RECOGNISING ACHIEVEMENT

ADVANCED GCE **4756/01**

MATHEMATICS (MEI)

Further Methods for Advanced Mathematics (FP2)

WEDNESDAY 9 JANUARY 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions in Section A and **one** question from Section B.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

Section A (54 marks)

Answer all the questions

1 **(a)** Fig. 1 shows the curve with polar equation $r = a(1 - \cos 2\theta)$ for $0 \leqslant \theta \leqslant \pi$, where a is a positive constant.

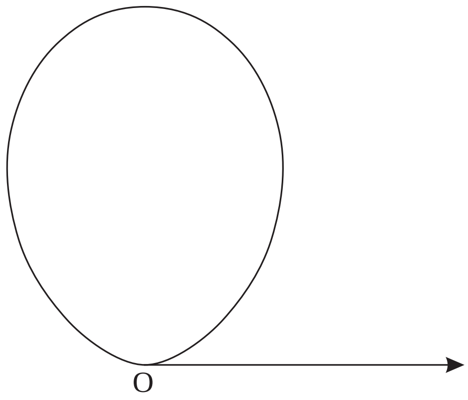

Fig. 1

Find the area of the region enclosed by the curve. [7]

(b) **(i)** Given that $f(x) = \arctan(\sqrt{3} + x)$, find $f'(x)$ and $f''(x)$. [4]

(ii) Hence find the Maclaurin series for $\arctan(\sqrt{3} + x)$, as far as the term in x^2. [4]

(iii) Hence show that, if h is small, $\int_{-h}^{h} x \arctan(\sqrt{3} + x)\,dx \approx \frac{1}{6}h^3$. [3]

2 **(a)** Find the 4th roots of 16j, in the form $re^{j\theta}$ where $r > 0$ and $-\pi < \theta \leqslant \pi$. Illustrate the 4th roots on an Argand diagram. [6]

(b) **(i)** Show that $(1 - 2e^{j\theta})(1 - 2e^{-j\theta}) = 5 - 4\cos\theta$. [3]

Series C and S are defined by

$$C = 2\cos\theta + 4\cos 2\theta + 8\cos 3\theta + \ldots + 2^n \cos n\theta,$$
$$S = 2\sin\theta + 4\sin 2\theta + 8\sin 3\theta + \ldots + 2^n \sin n\theta.$$

(ii) Show that $C = \dfrac{2\cos\theta - 4 - 2^{n+1}\cos(n+1)\theta + 2^{n+2}\cos n\theta}{5 - 4\cos\theta}$, and find a similar expression for S. [9]

3 You are given the matrix $\mathbf{M} = \begin{pmatrix} 7 & 3 \\ -4 & -1 \end{pmatrix}$.

(i) Find the eigenvalues, and corresponding eigenvectors, of the matrix **M**. [8]

(ii) Write down a matrix **P** and a diagonal matrix **D** such that $\mathbf{P}^{-1}\mathbf{M}\mathbf{P} = \mathbf{D}$. [2]

(iii) Given that $\mathbf{M}^n = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, show that $a = -\frac{1}{2} + \frac{3}{2} \times 5^n$, and find similar expressions for b, c and d. [8]

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

4 **(i)** Given that $k \geqslant 1$ and $\cosh x = k$, show that $x = \pm \ln\left(k + \sqrt{k^2 - 1}\right)$. [5]

(ii) Find $\displaystyle\int_1^2 \frac{1}{\sqrt{4x^2 - 1}}\,dx$, giving the answer in an exact logarithmic form. [5]

(iii) Solve the equation $6\sinh x - \sinh 2x = 0$, giving the answers in an exact form, using logarithms where appropriate. [4]

(iv) Show that there is no point on the curve $y = 6\sinh x - \sinh 2x$ at which the gradient is 5. [4]

[Question 5 is printed overleaf.]

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 A curve has parametric equations $x = \dfrac{t^2}{1+t^2}$, $y = t^3 - \lambda t$, where λ is a constant.

(i) Use your calculator to obtain a sketch of the curve in each of the cases

$$\lambda = -1, \quad \lambda = 0 \quad \text{and} \quad \lambda = 1.$$

Name any special features of these curves. [5]

(ii) By considering the value of x when t is large, write down the equation of the asymptote. [1]

For the remainder of this question, assume that λ is positive.

(iii) Find, in terms of λ, the coordinates of the point where the curve intersects itself. [3]

(iv) Show that the two points on the curve where the tangent is parallel to the x-axis have coordinates

$$\left(\frac{\lambda}{3+\lambda}, \pm\sqrt{\frac{4\lambda^3}{27}}\right).$$ [4]

Fig. 5 shows a curve which intersects itself at the point (2, 0) and has asymptote $x = 8$. The stationary points A and B have y-coordinates 2 and −2.

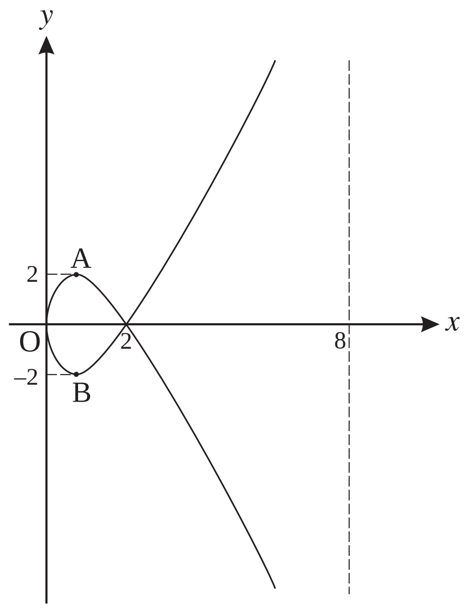

Fig. 5

(v) For the curve sketched in Fig. 5, find parametric equations of the form $x = \dfrac{at^2}{1+t^2}$, $y = b(t^3 - \lambda t)$, where a, λ and b are to be determined. [5]