ADVANCED GCE UNIT

4756/01

MATHEMATICS (MEI)

Further Methods for Advanced Mathematics (FP2)

THURSDAY 7 JUNE 2007

Morning

Time: 1 hour 30 minutes

Additional materials:
- Answer booklet (8 pages)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions in Section A and **one** question from Section B.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

[Turn over

Section A (54 marks)

Answer all the questions

1 **(a)** A curve has polar equation $r = a(1 - \cos\theta)$, where a is a positive constant.

(i) Sketch the curve. [2]

(ii) Find the area of the region enclosed by the section of the curve for which $0 \leqslant \theta \leqslant \frac{1}{2}\pi$ and the line $\theta = \frac{1}{2}\pi$. [6]

(b) Use a trigonometric substitution to show that $\displaystyle\int_0^1 \frac{1}{\left(4 - x^2\right)^{\frac{3}{2}}}\,\mathrm{d}x = \frac{1}{4\sqrt{3}}$. [4]

(c) In this part of the question, $\mathrm{f}(x) = \arccos(2x)$.

(i) Find $\mathrm{f}'(x)$. [2]

(ii) Use a standard series to expand $\mathrm{f}'(x)$, and hence find the series for $\mathrm{f}(x)$ in ascending powers of x, up to the term in x^5. [4]

2 **(a)** Use de Moivre's theorem to show that $\sin 5\theta = 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta$. [5]

(b) **(i)** Find the cube roots of $-2 + 2\mathrm{j}$ in the form $r\mathrm{e}^{\mathrm{j}\theta}$ where $r > 0$ and $-\pi < \theta \leqslant \pi$. [6]

These cube roots are represented by points A, B and C in the Argand diagram, with A in the first quadrant and ABC going anticlockwise. The midpoint of AB is M, and M represents the complex number w.

(ii) Draw an Argand diagram, showing the points A, B, C and M. [2]

(iii) Find the modulus and argument of w. [2]

(iv) Find w^6 in the form $a + b\mathrm{j}$. [3]

3 Let $\mathbf{M} = \begin{pmatrix} 3 & 5 & 2 \\ 5 & 3 & -2 \\ 2 & -2 & -4 \end{pmatrix}$.

(i) Show that the characteristic equation for $\mathbf{M}$ is $\lambda^3 - 2\lambda^2 - 48\lambda = 0$. [4]

You are given that $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ is an eigenvector of $\mathbf{M}$ corresponding to the eigenvalue 0.

(ii) Find the other two eigenvalues of $\mathbf{M}$, and corresponding eigenvectors. [8]

(iii) Write down a matrix $\mathbf{P}$, and a diagonal matrix $\mathbf{D}$, such that $\mathbf{P}^{-1}\mathbf{M}^2\mathbf{P} = \mathbf{D}$. [3]

(iv) Use the Cayley-Hamilton theorem to find integers a and b such that $\mathbf{M}^4 = a\mathbf{M}^2 + b\mathbf{M}$. [3]

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

4 **(a)** Find $\displaystyle\int_0^1 \frac{1}{\sqrt{9x^2+16}}\,\mathrm{d}x$, giving your answer in an exact logarithmic form. [5]

(b) **(i)** Starting from the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, prove that

$$\sinh 2x = 2\sinh x \cosh x.$$ [2]

(ii) Show that one of the stationary points on the curve

$$y = 20\cosh x - 3\cosh 2x$$

has coordinates $(\ln 3, \frac{59}{3})$, and find the coordinates of the other two stationary points. [7]

(iii) Show that $\displaystyle\int_{-\ln 3}^{\ln 3} (20\cosh x - 3\cosh 2x)\,\mathrm{d}x = 40$. [4]

[Question 5 is printed overleaf.]

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 The curve with equation $y = \dfrac{x^2 - kx + 2k}{x + k}$ is to be investigated for different values of k.

(i) Use your graphical calculator to obtain rough sketches of the curve in the cases $k = -2$, $k = -0.5$ and $k = 1$. [6]

(ii) Show that the equation of the curve may be written as $y = x - 2k + \dfrac{2k(k+1)}{x+k}$.

Hence find the two values of k for which the curve is a straight line. [4]

(iii) When the curve is not a straight line, it is a conic.

(A) Name the type of conic. [1]

(B) Write down the equations of the asymptotes. [2]

(iv) Draw a sketch to show the shape of the curve when $1 < k < 8$. This sketch should show where the curve crosses the axes and how it approaches its asymptotes. Indicate the points A and B on the curve where $x = 1$ and $x = k$ respectively. [5]