



ADVANCED GCE

MATHEMATICS (MEI)

Further Methods for Advanced Mathematics (FP2)

4756

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:

None

Monday 11 January 2010
Morning

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions in Section A and **one** question from Section B.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (54 marks)

Answer all the questions

- 1 (a) Given that $y = \arctan \sqrt{x}$, find $\frac{dy}{dx}$, giving your answer in terms of x . Hence show that

$$\int_0^1 \frac{1}{\sqrt{x}(x+1)} dx = \frac{\pi}{2}. \quad [6]$$

- (b) A curve has cartesian equation

$$x^2 + y^2 = xy + 1.$$

- (i) Show that the polar equation of the curve is

$$r^2 = \frac{2}{2 - \sin 2\theta}. \quad [4]$$

- (ii) Determine the greatest and least positive values of r and the values of θ between 0 and 2π for which they occur. [6]

- (iii) Sketch the curve. [2]

- 2 (a) Use de Moivre's theorem to find the constants a, b, c in the identity

$$\cos 5\theta \equiv a \cos^5 \theta + b \cos^3 \theta + c \cos \theta. \quad [6]$$

- (b) Let

$$C = \cos \theta + \cos\left(\theta + \frac{2\pi}{n}\right) + \cos\left(\theta + \frac{4\pi}{n}\right) + \dots + \cos\left(\theta + \frac{(2n-2)\pi}{n}\right),$$

$$\text{and } S = \sin \theta + \sin\left(\theta + \frac{2\pi}{n}\right) + \sin\left(\theta + \frac{4\pi}{n}\right) + \dots + \sin\left(\theta + \frac{(2n-2)\pi}{n}\right),$$

where n is an integer greater than 1.

By considering $C + jS$, show that $C = 0$ and $S = 0$. [7]

- (c) Write down the Maclaurin series for e^t as far as the term in t^2 .

Hence show that, for t close to zero,

$$\frac{t}{e^t - 1} \approx 1 - \frac{1}{2}t. \quad [5]$$

3

- 3 (i) Find the inverse of the matrix

$$\begin{pmatrix} 1 & 1 & a \\ 2 & -1 & 2 \\ 3 & -2 & 2 \end{pmatrix}$$

where $a \neq 4$.

Show that when $a = -1$ the inverse is

$$\frac{1}{5} \begin{pmatrix} 2 & 0 & 1 \\ 2 & 5 & -4 \\ -1 & 5 & -3 \end{pmatrix}. \quad [6]$$

- (ii) Solve, in terms of b , the following system of equations. [5]

$$\begin{aligned} x + y - z &= -2 \\ 2x - y + 2z &= b \\ 3x - 2y + 2z &= 1 \end{aligned}$$

- (iii) Find the value of b for which the equations

$$\begin{aligned} x + y + 4z &= -2 \\ 2x - y + 2z &= b \\ 3x - 2y + 2z &= 1 \end{aligned}$$

have solutions. Give a geometrical interpretation of the solutions in this case. [7]

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

- 4 (i) Prove, using exponential functions, that

$$\cosh 2x = 1 + 2 \sinh^2 x.$$

Differentiate this result to obtain a formula for $\sinh 2x$. [4]

- (ii) Solve the equation

$$2 \cosh 2x + 3 \sinh x = 3,$$

expressing your answers in exact logarithmic form. [7]

- (iii) Given that $\cosh t = \frac{5}{4}$, show by using exponential functions that $t = \pm \ln 2$.

Find the exact value of the integral

$$\int_4^5 \frac{1}{\sqrt{x^2 - 16}} dx. \quad [7]$$

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

- 5 A line PQ is of length k (where $k > 1$) and it passes through the point $(1, 0)$. PQ is inclined at angle θ to the positive x -axis. The end Q moves along the y -axis. See Fig. 5. The end P traces out a locus.

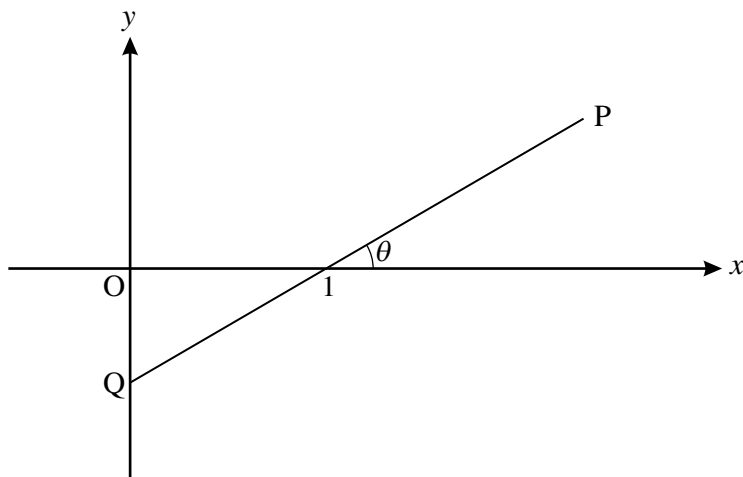


Fig. 5

- (i) Show that the locus of P may be expressed parametrically as follows. [3]

$$x = k \cos \theta \quad y = k \sin \theta - \tan \theta$$

You are now required to investigate curves with these parametric equations, where k may take any non-zero value and $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$.

- (ii) Use your calculator to sketch the curve in each of the cases $k = 2$, $k = 1$, $k = \frac{1}{2}$ and $k = -1$. [4]
- (iii) For what value(s) of k does the curve have
- (A) an asymptote (you should state what the asymptote is),
 - (B) a cusp,
 - (C) a loop? [3]
- (iv) For the case $k = 2$, find the angle at which the curve crosses itself. [2]
- (v) For the case $k = 8$, find in an exact form the coordinates of the highest point on the loop. [3]
- (vi) Verify that the cartesian equation of the curve is

$$y^2 = \frac{(x-1)^2}{x^2}(k^2 - x^2). \quad [3]$$

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations, is given to all schools that receive assessment material and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.