OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4756

Further Methods for Advanced Mathematics (FP2)

Monday **16 JANUARY 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions in Section A and **one** question from Section B.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 4 printed pages.

Section A (54 marks)

Answer all the questions

1 **(a)** A curve has polar equation $r = a\cos 3\theta$ for $-\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$, where a is a positive constant.

(i) Sketch the curve, using a continuous line for sections where $r > 0$ and a broken line for sections where $r < 0$. [3]

(ii) Find the area enclosed by one of the loops. [5]

(b) Find the exact value of $\displaystyle\int_0^{\frac{3}{4}} \frac{1}{\sqrt{3-4x^2}}\,\mathrm{d}x$. [5]

(c) Use a trigonometric substitution to find $\displaystyle\int_0^{1} \frac{1}{(1+3x^2)^{\frac{3}{2}}}\,\mathrm{d}x$. [5]

2 In this question, θ is a real number with $0 < \theta < \frac{1}{6}\pi$, and $w = \frac{1}{2}\mathrm{e}^{3\mathrm{j}\theta}$.

(i) State the modulus and argument of each of the complex numbers

$$w, \quad w^* \quad \text{and} \quad \mathrm{j}w.$$

Illustrate these three complex numbers on an Argand diagram. [6]

(ii) Show that $(1+w)(1+w^*) = \frac{5}{4} + \cos 3\theta$. [4]

Infinite series C and S are defined by

$$C = \cos 2\theta - \tfrac{1}{2}\cos 5\theta + \tfrac{1}{4}\cos 8\theta - \tfrac{1}{8}\cos 11\theta + \ldots,$$

$$S = \sin 2\theta - \tfrac{1}{2}\sin 5\theta + \tfrac{1}{4}\sin 8\theta - \tfrac{1}{8}\sin 11\theta + \ldots.$$

(iii) Show that $C = \dfrac{4\cos 2\theta + 2\cos\theta}{5 + 4\cos 3\theta}$, and find a similar expression for S. [8]

3 The matrix $\mathbf{M} = \begin{pmatrix} 1 & 2 & 3 \\ -2 & -3 & 6 \\ 2 & 2 & -4 \end{pmatrix}$.

(i) Show that the characteristic equation for **M** is $\lambda^3 + 6\lambda^2 - 9\lambda - 14 = 0$. [3]

(ii) Show that –1 is an eigenvalue of **M**, and find the other two eigenvalues. [3]

(iii) Find an eigenvector corresponding to the eigenvalue –1. [3]

(iv) Verify that $\begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 3 \\ -2 \end{pmatrix}$ are eigenvectors of **M**. [3]

(v) Write down a matrix **P**, and a diagonal matrix **D**, such that $\mathbf{M}^3 = \mathbf{PDP}^{-1}$. [3]

(vi) Use the Cayley-Hamilton theorem to express $\mathbf{M}^{-1}$ in the form $a\mathbf{M}^2 + b\mathbf{M} + c\mathbf{I}$. [3]

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

4 **(a)** Solve the equation

$$\sinh x + 4\cosh x = 8,$$

giving the answers in an exact logarithmic form. [6]

(b) Find the exact value of $\int_0^2 \mathrm{e}^x \sinh x \, \mathrm{d}x$. [4]

(c) **(i)** Differentiate $\operatorname{arsinh}\left(\frac{2}{3}x\right)$ with respect to x. [2]

(ii) Use integration by parts to show that $\int_0^2 \operatorname{arsinh}\left(\frac{2}{3}x\right) \mathrm{d}x = 2\ln 3 - 1$. [6]

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 A curve has equation $y = \dfrac{x^3 - k^3}{x^2 - 4}$, where k is a positive constant and $k \neq 2$.

(i) Find the equations of the three asymptotes. [3]

(ii) Use your graphical calculator to obtain rough sketches of the curve in the two separate cases $k < 2$ and $k > 2$. [4]

(iii) In the case $k < 2$, your sketch may not show clearly the shape of the curve near $x = 0$. Use calculus to show that the curve has a minimum point when $x = 0$. [5]

(iv) In the case $k > 2$, your sketch may not show clearly how the curve approaches its asymptote as $x \longrightarrow +\infty$. Show algebraically that the curve crosses this asymptote. [2]

(v) Use the results of parts **(iii)** and **(iv)** to produce more accurate sketches of the curve in the two separate cases $k < 2$ and $k > 2$. These sketches should indicate where the curve crosses the axes, and should show clearly how the curve approaches its asymptotes. The presence of stationary points should be clearly shown, but there is no need to find their coordinates. [4]