line NB AG	
5	(ii)	$x_2 = -0.72652$, $x_3 = -0.72611$ $\Rightarrow \alpha = -0.72611$	B1 B1 B1 **[3]**		NB $x_4 = -0.726109$
5	(iii)	Sketch plus at least one tangent  Converges to another root.	B1 B1 **[2]**	At least the first tangent and vertical line to curve Or positive root or, for e.g. "$x = 0$ is the wrong side of a turning point" **www**	Use of formula to find this root numerically is not acceptable

Question			Answer	Marks	Guidance	
6	(i)		Width of rectangles is $\frac{3}{n}$	B1	Statement about width	
			$\Rightarrow$ Sum of areas of rectangles	M1	Height or area of at least one rectangle	
			$=\frac{3}{n}\times\left(\ln(\ln 3)+\ln\left(\ln\left(3+\frac{3}{n}\right)\right)+......\right)$	A1	Correct conclusion **www**	
			$=\frac{3}{n}\times\sum_{r=0}^{n-1}\ln\left(\ln\left(3+\frac{3r}{n}\right)\right)$	**[3]**	1468 or	
6	(ii)		$=\frac{3}{n}\times\sum_{r=1}^{n}\ln\left(\ln\left(3+\frac{3r}{n}\right)\right)$	B1 **[1]**		
6	(iii)		$U-L=\frac{3}{n}\times\ln(\ln 6)-\frac{3}{n}\times\ln(\ln 3)$	M1*	Subtraction to obtain the difference of two terms	
			$=\frac{3}{n}(\ln(\ln 6)-\ln(\ln 3))=\frac{3}{n}\ln\left(\frac{\ln 6}{\ln 3}\right)$	A1		
			$\Rightarrow n>\frac{3}{0.001}\ln\left(\frac{\ln 6}{\ln 3}\right)\Rightarrow n>\frac{3}{0.001}\times\ln(1.6309)$	*M1	Dealing with inequality to obtain n dep on first M	
			$\Rightarrow$ least $n=1468$	A1 **[4]**	Accept $n\geq 1468$ or $n>1467$	
7	(i)		$x=-1$	B1	B1 for each	
			$x=7$	B1		
			$y=1$	B1 **[3]**	−1 for any extras	

Question			Answer	Marks	Guidance	
7	(ii)		$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x^2-6x-7)2x-(x^2+1)(2x-6)}{(x+1)^2(x-7)^2}$	M1 A1	Diffn using quotient rule	Or expand as partial fractions and use fn of fn rule
			$=0$ when $(x^2-6x-7)2x-(x^2+1)(2x-6)=0$ $3x^2+8x-3=0$	A1	Quadratic	
			$\Rightarrow x=-3,\ \frac{1}{3};\quad y=\frac{1}{2},\ -\frac{1}{8}$	A1	Both x values	Or: A1 one pair
			i.e. $\left(-3,\frac{1}{2}\right),\left(\frac{1}{3},-\frac{1}{8}\right)$	A1 **[5]**	Both y values	A1 other pair
7	(iii)		When $y=1,\ x^2-6x-7=x^2+1$	M1 A1		
			$\Rightarrow 6x=-8\Rightarrow x=-\frac{4}{3}\Rightarrow\left(-\frac{4}{3},1\right)$	A1 **[3]**	Coordinate pair needs to be seen.	
7	(iv)			B1 B1 B1 **[3]**	Left section, cutting asymptote and approaching $y = 1$ from below Right hand section Middle section all below x-axis labelling intercept on graph or by a statement	

Question			Answer	Marks	Guidance	
8	(i)		Substitute $r^2 = x^2 + y^2$, $x = r\cos\theta$ $\Rightarrow r^2 - r\cos\theta = r \Rightarrow r = 1 + \cos\theta$	M1 A1 A1 **[3]**	 **cao**	
8	(ii)			B1 B1 **[2]**	Cardioid (General shape) Correct shape at pole, r = 2 and symmetric	 e.g. cusp clearly at pole, vertical tangent at r = 2
8	(iii)		Line cuts curve at (0, 1) and (2, 0) Total area = $2 \times \frac{1}{2} \times \int_0^{\pi} (1 + \cos\theta)^2 \, d\theta$ $= \int_0^{\pi} (1 + 2\cos\theta + \cos^2\theta) \, d\theta = \int_0^{\pi} \left(1 + 2\cos\theta + \frac{1 + \cos 2\theta}{2}\right) d\theta$ $= \left[\frac{3}{2}\theta + 2\sin\theta + \frac{1}{4}\sin 2\theta\right]_0^{\pi} = \frac{3}{2}\pi$ area in 1st quadrant = $\frac{1}{2} \times \int_0^{\frac{1}{2}\pi} (1 + \cos\theta)^2 \, d\theta$ $= \frac{1}{2}\left[\frac{3}{2}\theta + 2\sin\theta + \frac{1}{4}\sin 2\theta\right]_0^{\frac{1}{2}\pi} = \frac{3}{8}\pi + 1$ Area under line in 1st quadrant = 1 $\Rightarrow$ Area enclosed by line and curve = $\frac{3}{8}\pi + 1 - 1 = \frac{3}{8}\pi$ $\Rightarrow$ ratio = $\left(\frac{3}{2}\pi - \frac{3}{8}\pi\right) : \frac{3}{8}\pi = 3:1$	B1 M1 A1 A1 M1 A1 **[6]**	 Formula for area used Or ratio 1 : 3	 Sight of expansion and attempt to integrate

Question			Answer	Marks	Guidance
1	(i)		$(20\sin\theta)^2 - 2g(2.44) = 0$ $\theta = 20.2$	M1 A1 **[2]**	Use $v^2 = u^2 + 2as$ vertically with $v = 0$ $\theta = 20.22908\ldots$
	(ii)		20 sin cv(θ)$t - 1/2gt^2 = 0$ AND range = 20 cv(t) cos cv(θ) Range = 26.5 m	M1 A1 **[2]**	Use $s = ut + ½at^2$ vertically with $s = 0$ OR use $v = u + at$ and doubles t AND horizontally with time found from vertical. (t = 1.4113… s or 1.4093…s (from 20.2)) Range = 26.48541… m or 26.45387….m (from 20.2)
		OR	$\dfrac{20^2 \sin(2\times \text{cv}(\theta))}{g}$ Range = 26.5 m	M1 A1 **[2]**	Use of range formula Range = 26.48541… m or 26.45387….m (from 20.2)
2	(i)		 $r/6 = \tan 21$ $r = 2.3(0)$	M1 A1 A1 **[3]**	Attempt to use trigonometry to form equation for r $r = 2.30318\ldots$
	(ii)		$\mu <$ cv(r)/6 or $\mu mg\cos 21 < mg\sin 21$ $\mu < 0.384$ or tan 21	M1 A1 **[2]**	Attempt comparison between weight comp and max friction. $\mu < 0.38386\ldots$ or 0.38333…. (from 2.3); allow $\leq$
3	(i)		CoM of triangle = $^1/_3$ x cv(12) from BD $(80 + 60)x_G$ $= 4(80) + 12(60)$ $x_G = 7.43$ cm	B1 M1 A1 A1 A1 **[5]**	OR $^2/_3$ x cv(12) from C. CoM of triangle Table of values idea 7.42857… or $^{52}/_7$ cm
	(ii)		$\tan\theta = (8 - x_G)/5$ $\tan\theta = 0.5714\ldots/5$ $\theta = 6.52°$	M1 A1ft A1 **[3]**	Using tan to find a relevant angle ft their x_G to target angle with the vertical 6.5198… Allow 6.5(0) from $x_G = 7.43$

Question			Answer	Marks	Guidance
4	(i)		 18(10) – T(20sinθ) + 3(6) = 0 T = 16.5 N	M1 A1 A1 **[3]**	Moments about P Need a value for sinθ or θ Exact
	(ii)		$X = T\cos\theta$ $Y + T\sin\theta - 18 - 3 = 0$ $R = \sqrt{(13.2^2 + 11.1^2)} = 17.2$ N	B1ft M1 A1ft A1 **[4]**	ft candidates value of T. Resolve horizontally (X = 13.2 N) or moments; Need a value for cosθ or θ Resolve vertically or moments ft candidates value of T. Y = 11.1 N; Need a value for sinθ or θ R = 17.2467…
	(iii)		μ = cv(|Y|)/cv(|X|) = 11.1/13.2 μ = 0.841	M1 A1 **[2]**	Use of $Fr = \mu R$ μ = 0.8409… ; allow $^{37}/_{44}$
5	(i)		Driving Force = 10000/20 (= 500) cv(10000/20) – 1300 + 800$g$sinα = 0 sinα = 5/49	B1 M1 A1 A1 **[4]**	 Attempt at N2L with 3 terms **AG** at least one more line of correct working (at least e.g. -800+800$g$sinα=0); allow verification (e.g. 500 – 1300 + 800 = 0)
	(ii)		800(22.1)$g$sinα 800(22.1)$g$sinα + 1300(22.1) + ½(800)(8^2) t = 3.6(0) s	B1 M1 A1 M1 A1 **[5]**	Work done against weight; Need a value for sinα or α Total work done, 3 terms needed Need a value for sinα or α; (72010 J) Time = work done(from at least one correct energy term)/power 'Exact' is 3.6005
6	(i)		 $(2m)(4) - (3m)(2) = 2mv_A + 3mv_B$ $(v_B - v_A)/(4 - -2) = 0.4$ Speed A = 1.04 m s^{-1}, Speed B = 1.36 m s^{-1}	*M1 A1 *M1 A1 Dep**M1 A1 **[6]**	Attempt at use of conservation of momentum Attempt at use of coefficient of restitution Solving for v_A and v_B Final answers must be positive

Question			Answer	Marks	Guidance
	(ii)		Energy before = ½(2*m*)(4^2) + ½(3*m*)(2^2) Energy after = ½(2*m*)(1.04^2) + ½(3*m*)(1.36^2) 22*m* – 3.856*m* 18.1*m*	B1ft B1ft M1 A1 **[4]**	Energy before or Loss in *A*'s KE Energy after or Loss in *B*'s KE Difference of total OR sum of differences (total kinetic energy must decrease) 18.144*m* (Exact)
		OR	$\frac{1}{2}\frac{m_1m_2}{m_1+m_2}(1-e^2)A^2$ $\frac{1}{2}\frac{(2m)(3m)}{2m+3m}(1-0.4^2)(4+2)^2$ 18.1*m*	*B1 Dep*M1 A1 A1 **[4]**	Loss of kinetic energy formula, where *A* = approach speed Substitution of values into quoted formula 18.144*m* (Exact)
	(iii)		2*m*(4) – 2*m*(−1.04) = 2.52 *m* = 0.25	M1 A1ft A1 **[3]**	Attempt at change in momentum and equate to impulse. Must use 2m or 3m Or 3*m*(2) – 3*m*(−1.36) = 2.52 Exact
7	(i)		*T*cos30 + *T*cos45 = 0.4*g* *T* = 2.49 N	M1 A1 A1 **[3]**	Resolve vertically (3 terms); may be different *T*'s at this stage *T* = 2.4918.…
	(ii)		cv(*T*)sin30 + cv(*T*)sin45 = $0.4v^2/0.5$ *v* = 1.94 m s^{-1}	M1 A1 A1 **[3]**	Resolve horizontally (3 terms); may be different *T*'s at this stage Or use acceleration = $0.5\omega^2$ *v* = 1.93904…
	(iii)		$(2AP =)\frac{0.5}{\sin 45}+\frac{0.5}{\sin 30}$ *AP* = 0.854 m	M1 A1 **[2]**	Reasonable attempt to use trigonometry to find total length of string **AG** (*AP* = 0.85355…m)

Question			Answer	Marks	Guidance
	(iv)		$2T\sin\theta = 0.4(0.854\sin\theta)(3.46^2)$	M1	θ angle with vertical. Resolve horizontally. Allow with T only. r = component of 0.854
			$T = 2.04$ N	A1	$T = 2.04474\ldots$ N using $AP = 0.854$ m, $T = 2.04367\ldots$ N using exact AP
			$2T\cos\theta = 0.4g$	M1	θ angle with vertical. Resolve vertically. Allow with T only
			$\theta = 16.5°$ or $16.6°$	A1	$\theta = 16.55377\ldots°$ using $AP = 0.854$ m, $\theta = 16.4526\ldots°$ using exact AP
				[4]	**SC** M1A0M1A1 for use of T instead of 2T throughout
8	**(i)**		$v_x = 12\cos 20$	*B1	11.27631…..
			$8 = 12t\cos 20$	B1	Using suvat to find expression in t only. ($t = 0.70945\ldots$)
				*M1	Attempt at use of $v = u + at$
			$v_y = 12\sin 20 - gcv(t)$	A1	-2.84838……
			$\tan\theta = v_y / v_x$	Dep**M1	Use trig to find a relevant angle
			14.2° below horizontal	A1	14.1763… (75.8° downward vertical)
				[6]	
	(ii)		$8 = Vt\cos 20$	B1	
				*M1	Attempt at use of s = $ut + ½ at^2$
			$1.5 = Vt\sin 20 - gt^2/2$	A1	
			Eliminate t	dep*M1	OR Eliminate V and solve for t
			Attempt to solve a quadratic for V	dep*M1	AND Sub value for t and solve for V
			$V = 15.9$	A1	$V = 15.8606\ldots$
				[6]	
		OR	$y = x\tan\theta - gx^2\sec^2\theta/2u^2$	*B1	Use equation of trajectory
			Substitute values for y, x, θ	dep*M1	
			$1.5 = 8\tan 20 - g8^2\sec^2 20/2V^2$	A1	
			Attempt to solve a quadratic for V	dep*M2	SC M1 for solving for V^2
			$V = 15.9$	A1	$V = 15.8606\ldots$
				[6]	