OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4756

Further Methods for Advanced Mathematics (FP2)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Monday 11 January 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions in Section A and **one** question from Section B.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (54 marks)

Answer all the questions

1 **(a)** Given that $y = \arctan\sqrt{x}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$, giving your answer in terms of x. Hence show that

$$\int_0^1 \frac{1}{\sqrt{x}(x+1)}\,\mathrm{d}x = \frac{\pi}{2}.$$ **[6]**

(b) A curve has cartesian equation

$$x^2 + y^2 = xy + 1.$$

(i) Show that the polar equation of the curve is

$$r^2 = \frac{2}{2 - \sin 2\theta}.$$ **[4]**

(ii) Determine the greatest and least positive values of r and the values of θ between 0 and 2π for which they occur. **[6]**

(iii) Sketch the curve. **[2]**

2 **(a)** Use de Moivre's theorem to find the constants a, b, c in the identity

$$\cos 5\theta \equiv a\cos^5\theta + b\cos^3\theta + c\cos\theta.$$ **[6]**

(b) Let

$$C = \cos\theta + \cos\left(\theta + \frac{2\pi}{n}\right) + \cos\left(\theta + \frac{4\pi}{n}\right) + \ldots + \cos\left(\theta + \frac{(2n-2)\pi}{n}\right),$$

$$\text{and}\quad S = \sin\theta + \sin\left(\theta + \frac{2\pi}{n}\right) + \sin\left(\theta + \frac{4\pi}{n}\right) + \ldots + \sin\left(\theta + \frac{(2n-2)\pi}{n}\right),$$

where n is an integer greater than 1.

By considering $C + \mathrm{j}S$, show that $C = 0$ and $S = 0$. **[7]**

(c) Write down the Maclaurin series for e^t as far as the term in t^2.

Hence show that, for t close to zero,

$$\frac{t}{\mathrm{e}^t - 1} \approx 1 - \tfrac{1}{2}t.$$ **[5]**

3 **(i)** Find the inverse of the matrix

$$\begin{pmatrix} 1 & 1 & a \\ 2 & -1 & 2 \\ 3 & -2 & 2 \end{pmatrix}$$

where $a \neq 4$.

Show that when $a = -1$ the inverse is

$$\frac{1}{5}\begin{pmatrix} 2 & 0 & 1 \\ 2 & 5 & -4 \\ -1 & 5 & -3 \end{pmatrix}.$$ **[6]**

(ii) Solve, in terms of b, the following system of equations. **[5]**

$$\begin{aligned} x + y - z &= -2 \\ 2x - y + 2z &= b \\ 3x - 2y + 2z &= 1 \end{aligned}$$

(iii) Find the value of b for which the equations

$$\begin{aligned} x + y + 4z &= -2 \\ 2x - y + 2z &= b \\ 3x - 2y + 2z &= 1 \end{aligned}$$

have solutions. Give a geometrical interpretation of the solutions in this case. **[7]**

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

4 **(i)** Prove, using exponential functions, that

$$\cosh 2x = 1 + 2\sinh^2 x.$$

Differentiate this result to obtain a formula for $\sinh 2x$. **[4]**

(ii) Solve the equation

$$2\cosh 2x + 3\sinh x = 3,$$

expressing your answers in exact logarithmic form. **[7]**

(iii) Given that $\cosh t = \frac{5}{4}$, show by using exponential functions that $t = \pm \ln 2$.

Find the exact value of the integral

$$\int_4^5 \frac{1}{\sqrt{x^2 - 16}}\,dx.$$ **[7]**

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 A line PQ is of length k (where $k > 1$) and it passes through the point $(1, 0)$. PQ is inclined at angle θ to the positive x-axis. The end Q moves along the y-axis. See Fig. 5. The end P traces out a locus.

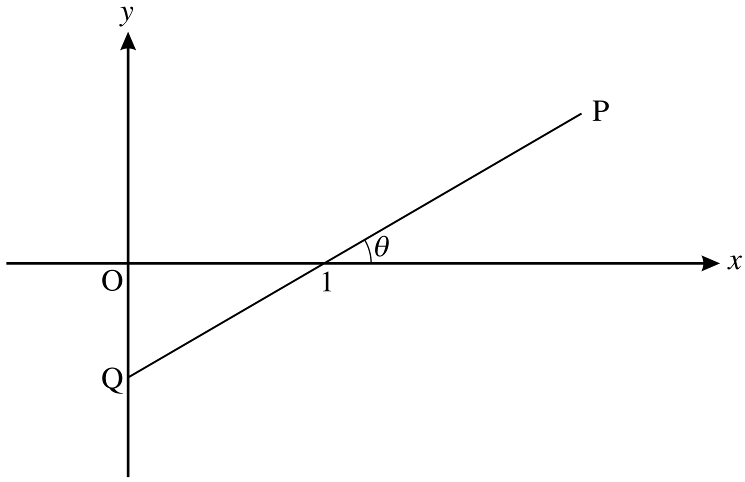

Fig. 5

(i) Show that the locus of P may be expressed parametrically as follows. **[3]**

$$x = k\cos\theta \qquad y = k\sin\theta - \tan\theta$$

You are now required to investigate curves with these parametric equations, where k may take any non-zero value and $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$.

(ii) Use your calculator to sketch the curve in each of the cases $k = 2$, $k = 1$, $k = \frac{1}{2}$ and $k = -1$. **[4]**

(iii) For what value(s) of k does the curve have

(*A*) an asymptote (you should state what the asymptote is),

(*B*) a cusp,

(*C*) a loop? **[3]**

(iv) For the case $k = 2$, find the angle at which the curve crosses itself. **[2]**

(v) For the case $k = 8$, find in an exact form the coordinates of the highest point on the loop. **[3]**

(vi) Verify that the cartesian equation of the curve is

$$y^2 = \frac{(x-1)^2}{x^2}(k^2 - x^2).$$ **[3]**