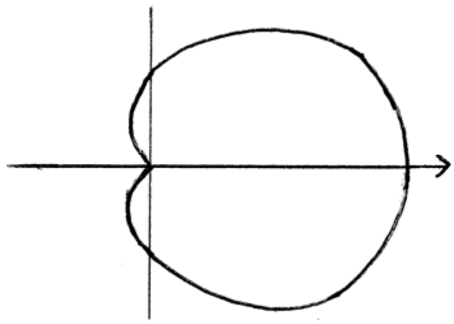
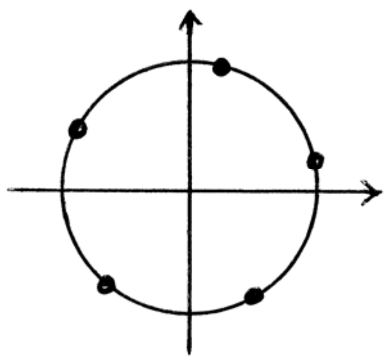
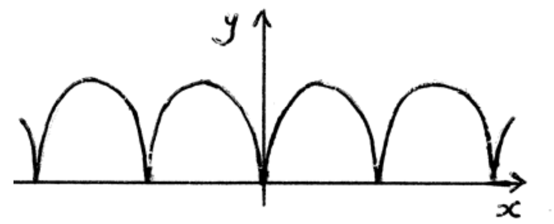
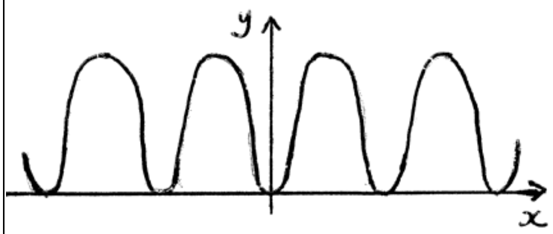
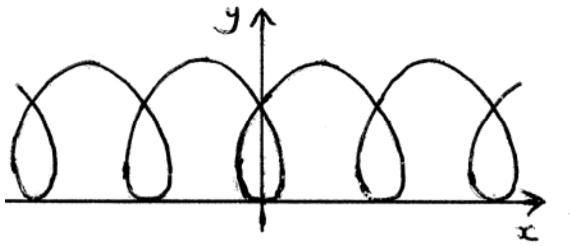
1(a)(i)		B1 B1 **2**	Correct shape for $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$ including maximum in 1st quadrant Correct form at O and no extra sections
(ii)	Area is $\int \frac{1}{2} r^2 \, d\theta = \int_{-\frac{3}{4}\pi}^{\frac{3}{4}\pi} \frac{1}{2} a^2 (\sqrt{2} + 2\cos\theta)^2 \, d\theta$ $= \int_{-\frac{3}{4}\pi}^{\frac{3}{4}\pi} a^2 (1 + 2\sqrt{2}\cos\theta + 1 + \cos 2\theta) \, d\theta$ $= \left[a^2 (2\theta + 2\sqrt{2}\sin\theta + \frac{1}{2}\sin 2\theta) \right]_{-\frac{3}{4}\pi}^{\frac{3}{4}\pi}$ $= 3(\pi + 1) a^2$	M1 A1 B1 B1B1 ft M1 A1 **7**	For integral of $(\sqrt{2} + 2\cos\theta)^2$ For a correct integral expression including limits *(may be implied by later work)* Using $2\cos^2\theta = 1 + \cos 2\theta$ Integration of $\cos\theta$ and $\cos 2\theta$ Evaluation using $\sin \frac{3}{4}\pi = (\pm)\frac{1}{\sqrt{2}}$
(b)(i)	$f'(x) = \sec^2(\frac{1}{4}\pi + x)$ $f''(x) = 2\sec^2(\frac{1}{4}\pi + x)\tan(\frac{1}{4}\pi + x)$ $f(0) = 1, \quad f'(0) = 2, \quad f''(0) = 4$ $f(x) = 1 + 2x + 2x^2 + \ldots$	B1 B1 M1 B1A1A1 **6**	Any correct form Evaluating $f'(0)$ or $f''(0)$
	OR $g'(u) = \sec^2 u$ (where $g(u) = \tan u$) B1 $g''(u) = 2\sec^2 u \tan u$ B1 $g(\frac{1}{4}\pi) = 1, \; g'(\frac{1}{4}\pi) = 2, \; g''(\frac{1}{4}\pi) = 4$ M1 $f(x) = g(\frac{1}{4}\pi + x) = 1 + 2x + 2x^2 + \ldots$ B1A1A1		Condone $\sec^2 x$ etc Evaluating $g'(\frac{1}{4}\pi)$ or $g''(\frac{1}{4}\pi)$
(ii)	$\int_{-h}^{h} x^2 (1 + 2x + 2x^2 + \ \ldots) \, dx$ $= \left[\frac{1}{3}x^3 + \frac{1}{2}x^4 + \frac{2}{5}x^5 + \ldots \right]_{-h}^{h}$ $\approx (\frac{1}{3}h^3 + \frac{1}{2}h^4 + \frac{2}{5}h^5) - (-\frac{1}{3}h^3 + \frac{1}{2}h^4 - \frac{2}{5}h^5)$ $= \frac{2}{3}h^3 + \frac{4}{5}h^5$	M1 A1 ft A1 (ag) **3**	Using series and integrating (ft requires three non-zero terms) Correctly shown Allow ft from $1 + kx + 2x^2$ with $k \neq 0$

2 (a)(i)	$z^n + \frac{1}{z^n} = 2\cos n\theta$, $z^n - \frac{1}{z^n} = 2\mathrm{j}\sin n\theta$	B1B1 **2**	
(ii)	$\left(z - \frac{1}{z}\right)^4 \left(z + \frac{1}{z}\right)^2 = 64\sin^4\theta\cos^2\theta$ $= z^6 - 2z^4 - z^2 + 4 - \frac{1}{z^2} - \frac{2}{z^4} + \frac{1}{z^6}$ $= 2\cos 6\theta - 4\cos 4\theta - 2\cos 2\theta + 4$ $\sin^4\theta\cos^2\theta = \frac{1}{32}\cos 6\theta - \frac{1}{16}\cos 4\theta - \frac{1}{32}\cos 2\theta + \frac{1}{16}$ $(A = \frac{1}{32},\ B = -\frac{1}{16},\ C = -\frac{1}{32},\ D = \frac{1}{16})$	B1 M1 A1 M1 A1 ft A1 **6**	Expansion $z^6 + \ldots + z^{-6}$ Using $z^n + \frac{1}{z^n} = 2\cos n\theta$ with $n = 2,\ 4$ or 6. Allow M1 if used in partial expansion, or if 2 omitted, etc
(b)(i)	$\lvert 4 + 4\mathrm{j}\rvert = \sqrt{32}$, $\arg(4 + 4\mathrm{j}) = \frac{1}{4}\pi$	B1B1 **2**	Accept $5.7;\ 0.79,\ 45°$
(ii)	$r = \sqrt{2}$ $\theta = -\frac{3}{4}\pi,\ -\frac{7}{20}\pi,\ \frac{1}{20}\pi,\ \frac{9}{20}\pi,\ \frac{17}{20}\pi$ 	B1 B3 B2 **6**	Accept $32^{\frac{1}{10}}$, 1.4, $\sqrt[5]{4\sqrt{2}}$ etc Accept $-2.4, -1.1, 0.16, 1.4, 2.7$ Give B2 for three correct Give B1 for one correct Deduct 1 mark (maximum) if degrees used $(-135°,\ -63°,\ 9°,\ 81°,\ 153°)$ $\frac{1}{20}\pi + \frac{2}{5}k\pi$ earns B2; with $k = -2,\ -1,\ 0,\ 1,\ 2$ earns B3 Give B1 for four points correct, or B1 ft for five points
(iii)	$\sqrt{2}e^{-\frac{3}{4}\pi\mathrm{j}} = \sqrt{2}\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\mathrm{j}\right)$ $= -1 - \mathrm{j}$ $p = -1,\ q = -1$	M1 A1 **2**	Exact evaluation of a fifth root Give B2 for correct answer stated or obtained by any other method

3 (i)	$\mathbf{M}^{-1}=\frac{1}{5-k}\begin{pmatrix}1 & 5k-13 & 5-2k\\ 1 & 52-8k & 3k-20\\ -1 & -12 & 5\end{pmatrix}$	M1 A1 M1 A1 M1 A1 **6**	Evaluating determinant For $(5-k)$ *must be simplified* Finding at least four cofactors At least 6 signed cofactors correct Transposing matrix of cofactors and dividing by determinant Fully correct
	OR Elementary row operations applied to **M** (LHS) and **I** (RHS), and obtaining at least two zeros in LHS M1 Obtaining one row in LHS consisting of two zeros and a multiple of $(5-k)$ A1 Obtaining one row in RHS which is a multiple of a row of the inverse matrix A1 Obtaining two zeros in every row in LHS M1 Completing process to find inverse M1A1		or elementary column operations
(ii)	$\begin{pmatrix}x\\ y\\ z\end{pmatrix}=-\frac{1}{2}\begin{pmatrix}1 & 22 & -9\\ 1 & -4 & 1\\ -1 & -12 & 5\end{pmatrix}\begin{pmatrix}12\\ m\\ 0\end{pmatrix}$ $x=-11m-6,\quad y=2m-6,\quad z=6m+6$	M1 M1 M1 A2 ft **5**	Substituting $k=7$ into inverse Correct use of inverse Evaluating matrix product Give A1 ft for one correct *Accept unsimplified forms or solution left in matrix form*
	OR e.g. eliminating *x* , $3y-z=-24$ $5y-z=4m-36$ M2 $y=2m-6$ M1 $x=-11m-6,\quad y=2m-6,\quad z=6m+6$ A2		Eliminating one variable in two different ways Obtaining one of *x* , *y* , *z* Give M3 for any other valid method leading to one of *x* , *y* , *z* in terms of *m* Give A1 for one correct
(iii)	Eliminating *x* , $3y+3z=-24$ $5y+5z=4p-36$ For solutions, $4p-36=-24\times\frac{5}{3}$	M2 A1 M1	Eliminating one variable in two different ways Two correct equations *Dependent on previous M2*
	OR Replacing one column of matrix with column from RHS, and evaluating determinant M2 determinant $12+12p$ or $-12-12p$ A1 For solutions, det $=0$ M1		*Dependent on previous M2*

	OR Any other method leading to an equation from which p could be found M3 Correct equation A1		
	$p=-1$ Let $z=\lambda$, $x=5-\lambda,\quad y=-8-\lambda,\quad z=\lambda$	A1 M1 (or M3) A1 **7**	Obtaining a line of solutions Give M3 when M0 for finding p or $x=13+\lambda,\quad y=\lambda,\quad z=-8-\lambda$ or $x=\lambda,\quad y=-13+\lambda,\quad z=5-\lambda$ Accept $x=5-z,\quad y=-8-z$ or $x=y+13=5-z$ etc

4 (i)	$1+2\sinh^2 x = 1+2[\tfrac{1}{2}(e^x - e^{-x})]^2$ $= 1+\tfrac{1}{2}(e^{2x} - 2 + e^{-2x})$ $= \tfrac{1}{2}(e^{2x} + e^{-2x})$ $= \cosh 2x$	B1 B1 B1 (ag) **3**	For $(e^x - e^{-x})^2 = e^{2x} - 2 + e^{-2x}$ For $\cosh 2x = \tfrac{1}{2}(e^{2x} + e^{-2x})$ For completion
(ii)	$2(1+2\sinh^2 x) + \sinh x = 5$ $4\sinh^2 x + \sinh x - 3 = 0$ $(4\sinh x - 3)(\sinh x + 1) = 0$ $\sinh x = \tfrac{3}{4},\ -1$ $x = \text{arsinh}(\tfrac{3}{4}) = \ln(\tfrac{3}{4} + \sqrt{\tfrac{9}{16}+1}) = \ln 2$ $x = \text{arsinh}(-1) = \ln(-1+\sqrt{1+1}) = \ln(\sqrt{2}-1)$	M1 M1 A1A1 A1 ft A1 ft **6**	Using (i) Solving to obtain a value of $\sinh x$ or $-\ln(\sqrt{2}+1)$ *SR* Give A1 for $\pm\ln 2,\ \pm\ln(\sqrt{2}-1)$
	OR $2e^{4x} + e^{3x} - 10e^{2x} - e^x + 2 = 0$ M2 $(e^x - 2)(2e^x + 1)(e^{2x} + 2e^x - 1) = 0$ A1A1 $x = \ln 2,\ \ln(\sqrt{2}-1)$ A1A1 ft		Obtaining a linear or quadratic factor For $(e^x - 2)$ and $(e^{2x} + 2e^x - 1)$
(iii)	$\int_0^{\ln 3} \tfrac{1}{2}(\cosh 2x - 1)\,dx$ $= \left[\tfrac{1}{4}\sinh 2x - \tfrac{1}{2}x\right]_0^{\ln 3}$ $= \dfrac{1}{8}\left(9 - \dfrac{1}{9}\right) - \dfrac{1}{2}\ln 3$ $= \dfrac{10}{9} - \dfrac{1}{2}\ln 3$	M1 A1A1 M1 A1 (ag) **5**	Expressing in integrable form or $\int \tfrac{1}{4}(e^{2x} - 2 + e^{-2x})\,dx$ or $(\tfrac{1}{8}e^{2x} - \tfrac{1}{8}e^{-2x}) - \tfrac{1}{2}x$ For $e^{2\ln 3} = 9$ and $e^{-2\ln 3} = \tfrac{1}{9}$ M0 for just stating $\sinh(2\ln 3) = \tfrac{40}{9}$ etc Correctly obtained
(iv)	Put $x = 3\cosh u$ when $x = 3,\ u = 0$ when $x = 5,\ u = \text{ar}\cosh\tfrac{5}{3} = \ln 3$ $\int_3^5 \sqrt{x^2 - 9}\,dx = \int_0^{\ln 3} (3\sinh u)(3\sinh u\,du)$ $= 9\int_0^{\ln 3} \sinh^2 u\,du$ $= 10 - \tfrac{9}{2}\ln 3$	M1 B1 A1 A1 **4**	Any cosh substitution For $\ln 3$ *Not awarded for* $\text{arcosh}\tfrac{5}{3}$ *Limits not required*

5 (i)		B2	At least two cusps clearly shown Give B1 for at least two arches
	Has cusps Periodic / Symmetrical in *y*-axis / Has maxima / Is never below the *x*-axis	B1 B1 **4**	Any other feature
(ii)		B2	At least two minima (zero gradient) clearly shown Give B1 for general shape correct (at least two cycles)
	The curve has no cusps	B1 **3**	For description of any *difference*
(iii)	(*A*) 	B2 **2**	At least two loops Give B1 for general shape correct (at least one cycle)
	(*B*) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sin\theta}{1-2\cos\theta}$	M1 A1 **2**	Correct method of differentiation *Allow M1 if inverted* Allow $\frac{\sin\theta}{1-k\cos\theta}$
	(*C*) $\frac{\mathrm{d}y}{\mathrm{d}x}$ is infinite when $1-2\cos\theta=0$ $\theta=\frac{1}{3}\pi$ $x=\frac{1}{3}\pi-2\sin\frac{1}{3}\pi$ $=-(\sqrt{3}-\frac{1}{3}\pi)$ Hence width of loop is $2(\sqrt{3}-\frac{1}{3}\pi)$ $=2\sqrt{3}-\frac{2\pi}{3}$	M1 A1 M1 M1 A1 (ag) **5**	 Any correct value of θ Finding width of loop Correctly obtained *Condone negative answer*
(iv)	$k=4.6$	B2 **2**	Give B1 for a value between 4 and 5 (inclusive)