OCR
Oxford Cambridge and RSA

Monday 22 June 2015 – Morning

A2 GCE MATHEMATICS (MEI)

4756/01 Further Methods for Advanced Mathematics (FP2)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4756/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (54 marks)

1 **(a)** **(i)** A curve has polar equation $r = 2a\cos\theta + 2b\sin\theta$, where $a > 0$ and $b > 0$.

Show, by considering its cartesian equation, that the curve is a circle which passes through the origin. Find the centre and radius of the circle in terms of a and b. **[5]**

(ii) For the case $a = b = 1$, use integration to show that the region bounded by a minor arc of the circle and the lines $\theta = \frac{\pi}{6}$ and $\theta = \frac{\pi}{3}$ has area $1 + \frac{\pi}{3}$. **[5]**

(b) Given that $\mathrm{f}(t) = \ln(1+t)$, obtain expressions for $\mathrm{f}'(t)$, $\mathrm{f}''(t)$ and $\mathrm{f}'''(t)$. Hence show that the Maclaurin series for $\ln(1+t)$ begins

$$t - \frac{t^2}{2} + \frac{t^3}{3} \ldots .$$

Deduce the first two non-zero terms of the Maclaurin series for $\ln\left(\frac{1+t}{1-t}\right)$. **[8]**

2 **(a)** **(i)** By considering $\left(z + \frac{1}{z}\right)^5$, where $z = \cos\theta + \mathrm{j}\sin\theta$, show that

$$\cos^5\theta = \tfrac{1}{16}(\cos 5\theta + 5\cos 3\theta + 10\cos\theta).$$ **[5]**

(ii) Use de Moivre's theorem to find an expression for $\cos 5\theta$ in terms of powers of $\cos\theta$. **[5]**

(b) **(i)** Obtain the roots of the equation $w^5 = 4\sqrt{2}$ in the form $r\mathrm{e}^{\mathrm{j}\theta}$. Show the points corresponding to these roots in an Argand diagram. **[4]**

(ii) For each root w, let $v = w\sqrt{2}\,\mathrm{e}^{\mathrm{j}\pi/10}$.

Show the points corresponding to the values of v on your Argand diagram.

Find, in simplified form, an equation for which the values of v are the roots. **[4]**

3 This question concerns the matrix $\mathbf{M}$ where $\mathbf{M} = \begin{pmatrix} 5 & -1 & 3 \\ 4 & -3 & -2 \\ 2 & 1 & 4 \end{pmatrix}$.

(i) Obtain the characteristic equation of $\mathbf{M}$.

Find the eigenvalues of $\mathbf{M}$. **[7]**

These eigenvalues are denoted by λ_1, λ_2, λ_3, where $\lambda_1 < \lambda_2 < \lambda_3$.

(ii) Verify that an eigenvector corresponding to λ_1 is $\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$ and that an eigenvector corresponding to λ_2 is $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$. Find an eigenvector of the form $\begin{pmatrix} a \\ 1 \\ c \end{pmatrix}$ corresponding to λ_3. **[5]**

(iii) Write down a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{M} = \mathbf{PDP}^{-1}$. (You are not required to calculate $\mathbf{P}^{-1}$.)

Hence write down an expression for $\mathbf{M}^4$ in terms of $\mathbf{P}$ and a diagonal matrix. You should give the elements of the diagonal matrix explicitly. **[3]**

(iv) Use the Cayley-Hamilton theorem to obtain an expression for $\mathbf{M}^4$ as a linear combination of $\mathbf{M}$ and $\mathbf{M}^2$. **[3]**

Section B (18 marks)

4 **(i)** Starting with the relationship $\cosh^2 t - \sinh^2 t = 1$, deduce a relationship between $\tanh^2 t$ and $\text{sech}^2 t$. **[1]**

You are given that $y = \text{artanh}\, x$.

(ii) Show that $\dfrac{dy}{dx} = \dfrac{1}{1-x^2}$. **[4]**

(iii) Show, by integrating the result in part (ii), that $y = \dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$. **[4]**

(iv) Show that $\displaystyle\int_0^{\frac{\sqrt{3}}{6}} \frac{1}{1-3x^2}\,dx = \frac{1}{\sqrt{3}}\text{artanh}\frac{1}{2}$. Express this answer in logarithmic form. **[4]**

(v) Use integration by parts to find $\displaystyle\int \text{artanh}\, x\, dx$, giving your answer in terms of logarithms. **[5]**

END OF QUESTION PAPER