Question			Answer	Marks	Guidance	
1	(a)	(i)	$\sin y = x \Rightarrow \cos y \dfrac{dy}{dx} = 1$	M1	Differentiating w.r.t. x or y	$\dfrac{dx}{dy} = \cos y$
			$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\cos y}$	A1		
			$\Rightarrow \dfrac{dy}{dx} = (\pm)\dfrac{1}{\sqrt{1-x^2}}$	A1(ag)	Completion www, but independent of B1	$\dfrac{dy}{dx} = \pm\dfrac{1}{\sqrt{1-x^2}}$ or ± not considered scores max. 3
			Taking + sign because gradient is positive	B1	Validly rejecting − sign. Dependent on A1 above	Or $-\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2} \Rightarrow 0 \le \cos y \le 1$
				[4]		
1	(a)	(ii)	(A) $\displaystyle\int_{-1}^{1} \frac{1}{\sqrt{2-x^2}}\,dx = \left[\arcsin\frac{x}{\sqrt{2}}\right]_{-1}^{1}$	M1	arcsin alone, or any appropriate substitution	
				A1	$\arcsin\dfrac{x}{\sqrt{2}}$ or $\int 1\,d\theta$ www	Condone omitted or incorrect limits
			$= \dfrac{\pi}{2}$	A1		
				[3]		
			(B) $\displaystyle\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\sqrt{1-2x^2}}\,dx = \frac{1}{\sqrt{2}}\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\sqrt{\frac{1}{2}-x^2}}\,dx$			
			$= \dfrac{1}{\sqrt{2}}\left[\arcsin\sqrt{2}x\right]_{-\frac{1}{2}}^{\frac{1}{2}}$	M1	arcsin alone, or any appropriate substitution	
				A1	$\dfrac{1}{\sqrt{2}}$ and $\sqrt{2}x$ or $\int \dfrac{1}{\sqrt{2}}\,d\theta$ www	
				M1	Using consistent limits in order and evaluating in terms of π. Dependent on M1 above	e.g. $\pm\dfrac{\pi}{4}$ with sub. $x = \dfrac{1}{\sqrt{2}}\sin\theta$
			$= \dfrac{\pi}{2\sqrt{2}}$	A1		
				[4]		

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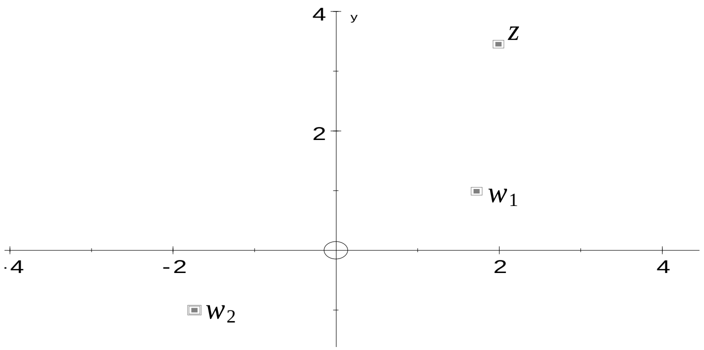
1	(b)		$r = \tan\theta$			
			$\Rightarrow x = r\cos\theta = \dfrac{\sin\theta}{\cos\theta} \times \cos\theta = \sin\theta$	M1 A1(ag)	Using $x = r\cos\theta$ o.e.	
			$\Rightarrow r^2 = \dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{\sin^2\theta}{1-\sin^2\theta} = \dfrac{x^2}{1-x^2}$	M1 A1(ag)	Obtaining r^2 in terms of x	
			$r^2 = x^2 + y^2 \Rightarrow x^2 + y^2 = \dfrac{x^2}{1-x^2}$			
			$\Rightarrow y^2 = \dfrac{x^2}{1-x^2} - x^2$	M1	Obtaining y^2 in terms of x	
			$\Rightarrow y^2 = \dfrac{x^2 - x^2(1-x^2)}{1-x^2} = \dfrac{x^4}{1-x^2}$			
			$\Rightarrow y = \dfrac{x^2}{\sqrt{1-x^2}}$	A1(ag)		Ignore discussion of ±
			Asymptote $x = 1$	B1 **[7]**	Condone $x = \pm 1$	$x \neq 1$, $x^2 = 1$ B0
2	(a)	(i)	$z^n + \dfrac{1}{z^n} = 2\cos n\theta$	B1	Mark final answer	
			$z^n - \dfrac{1}{z^n} = 2j\sin n\theta$	B1 **[2]**	Mark final answer	
2	(a)	(ii)	$\left(z + \dfrac{1}{z}\right)^4 = z^4 + 4z^2 + 6 + \dfrac{4}{z^2} + \dfrac{1}{z^4} = z^4 + \dfrac{1}{z^4} + 4\left(z^2 + \dfrac{1}{z^2}\right) + 6$	M1	Expanding by Binomial or complete equivalent	
			$\Rightarrow (2\cos\theta)^4 = 2\cos 4\theta + 8\cos 2\theta + 6$	M1	Introducing cosines of multiple angles	Condone lost 2s
				A1	RHS correct	Both As depend on both Ms
			$\Rightarrow \cos^4\theta = \dfrac{3}{8} + \dfrac{1}{2}\cos 2\theta + \dfrac{1}{8}\cos 4\theta$	A1ft **[4]**	Dividing both sides by 16. F.t. line above	$A = \dfrac{3}{8}$, $B = \dfrac{1}{2}$, $C = \dfrac{1}{8}$ Give SC2 for fully correct answer found "otherwise"

Question			Answer	Marks	Guidance	
2	(a)	(iii)	$\cos^4\theta = \frac{3}{8} + \frac{1}{2}\left(2\cos^2\theta - 1\right) + \frac{1}{8}\cos 4\theta$ $\Rightarrow \cos^4\theta = \cos^2\theta - \frac{1}{8} + \frac{1}{8}\cos 4\theta$ $\Rightarrow \cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$	M1 A1 **[2]**	Using (ii), obtaining cos 4θ and expressing cos 2θ in terms of $\cos^2\theta$ c.a.o.	Condone cos $2\theta = \pm 1 \pm 2\cos^2\theta$
2	(b)	(i)	$z = 4e^{\frac{j\pi}{3}}$ and $w^2 = z$: let $w = re^{j\theta} \Rightarrow w^2 = r^2 e^{2j\theta}$ $\Rightarrow r^2 = 4 \Rightarrow r = 2$ and $\theta = \frac{\pi}{6}, \frac{7\pi}{6}$ 	B1 B1B1 B1 B1 **[5]**	Or $-\frac{5\pi}{6}$ Roots with approx. equal moduli and approx. correct argument Dependent on first B1 z in correct position	Condone $r = \pm 2$ Award B2 for $\pi\left(k + \frac{1}{6}\right)$ Ignore annotations and scales $\le \pi/4$ Modulus and argument bigger
2	(b)	(ii)	$z = 4e^{\frac{j\pi}{3}} \Rightarrow z^n = 4^n e^{\frac{j\pi n}{3}}$ so real if $\frac{\pi n}{3} = \pi \Rightarrow n = 3$ Imaginary if $\frac{\pi n}{3} = \frac{\pi}{2} + k\pi \Rightarrow n = \frac{3}{2} + 3k$ which is not an integer for any k $w_1 = 2e^{\frac{j\pi}{6}} \Rightarrow w_1{}^3 = 8e^{\frac{j\pi}{2}} = 8j$ $w_2 = 2e^{\frac{7j\pi}{6}} \Rightarrow w_2{}^3 = 8e^{\frac{7j\pi}{2}} = -8j$	B1 M1 A1(ag) M1 A1 **[5]**	 $\cos\frac{\pi n}{3} = 0$ or $\frac{\pi n}{3} = \frac{\pi}{2}$... An argument which covers the positive and negative im. axis Attempting their w^3 in any form 8j, −8j	Ignore other larger values Must deal with mod and arg

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3	(i)		det (**M**) = 1(2*a* + 8) − 2(−2 − 12) + 3(2 − 3*a*) = 42 − 7*a* ⇒ no inverse if *a* = 6 $\mathbf{M}^{-1} = \frac{1}{42-7a}\begin{pmatrix} 2a+8 & -10 & 8-3a \\ 14 & -7 & -7 \\ 2-3a & 8 & a+2 \end{pmatrix}$	M1A1 A1 M1 A1 M1 A1 **[7]**	Obtaining det(**M**) in terms of *a* At least 4 cofactors correct (including one involving *a*) Six signed cofactors correct Transposing and ÷ by det(**M**). Dependent on previous M1M1	Accept unsimplified Accept $a \neq 6$ after correct det M0 if more than 1 is multiplied by the corresponding element Mark final answer
3	(ii)		$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{42}\begin{pmatrix} 8 & -10 & 8 \\ 14 & -7 & -7 \\ 2 & 8 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ $\Rightarrow x = \frac{6}{7}, y = \frac{1}{2}, z = -\frac{2}{7}$	M1 M1 A2 **[4]**	Substituting *a* = 0 Correct use of inverse Dependent on both M marks. Give A1 for one correct SC1 for *x* = 6, *y* = 3.5, *z* = −2	One correct element. Condone missing determinant After M0, give SC2 for correct solution and SC1 for one correct Answers unsupported score 0
3	(iii)		e.g. 7*x* − 10*y* = 10, 7*x* − 10*y* = 3*b* − 2 (or e.g. 4*x* + 5*z* = 5, 4*x* + 5*z* = *b* + 1) (or e.g. 8*y* + 7*z* = −1, 8*y* + 7*z* = 3 − *b*) For solutions, 10 = 3*b* − 2 ⇒ *b* = 4	M1 A1 M1 A1	Eliminating one variable in two different ways Two correct equations Validly obtaining a value of *b*	Or 7*x* − 10*y* = 2*b* + 2 Or 8*x* + 10*z* = 3*b* − 2 Or 16*y* + 14*z* = *b* − 6
			OR M2 A1 *b* = 4 A1		A method leading to an equation from which *b* could be found A correct equation	E.g. setting *z* = 0, augmented matrix, adjoint matrix, etc.
			$x = \lambda, y = 0.7\lambda - 1, z = 1 - 0.8\lambda$ Straight line	M1 A1 B1 **[7]**	Obtaining general soln. by e.g. setting one unknown = λ and finding other two in terms of λ Any correct form Accept "sheaf", "pages of a book", etc.	Accept unknown instead of λ $x = \frac{10}{7}\lambda + \frac{10}{7}, y = \lambda, z = -\frac{8}{7}\lambda - \frac{1}{7}$ $x = \frac{5}{4} - \frac{5}{4}\lambda, y = -\frac{7}{8}\lambda - \frac{1}{8}, z = \lambda$ Independent of all previous marks. Ignore other comments

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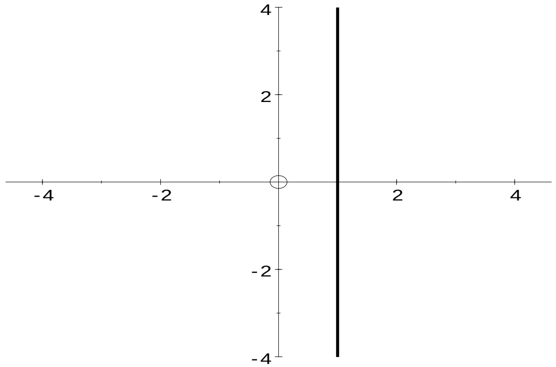
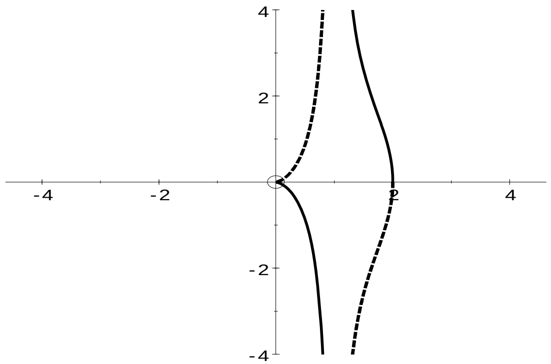
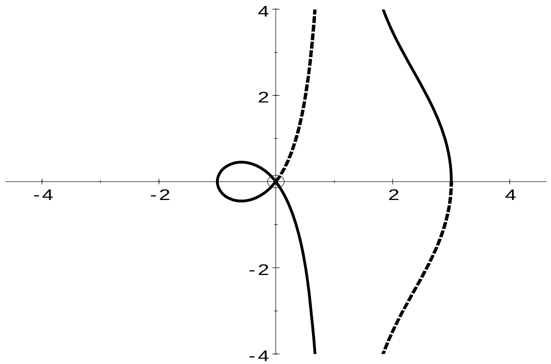
4	(i)		$\sinh u = \frac{e^u - e^{-u}}{2} \Rightarrow \sinh^2 u = \frac{e^{2u} - 2 + e^{-2u}}{4}$	B1	$\left(e^u - e^{-u}\right)^2 = e^{2u} - 2 + e^{-2u}$	Accept other or mixed variables
			$\Rightarrow 2\sinh^2 u + 1 = \frac{e^{2u} - 2 + e^{-2u}}{2} + 1 = \frac{e^{2u} + e^{-2u}}{2}$	B1	$\cosh 2u = \frac{e^{2u} + e^{-2u}}{2}$	
			$= \cosh 2u$	B1 **[3]**	Completion www	
4	(ii)		If $\cosh y = u$, $u = \frac{e^y + e^{-y}}{2}$	M1	Expressing u in exponential form	$\frac{1}{2}$, + must be correct
			$\Rightarrow e^y + e^{-y} = 2u \Rightarrow e^{2y} - 2ue^y + 1 = 0$			
			$\Rightarrow \left(e^y - u\right)^2 - u^2 + 1 = 0$			
			$\Rightarrow e^y = u \pm \sqrt{u^2 - 1}$	M1	Reaching e^y	Condone omitted ±
			$\Rightarrow y = \ln\left(u + \sqrt{u^2 - 1}\right)$	A1(ag)	Completion www; indep. of B1	$y = \ln\left(u \pm \sqrt{u^2 - 1}\right)$ or ± not considered scores max. 3
			$y \geq 0 \Rightarrow e^y = u + \sqrt{u^2 - 1}$	B1	Validly rejecting − sign Dependent on A1 above	
			OR $\ln\left(u + \sqrt{u^2 - 1}\right) = \ln\left(\cosh y + \sqrt{\cosh^2 y - 1}\right)$ M1		Substituting $u = \cosh y$	
			$= \ln\left(\cosh y + \sinh y\right)$			
			since $\sinh y > 0$ B1		Rejecting −ve square root	Dependent on A1
			$= \ln\left(e^y\right)$ M1		Reaching e^y	
			$= y$ A1		Completion www; indep. of B1	
				[4]		

Question			Answer	Marks	Guidance	
4	(iii)		$x=\frac{1}{2}\cosh u \Rightarrow \frac{dx}{du}=\frac{1}{2}\sinh u$	M1	Reaching integrand equivalent to $k\sinh^2 u$	
			$\int\sqrt{4x^2-1}\,dx=\int\sqrt{\cosh^2 u-1}\times\frac{1}{2}\sinh u\,du$			
			$=\int\frac{1}{2}\sinh^2 u\,du$	A1		
			$=\int\frac{1}{4}\cosh 2u-\frac{1}{4}\,du$	M1	Simplifying to integrable form. Dependent on M1 above	Or $\frac{1}{8}e^{2u}-\frac{1}{4}+\frac{1}{8}e^{-2u}$
			$=\frac{1}{8}\sinh 2u-\frac{1}{4}u+c$	A1A1	For $\frac{1}{8}\sinh 2u$ o.e. and $-\frac{1}{4}u$ seen	Or $\frac{1}{16}e^{2u}-\frac{1}{4}u-\frac{1}{16}e^{-2u}+c$
			$=\frac{1}{4}\sinh u\cosh u-\frac{1}{4}u+c$			Condone omission of + c throughout
			$=\frac{1}{4}\sqrt{4x^2-1}\times 2x-\frac{1}{4}\operatorname{arcosh} 2x+c$	M1	Clear use of $\sinh 2u = 2\sinh u\cosh u$ Dependent on M1M1 above	
			$=\frac{1}{2}x\sqrt{4x^2-1}-\frac{1}{4}\operatorname{arcosh} 2x+c$			
			$a=\frac{1}{2}$	A1		a, b need not be written separately
				[7]		
4	(iv)		$\int_{\frac{1}{2}}^{1}\sqrt{4x^2-1}\,dx=\left[\frac{1}{2}x\sqrt{4x^2-1}-\frac{1}{4}\operatorname{arcosh} 2x\right]_{\frac{1}{2}}^{1}$	M1	Using their (iii) and using limits correctly	
			$=\frac{\sqrt{3}}{2}-\frac{1}{4}\operatorname{arcosh} 2+\frac{1}{4}\operatorname{arcosh} 1$	A1ft	May be implied F.t. values of a and b in (iii)	$a\sqrt{3}-b\operatorname{arcosh} 2$. No decimals. Must have obtained values for a and b
			$=\frac{\sqrt{3}}{2}-\frac{1}{4}\ln\left(2+\sqrt{3}\right)+\frac{1}{4}\ln 1$	M1	Using (ii) accurately Dependent on M1 above	
			$=\frac{\sqrt{3}}{2}-\frac{1}{4}\ln\left(2+\sqrt{3}\right)$	A1	c.a.o. A0 if ln 1 retained Mark final answer	Correct answer www scores 4/4
				[4]		

Question			Answer	Marks	Guidance	
5	(i)		Undefined for $\theta = \dfrac{\pi}{2}$ and $\dfrac{3\pi}{2}$	B1B1 **[2]**		
5	(ii)		 $r = \sec\theta \Rightarrow r\cos\theta = 1$ $\Rightarrow x = 1$	B1 M1 A1 **[3]**	Vertical line through (1, 0) (indicated, e.g. by scale) Use of $x = r\cos\theta$	
5	(iii)		$a = 1$:  $a = -1$ gives same curve $a = 1$, $0 < \theta < \pi$ corresponds to $a = -1$, $\pi < \theta < 2\pi$ $a = -1$, $0 < \theta < \pi$ corresponds to $a = 1$, $\pi < \theta < 2\pi$	B1 B2 B1 B1 B1 **[6]**	Section through (2, 0) (indicated) Section through (0, 0) (give B1 for one error)	If asymptote included max. 2/3

Question			Answer	Marks	Guidance	
5	(iv)		Loop e.g. $a = 2$ 	B1 B2 **[3]**	Give B1 for one error	
5	(v)		$r = \sec\theta + a$ $\Rightarrow\quad r = \frac{r}{x} + a$ $\Rightarrow\quad r\left(1 - \frac{1}{x}\right) = a$ $\Rightarrow\quad \sqrt{x^2+y^2}\left(\frac{x-1}{x}\right) = a$ $\Rightarrow\quad \sqrt{x^2+y^2}\,(x-1) = ax$ $\Rightarrow\quad (x^2+y^2)(x-1)^2 = a^2x^2$	M1 M1 M1 A1(ag) **[4]**	Use of $x = r\cos\theta$ Use of $r = \sqrt{x^2+y^2}$ Correct manipulation	