



**ADVANCED GCE**

**MATHEMATICS (MEI)**

Further Methods for Advanced Mathematics (FP2)

**4756**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

**Other Materials Required:**

None

**Friday 5 June 2009**  
**Afternoon**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions in Section A and **one** question from Section B.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

## Section A (54 marks)

## Answer all the questions

- 1 (a) (i) Use the Maclaurin series for  $\ln(1+x)$  and  $\ln(1-x)$  to obtain the first three non-zero terms in the Maclaurin series for  $\ln\left(\frac{1+x}{1-x}\right)$ . State the range of validity of this series. [4]
- (ii) Find the value of  $x$  for which  $\frac{1+x}{1-x} = 3$ . Hence find an approximation to  $\ln 3$ , giving your answer to three decimal places. [4]
- (b) A curve has polar equation  $r = \frac{a}{1 + \sin \theta}$  for  $0 \leq \theta \leq \pi$ , where  $a$  is a positive constant. The points on the curve have cartesian coordinates  $x$  and  $y$ .
- (i) By plotting suitable points, or otherwise, sketch the curve. [3]
- (ii) Show that, for this curve,  $r + y = a$  and hence find the cartesian equation of the curve. [5]

- 2 (i) Obtain the characteristic equation for the matrix  $\mathbf{M}$  where

$$\mathbf{M} = \begin{pmatrix} 3 & 1 & -2 \\ 0 & -1 & 0 \\ 2 & 0 & 1 \end{pmatrix}.$$

Hence or otherwise obtain the value of  $\det(\mathbf{M})$ . [3]

- (ii) Show that  $-1$  is an eigenvalue of  $\mathbf{M}$ , and show that the other two eigenvalues are not real.

Find an eigenvector corresponding to the eigenvalue  $-1$ .

Hence or otherwise write down the solution to the following system of equations. [9]

$$\begin{aligned} 3x + y - 2z &= -0.1 \\ -y &= 0.6 \\ 2x + z &= 0.1 \end{aligned}$$

- (iii) State the Cayley-Hamilton theorem and use it to show that

$$\mathbf{M}^3 = 3\mathbf{M}^2 - 3\mathbf{M} - 7\mathbf{I}.$$

Obtain an expression for  $\mathbf{M}^{-1}$  in terms of  $\mathbf{M}^2$ ,  $\mathbf{M}$  and  $\mathbf{I}$ . [4]

- (iv) Find the numerical values of the elements of  $\mathbf{M}^{-1}$ , showing your working. [3]

## 3

- 3 (a) (i) Sketch the graph of  $y = \arcsin x$  for  $-1 \leq x \leq 1$ . [1]

Find  $\frac{dy}{dx}$ , justifying the sign of your answer by reference to your sketch. [4]

- (ii) Find the exact value of the integral  $\int_0^1 \frac{1}{\sqrt{2-x^2}} dx$ . [3]

- (b) The infinite series  $C$  and  $S$  are defined as follows.

$$C = \cos \theta + \frac{1}{3} \cos 3\theta + \frac{1}{9} \cos 5\theta + \dots$$

$$S = \sin \theta + \frac{1}{3} \sin 3\theta + \frac{1}{9} \sin 5\theta + \dots$$

By considering  $C + jS$ , show that

$$C = \frac{3 \cos \theta}{5 - 3 \cos 2\theta},$$

and find a similar expression for  $S$ . [11]

### Section B (18 marks)

#### Answer one question

#### Option 1: Hyperbolic functions

- 4 (i) Prove, from definitions involving exponentials, that

$$\cosh 2u = 2 \cosh^2 u - 1. \quad [3]$$

- (ii) Prove that  $\operatorname{arsinh} y = \ln(y + \sqrt{y^2 + 1})$ . [4]

- (iii) Use the substitution  $x = 2 \sinh u$  to show that

$$\int \sqrt{x^2 + 4} dx = 2 \operatorname{arsinh} \frac{1}{2}x + \frac{1}{2}x\sqrt{x^2 + 4} + c,$$

where  $c$  is an arbitrary constant. [6]

- (iv) By first expressing  $t^2 + 2t + 5$  in completed square form, show that

$$\int_{-1}^1 \sqrt{t^2 + 2t + 5} dt = 2(\ln(1 + \sqrt{2}) + \sqrt{2}). \quad [5]$$

[Question 5 is printed overleaf.]

## Option 2: Investigation of curves

This question requires the use of a graphical calculator.

- 5 Fig. 5 shows a circle with centre  $C(a, 0)$  and radius  $a$ .  $B$  is the point  $(0, 1)$ . The line  $BC$  intersects the circle at  $P$  and  $Q$ ;  $P$  is above the  $x$ -axis and  $Q$  is below.

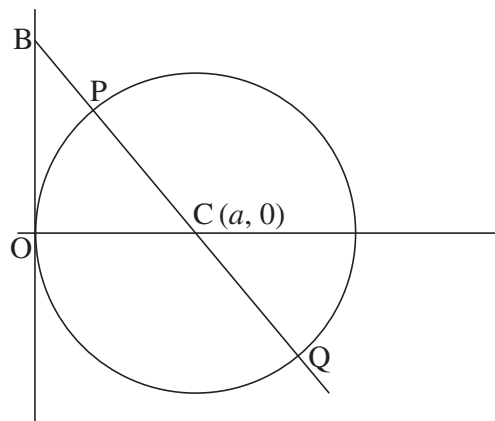


Fig. 5

- (i) Show that, in the case  $a = 1$ ,  $P$  has coordinates  $\left(1 - \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ . Write down the coordinates of  $Q$ . [3]

- (ii) Show that, for all positive values of  $a$ , the coordinates of  $P$  are

$$x = a \left(1 - \frac{a}{\sqrt{a^2 + 1}}\right), \quad y = \frac{a}{\sqrt{a^2 + 1}}. \quad (*)$$

Write down the coordinates of  $Q$  in a similar form. [4]

Now let the variable point  $P$  be defined by the parametric equations  $(*)$  for all values of the parameter  $a$ , positive, zero and negative. Let  $Q$  be defined for all  $a$  by your answer in part (ii).

- (iii) Using your calculator, sketch the locus of  $P$  as  $a$  varies. State what happens to  $P$  as  $a \rightarrow \infty$  and as  $a \rightarrow -\infty$ .

Show algebraically that this locus has an asymptote at  $y = -1$ .

On the same axes, sketch, as a dotted line, the locus of  $Q$  as  $a$  varies. [8]

(The single curve made up of these two loci and including the point  $B$  is called a *right strophoid*.)

- (iv) State, with a reason, the size of the angle  $POQ$  in Fig. 5. What does this indicate about the angle at which a right strophoid crosses itself? [3]

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